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Complexity Results for Wireless Sensor Network Scheduling

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Abstract

We study the problem of scheduling multiple sensors to visit and observe a group of sites at discrete time points over a planning horizon of given length. We show that scheduling under a given number of visits for each site and in each period is an NP-complete problem by providing equivalence with a problem in discrete tomography. We also give a polynomial time algorithm to schedule the sensors under a given number of visits in each period.

Keywords: Discrete Tomography, Sensor Scheduling, Combinatorial Optimization

1. Introduction

A wireless sensor is a small physical device having a battery with a limited capacity and having a transmission, a reception capability of limited range and capable to take various measurements of its neighborhood. These measurements include temperature, acoustic, solar radiation, etc. Since a single sensor is not able to monitor a hole field, a group of sensors should be deployed and frequently interact with each other to exchange information. This group of distributed sensors forms the wireless sensor networks (WSN). Generally, the basic goal of a WSN is to ensure the surveillance of a given region with a limited number of sensors and eventually transmit the sensed data to a processing unit.

The management and the organization of WSN have been investigated by many researchers. Depending on the application and the goal of researchers, several combinatorial optimization models and solution techniques have been proposed [1]. These models and problems include, among others, sensor localization and tracking [2-5], sensor scheduling [6-11] communication neutralization [12,13] and energy consumption [14,15]. There are a wide variety of methods and models because on one had several issues are considered in WSN. On the other hand, the most addressed problems are NP-complete, *i.e.*, there is no polynomial time algorithm to solves these problems unless P = NP.

This paper is concerned with WSN scheduling to periodically monitor and observe an environment. WSN scheduling is needed in various applications such as traffic monitoring at roads, military, medical, pollution detection, etc. In this paper, we show the complexity results of WSN scheduling by supposing that the number of required visits for each period are given. We will use a reduction from an independent problem of discrete tomography. For clarity, the next section is devoted to the area of discrete tomography.

The remainder of this paper is organized as follows. In Section 2, the problem of discrete tomography and binary matrices reconstruction will be presented. In Section 3, Sensor scheduling with a given numbers of visits per period and per site will be studied. In Section 4, sensor scheduling with a given numbers of visits per period is addressed.

2. Discrete Tomography

Discrete Tomography (DT) is an emergent area of computer science (refer to the books of Hermann and Kuba [16,17] for further information on the theory, algorithms and applications). It deals with the reconstruction of binary matrices and images from horizontal and vertical projections. Let A be a binary matrix of size m × n, we denote by $h_i = \sum_{j=1}^{n} A_{ij}$ the number of ones on row *i* and by $v_j = \sum_{i=1}^{m} A_{ij}$ the number of ones on column *j*.

The vectors $H=\left(h_1,\cdots,h_m\right)$ and $V=\left(v_1,\cdots,v_n\right)$ are called the horizontal and the vertical projection respectively. The problem of reconstructing a binary matrix from orthogonal projections, denoted MB(H,V), is defined as follows: given two vectors H and V, we search to reconstruct a binary matrix consistent with these projections or to report that such a matrix does not exist. It is well known that this problem is polynomial [18] (**Figure 1**). However, it becomes NP-complete by imposing a maximal length of sequence of zeros between the consecutive ones [19], *i.e.*, we impose a maximal number of zeros between two consecutive ones.

h_i \ v_j	3	1	3	1	4
2	0	0	1	0	1
2	1	0	0	0	1
5	1	1	1	1	1
3	1	0	1	0	1

Figure 1. A binary matrix with horizontal projection *H* = (2, 2, 5, 3) and vertical projection *V* = (3, 1, 3, 1, 4); maximal length of sequences of zeros is 3.

3. WSN Scheduling with Given Number of Visits per Period and Site SSHV(a,b)

We consider the following problem SSHV(a, b) of sensors scheduling. There are m sites to observe $s_i, i\in\left\{1,...,m\right\}$, a set of m sensors and a scheduling horizon (interval) T composed of n periods $T=\left\{1,...,n\right\}$. During period t_j of the time interval T, exactly v_j sites should be observed by v_j sensors. Each site s_i, should be observed during h_i periods. When site s_i is not observed at period t_j, a non visiting penalty a_{ij} is incurred and an information loss penalty $b_{ij}l_{ij}$ is also incurred. We suppose that a_{ij} and b_{ij} are given non negative and l_{ij} is the number of elapsed periods since last visiting site s_i. Note that the information loss penalty is proportional to the time interval when the site is not observed. The problem now is to determine a surveillance schedule, *i.e.*, to decide for each period which site to observe minimizing the maximum non visiting penalties and information loss penalties.

The problem SSHV(a,b) can be reformulated as a mixed integer linear program inspired from [11]. We introduce the binary decision variables x_{ij} and the real variables y_{ij} such that x_{ij} = 1 and if the site s_i is observed at time slot j. The real variable y_{ij} represents the last time where site s_i was observed before period j. This gives us the following MIP model:

$$P=\begin{cases}\min C \\ s.t. \\ C\ge a_{ij}\left(1-x_{ij}\right)+b_{ij}\left(\mathrm{j}-y_{ij}\right), \quad \forall i;\forall j & (1)\\ \sum_{i=1}^{m} x_{ij}=v_j, \quad \forall j & (2)\\ \sum_{j=1}^{n} x_{ij}=h_i, \quad \forall i & (3)\\ 0\le y_{ij}-y_{i,j\text{-}1}\le jx_{ij}, \quad \forall i;\forall j & (4)\\ jx_{ij}\le y_{ij}\le j, \quad \forall i;\forall j & (5)\\ y_{i1}=0, \quad \forall i & (6)\\ x_{ij}\in\left\{0,1\right\}, y_{ij}\in R \end{cases}$$

The constraint (1) ensures that the objective function (C) consists in minimizing the maximum penalty. The constraint (2) ensures that in each period t_j there are v_j visited sites. The constraints (3) ensure that each site s_i is visited in h_i periods. Constraints (4) and (5) together ensure that y_{ij} is modified only if site s_i is visited (x_{ij} = 1) and takes the value j. Constraint (6) implies that all the sites are not observed before the first period.

A solution to the problem SSHV(a,b) will be presented by an $m\times n$ binary matrix A such that a_{ij} = 1 only if site s_i is visited at period t_j.

Consider the subproblem SSHV(0,1) with a_{ij} = 0 and b_{ij} = 1 for all the sites over all the periods. Thus for each couple of site and period (s_i,t_j) the total penalty is (j-y_{ij}) which is the number of consecutive non visiting periods before time slot t_j. If we consider the schedule as a binary matrix, the total penalty is exactly the length of the sequence of zeros before the entry (i,j). Hence the subproblem SSHV(0,1) is equivalent to reconstructing a binary matrix with horizontal projection H and vertical projection V and minimizing the length of the longest sequence of zeros which is NP-complete (see Section 2). So the general problem SSHV(a,b) is also NP-complete.

Proposition 1

The problem SSHV(a,b) is NP-complete.

4. WSN Scheduling with a Given Number of Visits per Period

In this scheduling problem, we search for a sensor scheduling respecting the periodically numbers of visits, *i.e.*, in period j there are v_j visits while minimizing the maximal penalty. The problem SSV(a,b) can be considered as a relaxation of the problem SSHV(a,b) where the number of visits for each site is omitted. Yavuz and Jeffcoat [11] studied an NP-complete problem very close to SSV(a,b).

They supposed that in each period, at most m sites was observed instead of exactly v_j sites as in SSV(a,b). In this section, we show that the problem SSV(0,1) can be solved by a polynomial time algorithm.

As in the previous section, a solution to the problem SSV(0,1) will be presented by an $m \times n$ binary matrix A such that $a_{ij} = 1$ only if site s_i is visited at period t_j. Thus solving SSV(0,1) with vector of visits V is equivalent to reconstructing a binary matrix $m \times n$ respecting the vertical projection V and minimizing the maximal distance. We recall that the distance is the length of a sequence of consecutive zeros. We propose the following cyclic algorithm to solve SSV(0,1) where the rows are associated to the sites and the columns are associated to the periods.

Algorithm A-SSV

- Assign a number from 1 to m to each site
- Assign the visits on column 1 to the first v_1 rows
- Assign the visits on column 2 to the next v_2 rows
- This process is continued cyclically with row 1 being treated as the next row after row m.

We state the following result:

Proposition 2

The algorithm A-SSV solves SSV(0,1) in O(mn).

Proof The algorithm A-SSV provides a solution S* respecting the vertical projection since at each step j v_j ones are assigned to the rows.

Let us show that S* minimizes the maximal distance. Suppose that in S*, the maximal distance is reached between two 1s placed on cells (i,j) and (i,j') with $j < j'$. Since on row i, there is no 1 placed between columns $j + 1$ and $j' - 1$ then $\sum_{k=j}^{j'-1} v_k < m$. If S* is not optimal then there exists a solution S with a maximal distance less than that of S*. On each row of S, there is at least an *1* placed between columns $j + 1$ and $j' - 1$. Hence $\sum_{k=j}^{j'-1} v_k \geq m$, contrary with $\sum_{k=j}^{j'-1} v_k < m$.

Note that in both addressed problems SSV(0,1) and SSHV(a,b), once the schedule is established, the visits should be cyclically assigned to the sensors to improve the lifetime of WSN.

5. Conclusions

In this paper, we have studied the wireless sensor network scheduling problem. We have proved that scheduling with a given number of visits for each site and each period with a minimum maximal penalty is an NP-complete problem. We have also proposed a polynomial time algorithm to schedule sensors under only a given number of visits for each period. The problems studied in this paper belong to a rich and relatively unexplored area. Investigating the problems under more real constraints and designing heuristic to solve the hard problems are possible research directions.

6. References

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