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Target Detection in Three-Dimension Sensor Networks Based on Clifford Algebra

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Abstract

The three-dimensional sensor networks are supposed to be deployed for many applications. So it is significant to do research on the problems of coverage and target detection in three-dimensional sensor networks. In this paper, we introduced Clifford algebra in 3D Euclidean space, developed the coverage model of 3D sensor networks based on Clifford algebra, and proposed a method for detecting target moving. With Clifford Spinor, calculating the target moving formulation is easier than traditional methods in sensor node's coverage area.

1. Introduction

Three-dimensional sensor networks [1] could enable a broad range of applications: Video Surveillance, Ocean Sampling, Environmental Monitoring, Assisted Navigation, and etc. As an emerging technology which can put the information field to a new stage, the theory and applications of three-dimensional intelligent sensor networks have become a key research aspect. With the coming availability of low cost, short range radios along with advances in wireless networking, it is expected that wireless ad hoc sensor networks will become commonly deployed. So it is useful to study three-dimensional intelligent sensor network systems. The coverage problem and target detection problem are the fundamental issues in 3D intelligent sensor network systems. Studies on sensor networks include distributed network, distributed information acquisition, distributed intelligent information fusion and so on. Xie [2,3] proposed a coverage analysis approach for sensor networks based on Clifford algebra. In a 2-dimensional plane, a homogeneous computational method of distance measures has been provided for points, lines and areas, and a homogeneous coverage analysis model also has been proposed for targets with hybrid types and different dimensions. Thus, an analysis framework has been established for sensor net-

works in Clifford geometric space. To evaluate the quality of network coverage, Megerian [4] used Voronoi diagram and Delaunay triangulation respectively to define the worst and best-case coverage in sensor networks. There are also a lot of improved methods to solve these problems [5−8].

The target detection problem in sensor networks also has been a topic of extensive study under different metrics and assumptions [9−11]. There are already some related theories and algorithms proposed for solving the problems of target detection in sensor networks. The work of target detection in sensor networks includes many aspects, as the following:

The Traversing Path Detection [12]: A traversing path without being detected should not intersect the sensing areas of any sensors [13]. The detection rate of a sensor network is interested in application scenarios such as vehicles crossing a battlefield in military operations. Meguerdichian mentioned a novel coverage model for the target detection [14], and proposed an approximate value algorithm for calculating the traversing path [15]. There are also some distributed algorithms to calculate the efficient value in the sensor networks for detecting target [16]. Another way to solve the problem is the localized algorithm with lower computational complexity [17]. It uses polar coordinates to detect target and calcu-

late the path, and refers the Euler equation to calculate the minimal exposure traversing path.

The Exposure of Target [18]: The exposure of target detection is another aspect of the related work. The most popular definition is the information exposure to the estimation of target parameters [18]. With the information exposure formulation and grid graph, the minimal exposure traversing path in detecting target could be achieved. Meanwhile, some heuristic algorithms were also proposed for nodes deployment according to the degree of information exposure [8,15].

The Deployment of Nodes Density [19]: The nodes density is used for ensuring the probability of target detection[19]. It is assumed that the motive target could traverse the deployed area of sensor networks with a fixed velocity and a line path. The studies for deployment of nodes density include the probability sensor model and exposure model [20], grid deployment and random deployment in wireless sensor networks [21], and The critical nodes density based on continuum percolation [22]. The all-sensor field intensity can be modeled as a two dimensional Poisson shot noise process for large-scale sensor networks under the general sensing model [23].

Barrier Coverage [24]: Barrier coverage was proposed by Kumar [24] who mentioned it as an appropriate notion of coverage when a sensor network is deployed to detect targets traversing a protected region, which represents a promising and popular class of applications for wireless sensor networks. There are also some studies for the problem such as minimum segment barrier coverage [25], and double barrier coverage [11].

The coverage and target detection problem can be solved optimally in 2D plane by dividing the polygon into non-overlapping triangles, but it becomes NP-hard in 3D space. In this paper, we present a method for calculating target moving in 3D sensor networks with the coverage analysis approach based on Clifford algebra, which establishes a coordinate-free, homogeneous coverage model for different dimensional spaces and targets with hybrid types. This approach gives the intact relative geometric description between sensor node and target. With the Clifford Spinor, calculating target moving formulation will be more simply and effectively than traditional method for sensor networks.

The paper is organized as follows. In Section 2, we state relevant background of Clifford algebra. In Section 3, we present our model and target moving formulation. In Section 4, the algorithm based on Clifford algebra is proposed. In Section 5, we present our conclusions.

2. Clifford Algebra in 3D Euclidean Space

Rectangular Cartesian coordinate system is adopted in 2D Euclidean space, and any vector **a** can be written as $\mathbf{a} = a_1 e_1 + a_2 e_2$, where e_1, e_2 are the unit vectors of x, y directions, respectively. Hereby the geometric product between vector **a** and **b** can be calculated as:

$$\begin{aligned}\mathbf{ab} &= (a_1 e_1 + a_2 e_2)(b_1 e_1 + b_2 e_2)\\ &= a_1 b_1 + a_2 b_2 + a_1 b_2 e_1 e_2 + a_2 b_1 e_2 e_1\end{aligned}$$

Here $e_1 e_2 = e_2 e_1 = 1$. If we appoint $e_1 e_2$ as the area unit with direction, and let $e_1 e_2 = -e_2 e_1$, so the geometric product is

$$\begin{aligned}\mathbf{ab} &= a_1 b_1 + a_2 b_2 + (a_1 b_2 - a_2 b_1) e_1 e_2\\ &= \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \wedge \mathbf{b}\end{aligned} \tag{1}$$

$\mathbf{a} \bullet \mathbf{b}$ is inner product, and $\mathbf{a} \wedge \mathbf{b}$ is outer product. The inner product is always coincident with dot product in vector algebra, while outer product is the measurement of parallelogram area composed by adjacent borders **a**, **b**. If **a** is rotated counter-clockwise with an angle which is no more than π to overlap **b**, the measurement of parallelogram area is larger than zero. Otherwise it is less than zero. The area uses $e_1 e_2$ as a measurement unit, and its absolute value is $\mathbf{ab} \sin\theta$, where θ is the angle between **a** and **b**. Here $\mathbf{a} \bullet \mathbf{b} = \mathbf{ab} \cos\theta$.

Suppose that $e_1 e_2 = i$, then $e_2 e_1 = -i$, and

$$i^2 = e_1 e_2 e_1 e_2 = -e_1 e_1 e_2 e_2 = -1$$

Because $e_1 e_2 = -e_2 e_1$, exchanging the position of $e_1 e_2$ will appear a minus, and exchanging the position n times should multiply $(-1)^n$. We call it negative exchange, and should take care of it in Clifford algebra.

After confirming the $e_1 e_2 = i$, we will have

$$e_1 i = e_1 e_1 e_2 = e_2, \quad e_2 i = e_2 e_1 e_2 = -e_1 e_2 e_2 = -e_1,$$

$$\mathbf{a}i = (a_1 e_1 + a_2 e_2)e_1 e_2 = a_1 e_2 - e_2 e_1$$

So each vector multiplies i on right side to let the vector be rotated counter-clockwise with $\pi/2$. And

$$\begin{aligned}\mathbf{a}e^{i\theta} &= (a_1 e_1 + a_2 e_2)(\cos\theta + i\sin\theta)\\ &= a_1 e_1 \cos\theta + a_1 e_1 i\sin\theta + a_2 e_2 \cos\theta + a_2 e_2 i\sin\theta\\ &= (a_1 \cos\theta - a_2 i\sin\theta)e_1 + (a_1 \sin\theta + a_2 \cos\theta)e_2\end{aligned}$$

It definitely describes that the ordinate unit vectors $e_1 e_2$ are fixed, vector **a** is rotated counter-clockwise with θ. It is obvious that i is not only just as -1, but also has specific geometric significant. The $\mathbf{a}ze^{i\theta}$ also denotes that **a** is rotated with θ, and magnified z times. z is the module of complex number.

The nodes in sensor networks need scalar and directed quantity to be described together, so we use Equation (1) for calculating. The quantity $\mathbf{a} \wedge \mathbf{b}$ is called dual vector

or bivector, its unit is $e_1 e_2 = i = e_1 \wedge e_2$. According to (1), $e_1 e_2 = e_1 \bullet e_2 + e_1 \wedge e_2 = e_1 \wedge e_2$ due to $e_1 \bullet e_2 = 0$.

The vector **a** in 3D Euclidean space $\Re^3$ can be written as $\mathbf{a} = a_1 e_1 + a_2 e_2 + a_3 e_3$, where e_1, e_2, e_3 are orthonormal vectors. So

Definition 1: The geometric product of vector **a** and **b** in 3D space is

$$\begin{aligned}
\mathbf{ab} &= (a_1 e_1 + a_2 e_2 + a_3 e_3)(b_1 e_1 + b_2 e_2 + b_3 e_3) \\
&= a_1 b_1 + a_2 b_2 + a_3 b_3 + (a_1 b_2 - a_2 b_1)e_1 e_2 \\
&\quad + (a_2 b_3 - a_3 b_2)e_2 e_3 + (a_3 b_1 - a_1 b_3)e_3 e_1 \\
&= \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \wedge \mathbf{b}
\end{aligned} \tag{2}$$

Let $e_1 e_2 e_3 = i$, and $e_1 e_2 = e_1 e_2 e_3 e_3 = ie_3$, $e_2 e_3 = ie_1$, $e_3 e_2 = ie_2$. So

$$\begin{aligned}
\mathbf{a} \wedge \mathbf{b} &= (a_1 b_2 - a_2 b_1)ie_3 + (a_2 b_3 - a_3 b_2)ie_1 \\
&\quad + (a_3 b_1 - a_1 b_3)ie_2 \\
&= i(\mathbf{a} \times \mathbf{b})
\end{aligned}$$

where $\mathbf{a} \times \mathbf{b}$ is the scalar product in vector algebra.

Suppose that $ie_k = i_k (k = 1,2,3)$, $i_1 = e_2 e_3$, $i_2 = e_3 e_1$, $i_3 = e_1 e_2$, and the 3D vector i_k is the product of the pseudo-scalar i and the three basis vectors e_k. Here $i_1 e_2 = e_2 e_3 e_2 = -e_3$, and $e_2 i_1 = e_2 e_2 e_3 = e_3 = -i_1 e_2$. The i is direct volume unit in 3D space.

In 3D space, any rotation is denoted as the result of two vector reflections because the vectors are not always coplanar. The vector x_1 is from the reflection of vector x with the plane I whose normal is basis vector u, depicted in figure 1. And the vector x' is from the reflection of vector x_1 with the plane II whose normal is basis vector v. So vector x rotates to x' with the angle θ, and $\theta = 2\alpha$ where α is the angle between u and v. With Clifford algebra, it is shown as

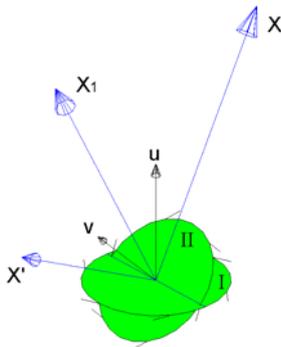$$x' = -v(-uxu)v = vuxuv$$

Figure 1. Vector rotation in 3D space.

Let $R = uv$, so

$$x' = R^T xR \tag{3}$$

where R^T is the reversion of R. R is called Spinor [26] which is composed of scalar $u \cdot v$ and bivector $u \wedge v$. In general

$$\begin{aligned}
R &= u \cdot v + u \wedge v = u \cdot v + i(u \times v) \\
&= \cos\alpha + in\sin\alpha \\
&= e^{in\alpha}
\end{aligned} \tag{4}$$

where n is the basis vector whose direction is decided by $u \times v$. α is the angle between u and v. $R = e^{in\alpha}$ can be written as $R = \alpha + ib$, where α is scalar and b is vector, here $\alpha^2 + b^2 = 1$. Any bivector can be written as the product of i and a vector due to $e_1 e_2 = ie_3$, so ib is a bivector. $R^T = e^{-in\alpha}$ if $R = e^{in\alpha}$ while $R^T = \alpha - ib$ if $R = \alpha + ib$ and $R^T = vu$ if $R = uv$. That means $RR^T = R^T R = 1$. $R = e^{in\theta/2}$ if the angle between x and x' is θ.

3. Target Detection in 3D Sensor Networks

Coverage analysis in sensor networks is essentially to determine whether an arbitrary point in a space is perceived by sensor nodes. Previous methods use models of different dimensional geometric targets to determine whether the interested points covered or not in different dimensional space, respectively. For sensor networks with hybrid types of targets, those methods cannot provide a homogeneous and effective coverage analysis in different dimensional subspaces. The geometric operation in Clifford algebra is independent of coordinates with a determinate dimension. Hence, Xie [2] proposed a coverage model based on the rotation operator in 3D space for sensor networks.

Definition 2: Set an omnidirectional sensor node $S = (\mathbf{v}_s, \rho e_1 \wedge e_2 \wedge e_3)$ with perceived radius ρ, the coverage range of S is $\mathbf{D} = \left\{ x \mid \|x - S\| \le \rho \right\}$, here vector $\mathbf{v}_s \in S$, so the coverage discrimination model for task region **B** in space $\mathcal{G}_3$ is given by

$$C(\mathbf{B}) = \begin{cases} 1, & \forall x \in \mathbf{B}, \|\mathbf{R}\|_x \le 1 \\ 0, & \forall x \in \mathbf{B}, \|\mathbf{R}\|_x > 1 \end{cases} \tag{5}$$

where the Spinor $\mathbf{R} = x / \mathbf{v}_s$. Figure 2 illustrates the coverage model.
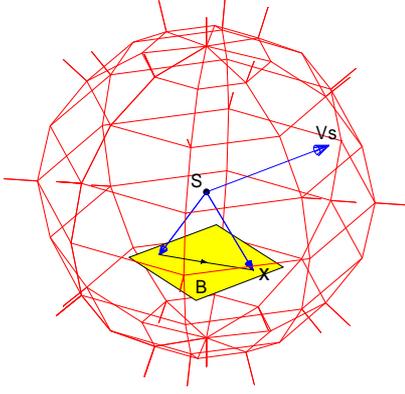
Figure 2. Coverage model based on Clifford Spinor in 3D space.

The relationship between sensor node and target is always depicted by Euler angle, and it would be illustrated by Clifford algebra without coordinates. So the Clifford algebra can present 3×3 Spinor matrix, and the angle transformation is

$$\mathbf{R}x = R_\phi^T R_\theta^T R_\varphi^T x R_\varphi R_\theta R_\phi \qquad (6)$$
$$= e^{-ie_3\phi/2} e^{-ie_3\theta/2} e^{-ie_3\varphi/2} x e^{ie_3\varphi/2} e^{ie_3\theta/2} e^{ie_3\phi/2}$$

Let $f_k = \mathbf{R}e_k$, and f_k is the new vector that is isomorphic with e_k in new position after transformation.

Theorem 1: The vector ξ which is perpendicular with axis rotates to ξ', so

$$\xi' = \mathbf{R}\xi = R^T \xi R = \xi R^2 \qquad (7)$$

Proof: ξ is perpendicular with vector b in $R = \alpha + ib$ whose direction is same as axis, so $b\xi = b\cdot\xi + b\wedge\xi = b\wedge\xi = -\xi\wedge b = -\xi b$, that means the negative exchange between vectors which are perpendicular with each other. So

$$\xi' = R^T \xi R = (\alpha - ib)\xi(\alpha + ib)$$
$$= (\alpha - ib)(\xi\alpha + i\xi b)$$
$$= \alpha^2\xi - i\alpha b\xi + i\alpha\xi b + b\xi b$$
$$= \xi(\alpha^2 + i\alpha b + i\alpha b - b^2)$$
$$= \xi(\alpha + ib)(\alpha + ib) = \xi R^2$$

Here the exchange among vector ξ, pseudo-scalar i and α is positive, but is negative between ξ and vector b.

With $\xi' = \xi R^2$, consider $R_\phi^T R_\theta^T R_\varphi^T e_1 R_\varphi R_\theta R_\phi$, here

$$R_\varphi^T e_1 R_\varphi = e_1 R_\varphi^2 = e_1 e^{ie_3\varphi}$$
$$= e_1(\cos\varphi + ie_3\sin\varphi) \qquad (8)$$
$$= e_1\cos\varphi + e_2\sin\varphi$$

And

$$R_\theta^T(e_1\cos\varphi + e_2\sin\varphi)R_\theta$$
$$= e_1\cos\varphi + e_2\sin\varphi e^{ie_1\theta} \qquad (9)$$
$$= e_1\cos\varphi + e_2\sin\varphi\cos\theta + e_3\sin\varphi\sin\theta$$

So

$$f_1 = R_\phi^T(e_1\cos\varphi + e_2\sin\varphi\cos\theta + e_3\sin\varphi\sin\theta)R_\phi$$
$$= e_1\cos\varphi(e^{ie_3\phi}) + e_2\sin\varphi\cos\theta(e^{ie_3\phi}) + e_3\sin\varphi\sin\theta$$
$$= e_1\cos\varphi\cos\phi + e_2\cos\varphi\sin\phi + e_2\sin\varphi\cos\theta\cos\phi$$
$$\quad - e_1\sin\varphi\cos\theta\sin\phi + e_3\sin\varphi\sin\theta$$
$$\qquad (10)$$

Meanwhile

$$f_2 = e_2\cos\varphi\cos\theta\cos\phi - e_1\cos\varphi\sin\theta\sin\phi - e_1\sin\varphi\cos\phi$$
$$\quad - e_2\sin\varphi\sin\phi + e_3\cos\varphi\sin\theta$$
$$\qquad (11)$$

$$f_3 = e_3\cos\theta - e_2\sin\theta\cos\phi + e_1\sin\theta\sin\phi \qquad (12)$$

The Spinor matrix $R_{jk} = (f_j\cdot e_k)$ is

$$R_{jk} = \begin{bmatrix} \cos\varphi\cos\phi - \sin\varphi\cos\theta\sin\phi & \cos\varphi\sin\phi + \sin\varphi\cos\theta\cos\phi & \sin\varphi\sin\theta \\ -\cos\varphi\sin\theta\sin\phi - \sin\varphi\cos\phi & \cos\varphi\cos\theta\cos\phi - \sin\varphi\sin\phi & \cos\varphi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix} \qquad (13)$$

With the method of Clifford algebra, we can easily calculate the 3×3 Spinor matrix of arbitrary angle θ rotating around any vector n. The element of this matrix is

$$R_{jk} = f_j \bullet e_k = (R'e_j R)\bullet e_k \qquad (14)$$

Generally, it is more suitable that the element under this circumstance can be written as $R_{jk} = <(R^T e_j R)\bullet e_k>_0$. Here $<\;>_0$ denotes the vector of grade zero which is scalar part, because the result will normally be $<\;>_0 + <\;>_1 + <\;>_2 + <\;>_3$. We have known that the established R_{jk} just is a simple scalar quantity, so we don't have to denote its vector of grade zero, where

$$R^T e_j R = (\alpha - ib)e_j(\alpha + ib)$$
$$= (\alpha - ib)(\alpha e_j + ie_j b)$$
$$= \alpha^2 e_j - i\alpha b e_j + i\alpha e_j b + b e_j b$$

With

$$i\alpha(e_j b - b e_j) = 2i\alpha(e_j \wedge b) = 2i\alpha i(e_j \times b) = -2\alpha(e_j \times b),$$

$$b e_j b = b \bullet (e_j \bullet b + e_j \wedge b) + b \wedge (e_j \bullet b + e_j \wedge b)$$
$$= b \bullet e_j b - b \bullet b e_j + e_j \bullet b b = 2b \bullet e_j b + b^2 e_j$$

Hence

$$R^T e_j R = (\alpha^2 - b^2)e_j - 2\alpha(e_j \times b) + 2b \bullet e_j b,$$

And

$$(R^T e_j R) \bullet e_j = (\alpha^2 - b^2)\delta_{jk} - 2\alpha(e_j \times b) \bullet e_j + 2(b \bullet e_j)(b \bullet e_k)$$

where $\alpha = \cos\dfrac{\theta}{2}$, $b = n\sin\dfrac{\theta}{2}$, θ is the angle, n is the unit vector for rotating. (3.10) can be written as

$$R_{jk} = \cos\theta \bullet \delta_{jk} - \sin\theta(e_j \times n) \bullet e_k + (1 - \cos\theta)n_j n_k \quad \textbf{(15)}$$

Here δ_{jk} is Kronecker symbol. When $j = k$, $\delta_{jk} = 1$, otherwise $\delta_{jk} = 0$. The second part in (15) is a general vector operation which can be easily calculated by specific j, k because $(e_j \times n) \bullet e_k = -(e_j \times e_k) \bullet n$. For example, $j = 2$, $k = 3$, and $e_2 \times e_3 = e_1$, so $(e_j \times e_k) \bullet n = e_1 \bullet n = n_1$. Therefore, the second part is $n_1 \sin\theta$ here which can infer the other parts. The final result is

$$R_{jk} = \begin{bmatrix} \cos\theta + n_1^2(1 - \cos\theta) & n_1 n_2(1 - \cos\theta) + n_3\sin\theta & n_1 n_3(1 - \cos\theta) - n_2\sin\theta \\ n_1 n_2(1 - \cos\theta) - n_3\sin\theta & \cos\theta + n_2^2(1 - \cos\theta) & n_2 n_3(1 - \cos\theta) + n_1\sin\theta \\ n_3 n_1(1 - \cos\theta) + n_2\sin\theta & n_3 n_1(1 - \cos\theta) - n_1\sin\theta & \cos\theta + n_3^2(1 - \cos\theta) \end{bmatrix}$$

The conventional methods can also achieve this result, but this method is straight and do not need the graph for help. Hence the target moving can be detected in coverage area of sensor networks by Clifford algebra, as shown in figure 3. If the target moves from x to x', that is

$$x' = R^T x R + a = \mathbf{R}x + a \quad \textbf{(16)}$$

where x is the position of target.

Each element in (16) would be time function, so the target moving formulation is

$$\dot{x}' = \dot{\mathbf{R}}x + \mathbf{R}\dot{x} + \dot{a} \quad \textbf{(17)}$$

Theorem 2: In (17), $\dot{\mathbf{R}}x = \dot{R}^T x R + R^T x \dot{R}$ can be calculated by (18):

$$\dot{\mathbf{R}}x = R^T(\omega \times x)R \quad \textbf{(18)}$$

where ω is target's angular velocity to sensor node.

Proof: Suppose that

$$\dot{R}x = \frac{1}{2}R\Omega$$

And

$$\dot{R}^T R + R^T \dot{R} = 0$$

$$\frac{1}{2}\Omega^T R^T R + \frac{1}{2}R^T \dot{R}\Omega = 0$$

$$\Omega^T = -\Omega, \dot{R}^T = -\frac{1}{2}\Omega R^T$$

Here $R^T R = 1$, $R^T = (\dot{R})^T$. Ω is bivector, and can be denoted as $\Omega = i\omega$. So $\Omega = 2R^T \dot{R}$ or $i\omega = 2R^T \dot{R}$.

Thus

$$\dot{\mathbf{R}}x = \dot{R}^T x R + R^T x \dot{R} = -\frac{1}{2}\Omega R^T x R + \frac{1}{2}R^T x R\Omega = \frac{1}{2}R^T(-R\Omega R^T x + x R\Omega R^T)R$$

and

$$\dot{\mathbf{R}}x = R^T \frac{1}{2}(x\Omega - \Omega x)R = \mathbf{R}x \cdot \Omega = R^T(x \cdot \Omega)R$$

Or

$$\dot{\mathbf{R}}x = R^T \frac{i}{2}(x\omega - \omega x)R = R^T(\omega \times x)R$$

With $\omega \wedge x = i(\omega \times x)$ and $\omega \times x = -i(\omega \wedge x)$,

$$\dot{x}' = R^T[(\omega \times x) + \dot{x}]R + \dot{a} \quad \textbf{(19)}$$

The target moving formulation can further be denoted as

$$\ddot{x}' = \dot{\mathbf{R}}[(\omega \times x) + \dot{x}] + \mathbf{R}[(\dot{\omega} \times x) + (\omega \times \dot{x}) + \ddot{x}] + \ddot{a} \quad \textbf{(20)}$$
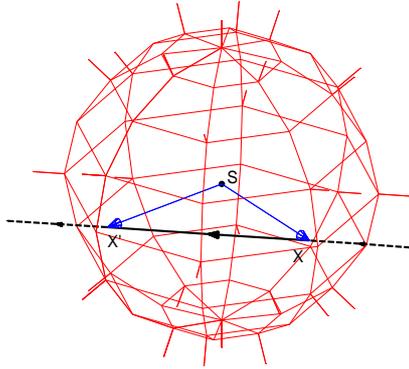
Figure 3. The target moving path detected by sensor node.

where

$$\dot{\mathbf{R}}(\omega \times x) = \dot{R}^T(\omega \times x)R + R^T(\omega \times x)\dot{R}$$

$$= -\frac{1}{2}i\omega' R^T(\omega \times x)R + \frac{1}{2}R^T(\omega \times x)Ri\omega'$$

$$= R^T \frac{i}{2}[(\omega \times x)R\omega' R^T - R\omega' R^T(\omega \times x)]R$$

$$= R^T \frac{i}{2}[(\omega \times x)\omega - \omega(\omega \times x)]R$$

$$= R^T \omega \times (\omega \times x)R$$

$$= R^T(x \cdot \Omega) \cdot \Omega R$$

And

$$\dot{\mathbf{R}}\dot{x} = \dot{R}^T \dot{x}R + R^T \dot{x}\dot{R} = R^T(\omega \times \dot{x})R = R^T \dot{x} \cdot \Omega R .$$

Therefore,

$$\ddot{x}' = R^T[\ddot{x} + 2(\omega \times \dot{x}) + \omega \times (\omega \times x) + \dot{\omega} \times x]R + \ddot{a} \quad (21)$$

In the 3D space, the movement formulation of the target m is

$$m\ddot{x}' = g' ,$$

where is the power of the target so

$$R^T m[\ddot{x} + 2(\omega \times \dot{x}) + \omega \times (\omega \times x) + \dot{\omega} \times x]R + m\ddot{a} = g'$$

$$m[\ddot{x} + 2(\omega \times \dot{x}) + \omega \times (\omega \times x) + \dot{\omega} \times x] + Rm\ddot{a}R^T = Rg'R^T = g$$

$$m\ddot{x} = g - m[2(\omega \times \dot{x}) + \omega \times (\omega \times x) + \dot{\omega} \times x] - Rm\ddot{a}R^T$$

We can achieve the movement formulation of the target in sensor networks, which will help us to analyze the state of the detected target.

4. Algorithm

With the movement formulation of the target in 3D sensor networks, we propose a motive target detection algorithm based on Clifford Spinor. We assume that the mo-

tive target in sensor networks can be detected by some nodes, and its positions in the range of these nodes also can be calculated. Because the surveillance range of a node is always small, the track of the target in the range will be considered as a line approximately. Our algorithm for tracking the motive target is following:

1) Detect the two vertexes of the track when the target is in the range of node;

2) Use (17) or (20) to calculate the movement formulation of the target;

3) Use the movement states of target to estimate the direction and velocity so as to inform the next correlative node turning on and starting to detect the target;

4) Collect all information from each node, and achieve the track of the motive target in the sensor networks.

Figure 4 shows the motive target detection using moving formulation calculated by Clifford Spinor in 3D sensor networks, where (a) is the target's actual path to traverse sensor networks, and (b) is the target's traversing path achieved by moving formulation. The target positions in sensor's coverage area can be used to calculate the moving formulation to get the traversing path in this area. Connecting all of these paths which are achieved from each node can get the target's track in sensor networks. It is helpful for target tracking and forecasting.
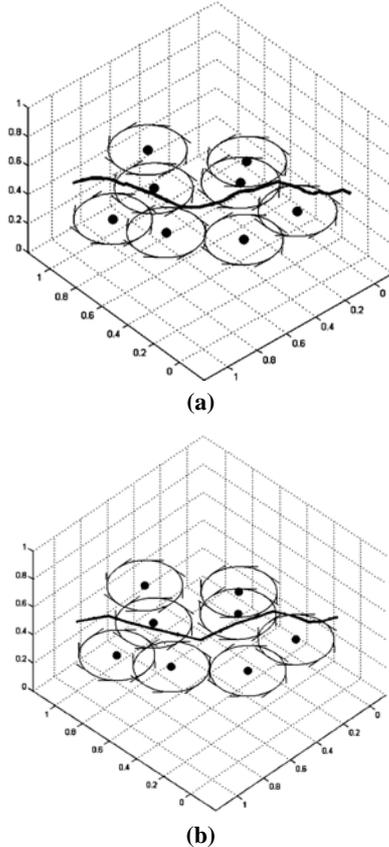

(a)

(b)

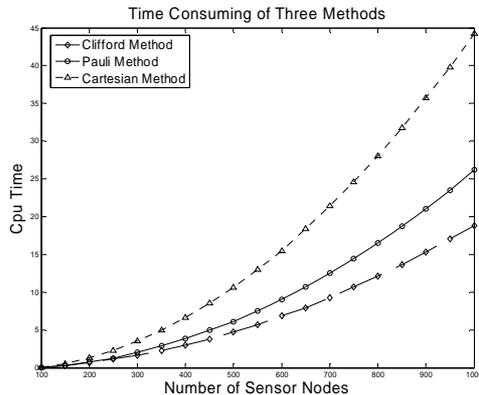**Figure 4. Target detection in 3D sensor networks.**

Figure 5. The comparison of time consuming among three methods.

Figure 5 denotes the time consuming to calculate target moving formulation using Cartesian method, polar coordinates method and Clifford method, respectively. It is obvious that the time consuming using Clifford method is lower than that of the other two methods when the number of sensor nodes increases. This is because the Cartesian method should use the information of three axes in 3D space and the polar coordinates method would calculate three 3×3 Pauli matrices. The quantity of calculation is decreased in Clifford method due to just using Spinor equation to get the moving formulation without axes information in 3D space.

5. Conclusions

In this paper, we developed the coverage model of 3D sensor networks with the coverage analysis approach based on Clifford algebra, which establishes a coordinate-free, homogeneous coverage model for different dimensional spaces and targets with hybrid types, and proposed the method for detecting target moving. With Clifford Spinor, calculating the target moving formulation is easier than traditional methods in sensor node's coverage area. It is helpful to the research of coverage analysis in sensor networks.

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7. References

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