Permittivity Measurement of Low-Loss Substrates Based on Split Ring Resonators

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Abstract

In this paper, we present the complex permittivity measurement of low-loss substrates based on a microstrip-line-excited split-ring resonator (SRR). Permittivity of an unknown substrate is calculated based on the change in oscillation frequency of SRR caused by the material-under-test (MUT) above the SRR. Theoretical analysis and results of the simulations and experiments demonstrate the microstrip-line-excited SRR can be used to effectively improve measurement sensitivity. Simple equations for measurement of low-loss substrates using SRR are proposed and experimentally verified.

Keywords

Split-Ring Resonators (SRR), Permittivity, Non-Contact Measurement

1. Introduction

Complex permittivity of the material is an important parameter to reflect the interactions of matter and magnetic fields and accurate measurement of complex permittivity is widely applied in fabrication process, quality control and biosensing.

Many methods have been proposed to measure the complex permittivity such as free-space methods, transmission-line methods and resonance methods. Free-space methods are based on the measurement of the free-space reflection and transmission coefficients of the sample placed between the antennas. This method is nondestructive and contactless; however it is more suitable for frequency higher than 30 G and the diffraction effects around the sample edges are difficult to eliminate [1] [2] [3]. Compared to the free-space methods, transmission-line methods are much cheaper and more suitable for wideband measurements. The most common structures of transmission-line methods are rectangular waveguide and coaxial
The resonance methods are the most accurate methods to measure the complex permittivity by measuring the shift in the resonance frequency and the change in the quality factor. The resonance method offers high accuracy but have a limited bandwidth [7] [8].

In this paper, we present a non-invasive permittivity measurement of low-loss substrates based on a microstrip-line-excited split-ring resonator (SRR). Permittivity of an unknown substrate is calculated based on the change in oscillation frequency of SRR caused by the material-under-test (MUT) above the SRR. The microstrip-line-excited SRR has small size and low requirement of the sample’s shape. Theoretical analysis and results of the simulations and experiments demonstrate the new sensor can be used to effectively improve the measurement sensitivity.

2. Theory

The corresponding equivalent circuit is shown in Figure 1 [9]. Based on circuit theory, the input impedance for the SRR without the SUT is

\[ Z_0 = R_{\text{ring}} + j\omega L_{\text{ring}} + 1/j\omega C_{\text{ring}} \]  

(1)

The relationship between the impedance perturbation and the SUT’s permittivity has been well investigated in many papers [10]. Assume the SRR carries a current with a magnitude \( I \), the impedance perturbation caused by the SUT is

\[ \Delta Z_L = \left[ \left( j\omega / I^2 \right) \int \int \int \left( \vec{E}_{\text{inc}}(\vec{r}) \cdot \left( \vec{E}_{\text{tot}}(\vec{r}) - 1 \right) \right) dV \right] \]

(2)

where \( \vec{E}_{\text{inc}} \) and \( \vec{E}_{\text{tot}} \) are the electric fields with the SUT and without the SUT. \( \vec{E}_{\text{inc}} \) denotes the electric field induced by the resonance current \( I \) in the load coil, and \( \vec{E}_{\text{tot}} \) denotes the total field. Thus the input impedance for the SRR with the SUT is

\[ Z = R_{\text{ring}} + j\omega L_{\text{ring}} + 1/j\omega C_{\text{ring}} + \Delta Z_L \]  

(3)
Equation (2) is exact for the change in impedance due to SUT perturbations, but is not in a very usable form since we generally do not know $E_{tot}$. So the relationship between $\Delta Z_L$ and $\tilde{\varepsilon}_r$ is not intuitive, however we can make some approximation in some restrictions.

1) **Contact methods**

In this section, we provide a simple setup for the contact measurement with high sensitivity, as in Figure 2. If the permittivity of the SUTs is small ($\varepsilon_r \leq 10$), then we can approximate the $E_{tot}$ by the original fields $E_{inc}$. Equation (2) can be simplified to

$$\Delta Z_L \approx j\alpha \cdot (\tilde{\varepsilon}_r - 1)$$

(4)

Substituting (4) into (3), we can get

$$Z = R_{ring} + j\omega L_{ring} + 1 / j\omega C_{ring} + j\alpha(\tilde{\varepsilon}_r - 1)$$

(5)

For a given configuration, $\alpha$ is a pure real constant. When the SRR resonates, imaginary part of the impedance is zero. The SUTs we measured are low-loss substrates, so we can approximate the complex permittivity $\tilde{\varepsilon}_r$ by $\varepsilon_r$ (the real part of $\tilde{\varepsilon}_r$). According such conditions, we can achieve

$$\omega L_{ring} - 1 / \omega C_{ring} + \alpha(\varepsilon_r - 1) = 0$$

(6)

$$\omega L_{ring} - 1 / \omega C_{ring} = 0$$

(7)

where $\omega$ is the resonant frequency of the SRR with SUT and $\omega_b$ is the resonant frequency without SUT. Substituting (7) into (6), we can obtain

$$\Delta f^2 \approx A(\varepsilon_r - 1)$$

(8)

2) **Non-contact methods**

In theory, for all the setup configuration the impedance perturbation is obtained by (2). In order to simplify (2), a contactless measurement for low-loss and small size SUT is provided, as in Figure 3. Considering the size of the SUT is much smaller than the corresponding wavelength, the Rayleigh approximation can be used to calculate the total field

$$E_{tot}(\vec{r}) \approx \left[3 / (\tilde{\varepsilon}_r + 2)\right] \cdot E_{inc}(\vec{r})$$

(9)

Substituting (9) into (2), we get

$$\Delta Z_{L,\text{Rayleigh}} \approx j\beta \left[\left(\tilde{\varepsilon}_r - 1\right) \cdot \left(\varepsilon_r + 2\right)\right]$$

(10)

Then, we can also obtain a linear equation

$$\Delta f^2 \approx B(\varepsilon_r - 1) / (\varepsilon_r + 2)$$

(11)

Figure 2. A typical setup of contact measurement.
3. Experiment

In this section, we use two setups to measure the permittivity of the SUTs, one is the contact measurement for high sensitivity, the other is the contactless one. The microstrip-line-excited SRR is fabricated using printed circuit board technology. The implementation of the sensor is shown in Figure 4. CST microwave studio is used to simulate the mode and a VNA (AnritsuS331E) is used to measure the resonate frequency. Figure 5 shows the simulation results and the measured reflection coefficients of the SRR without SUTs. The divergence between the simulation and measurement is mainly due to errors in the fabrication process. Four substrates with the same size were prepared for the permittivity measurements.

Figure 6 shows the contact measurement result when the SUT changes and Figure 7 shows the contactless measurement results. The resonant frequency reduces as the sample permittivity increases in agreement with the theoretical analysis and simulation results.
Figure 5. S-parameters of the microstrip-line-excited split-ring resonator without SUTs.

Figure 6. (a) Contact measured $S_{11}$ for different SUTs; and (b) The measured frequency with respect to permittivity.
Figure 7. (a) Contactless measured $S_{11}$ for different SUTs; and (b) The measured frequency with respect to permittivity.

4. Conclusion

In conclusion, the theoretical analysis and results of the simulations and experiments demonstrate the microstrip-line-excited SRR can be used to effectively improve the measurement sensitivity. Simple equations for measurement of low-loss substrates using SRR are proposed and experimentally verified.

References


