A Simple Model of Currency Notes Withdrawal

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Abstract
In the backdrop of the demonetization move by the Government of India, this paper proposes a model of optimal currency holding when there is a possibility of currency withdrawal. Our results indicate that if the perceived probability of withdrawal of higher denomination currency is very high, then agents would eventually hold cash in lower denomination currency only, thereby making the higher currency notes redundant. Thus, one of the targets of demonetization, which is less holding of higher currency notes, can be achieved without actually implementing demonetization.

Keywords
Demonetization, Expected Payoff, Risky Asset, Optimal Portfolio

1. Introduction
In the evening of November 8, 2016, the Prime Minister of India made a sudden, unanticipated and rare economic policy move when he announced that the currency notes of higher denomination *i.e.* Rs.500 and Rs.1000, would be rendered invalid at the stroke of the midnight (except for some essential services that would continue to accept these currency notes for some more time as stipulated by the government). With almost 85% of the total currency in circulation being in these two denominations, this news sparked widespread panic in the economy despite repeated assurance from the government regarding redundant currency exchange.

Since then, this “demonetization” move, as dubbed by the media, has been a topic of intense discussion and debate ranging from the motive behind this step and the inconvenience faced by the general public to its impact on the economy.°

°The policy move of withdrawal of old 500 and 1000 currency notes is known as demonetization in India even though the Government of India gradually introduced a new series of 200, 500 and 2000 currency notes in exchange. Such a policy move can also be called as a “policy of currency note exchange”. The objective of our paper is to analyze optimal portfolio choice comprising of risky high denomination currency notes and non-risky lower denomination currency notes. Using either of the terminology does not affect the objective our paper. We are using the terminology “demonetization” as it has been used by media, policy makers and economists in India.
Some recent articles in this context have focused on the economic rationale and consequences of demonetization, as in Gandhy [1], Kohli and Ramkumar [2], and Kumar [3]; touching upon issues like the economic costs of demonetization, the success (or the lack of it) of tackling counterfeit currency in the economy, the effects on the informal sector etc. From the point of view of macroeconomic modelling of demonetization, Dasgupta [1] describes demonetization within the IS-LM framework and looks at its effects on various macroeconomic variables. Waknis [4] adds to the commentary on demonetization by analyzing the money-multiplier theory and segmented markets theory in economics.2

In this paper, we take up only one particular aspect of this policy, which is, what happens to optimal currency holding of an agent when higher currency notes withdrawal is a possibility but the timing is unanticipated. We show that if this policy is exercised frequently beyond a certain critical threshold, then agents would shift their entire currency holding to smaller denominations, thereby defeating the purpose of withdrawal of bigger currency notes. Alternately, if the perceived probability of withdrawal of higher denomination currency is very high, then one of the targets of demonetization, that is, less holding of higher currency notes, can be achieved without actually adopting demonetization.

2. The Model3

We begin with an agent who holds a portfolio, \( m \), consisting of lower denominated currency, \( x_1 \) and higher denominated currency, \( x_2 \), thereby \( x_1 < x_2 \).4 The proportion of lower denominated currency in portfolio is \( (1 - \alpha) \), \( 0 < \alpha < 1 \). Therefore,

\[
m = x_1 + \alpha (x_2 - x_1).
\]

We assume that the lower denomination currency is risk free in the sense that, it would never be withdrawn from the market.5 As a result this currency can purchase \( x_1 \) unit of good whose price is normalized to unity. Thus the expected payoff of holding lower denomination currency is, \( x_1 \).

However, holding higher denomination currency is risky since it is public knowledge that this currency can be withdrawn from the market by the govern-

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2Also see, Rogoff [5] for detail discussion about cost and benefit of less cash economy.
3We have drawn inspiration from Chapter 13 of Varian [6] to sketch the outline of this model.
4We deliberately do not include bank deposits or purchase of bonds as a part of the portfolio since in a developing country like India, only 400 million out of 1.25 billion had bank accounts in 2013. Although due to the Pradhan Mantri Jan Dhan Yojana of the government of India, the number of bank accounts rose sharply in the last two years, a World Bank survey found that most of these accounts were dormant with no cash deposits or holdings. Due to low literacy rates, bank account operation or ownership of credit cards etc. eludes the common man.
5Higher denomination currency notes are risky as there is a possibility of its withdrawal to reduce tax evasion, corruption and terrorism funding etc. Withdrawal of higher denomination currency is also required to promote cashless economy which induces transparency and increase revenue. Lower denomination currency notes on the other hand are non-risky. It is assumed to be non-withdrawable and economy is assumed to continue with the lower denomination currency notes perpetually.
ment at any point of time with a probability \( (1 - p) \), \( 0 < p < 1 \). If withdrawn, an individual can convert this higher denomination currency to lower denomination currency but this conversion is valid for \( 0 \leq \gamma < 1 \) fraction of his higher denomination currency holding.\(^6\) Note that the conversion is costly when \( \gamma \) close to zero. As a result, the individual can purchase, \( (p + (1 - p) \gamma)x_2 \) unit of goods from higher denomination currency. Therefore, the expected payoff of holding higher denomination currency is, \( (p + (1 - p) \gamma)x_2 \). We further assume that holding currency notes has a cost. The cost includes foregone interest rate, carrying cost, cost for taking precautionary measures of being theft, etc. We assume that, cost of holding currency notes is \( 0 < \beta < 1 \) of the expected pay-off of the portfolio.

As a result, from Equation (1), we can calculate the net expected payoff of the portfolio as,

\[
\hat{r}_m = (1 - \beta) \left[ x_1 + \alpha \left( (p + (1 - p) \gamma)x_2 - x_1 \right) \right]
\]

(2)

This further implies that the risk associated with portfolio as,

\[
\sigma_m^2 = \alpha^2 \sigma_x^2, \quad \text{where,} \quad \sigma_x^2 = (1 - \gamma)^2 p(1 - p)x_2^2.
\]

(3)

Therefore, the proportion of the higher denomination currency in the portfolio is,

\[
\frac{\alpha}{(1 - \gamma)x_2 \sqrt{p(1 - p)}},
\]

(3)

Substituting Equation (3) to Equation (2) gives,

\[
\hat{r}_m = (1 - \beta) \left[ x_1 + \frac{\sigma_m}{(1 - \gamma)x_2 \sqrt{p(1 - p)}} \left( (p + (1 - p) \gamma)x_2 - x_1 \right) \right]
\]

(4)

Equation (4) denotes the relationship between expected payoff and risk associated with the portfolio, \( m \). This acts as a constraint to the individual holding this portfolio, \( m \). This is the portfolio constraint and its slope is given by:

\[
\frac{\partial \hat{r}_m}{\partial \sigma_m} = \frac{(1 - \beta)\left( (p + (1 - p) \gamma)x_2 - x_1 \right)}{(1 - \gamma)x_2 \sqrt{p(1 - p)}}.
\]

(4)

It is to be noted that the slope of the constraint given in Equation (4) is either,

1) positive when expected pay-off of higher denomination currency is more than the expected pay-off of lower denomination currency,

\(^6\)We can think of several interpretations as to why an agent might not be able to convert the entire amount of his high denomination currency holding. This might be due to holding of excessive cash, a part of which might be unaccounted for and hence comes under the income tax radar, or it might simply be interpreted as the cost of conversion of currency that includes standing in long bank queues, making multiple trips to the bank for conversion, a limit on the amount of daily conversions etc. Cost of conversion may even increase with the amount of higher denomination currency held by the individual. In this case, \( \gamma(x_2); \gamma(x_2) < 0 \).

\(^7\) \( \alpha_x^2 = [p(1 - p) + (1 - p) \gamma^2 x_2^2] - [p(1 - p) + (1 - p) \gamma x_2] = p(1 - p)x_2^2 + p(1 - p) \gamma x_2^2 - 2p(1 - p) \gamma x_2^2 \). Simplifying the above expression gives, \( \sigma_x^2 = (1 - \gamma)^2 p(1 - p)x_2^2 \).
\[(p + (1 - p)\gamma)x_2 - x_1 > 0,\] 2) zero when expected return from higher denomination currency equals to the expected return of lower denomination currency, 
\[(p + (1 - p)\gamma)x_2 - x_1 = 0,\] or 3) negative when expected pay-off of higher denomination currency is less than the expected pay-off of lower denomination currency, 
\[(p + (1 - p)\gamma)x_2 - x_1 < 0.\]

To explain individual preference, we assume that an individual is risk averse and derives utility from portfolio return and risk. The utility function of the individual is, 
\[u = u(r_m, \sigma_m),\]
with \(u_1 > 0\) and \(u_2 < 0\). The slope of the indifference curve of the individual is,
\[
\frac{\partial r_m}{\partial \sigma_m} = -\frac{u_2}{u_1} > 0.
\]

The objective of the individual is to,
\[
\max_{(r_m, \sigma_m)} u = u(r_m, \sigma_m)
\]
Subject to,
\[
r_m = (1 - \beta) \left[ x_1 + \frac{\sigma_m}{(1 - \gamma)x_2\sqrt{p(1 - p)}} \left( (p + (1 - p)\gamma)x_2 - x_1 \right) \right]
\]

The above optimization exercise solves for optimal expected pay-off of the individual’s portfolio, \(r_m^* = r_m(p, x_1, x_2)\) and optimum risk \(\sigma_m^* = \sigma_m(p, x_1, x_2)\). This allows the individual to determine the composition of optimal portfolio by solving the proportion of higher denominated currency as,
\[
\alpha^* = \frac{\sigma_m^*}{(1 - \gamma)x_2\sqrt{p(1 - p)}}.
\]

Let us analyze the optimization problem graphically instead of a full blown analytical solution. The graphical analysis is sufficient to understand the intuition of the model.

**Case 1:** \((1 - \gamma)^{-1} \left[ \frac{x_1}{x_2} - \gamma \right] < p < 1\)

The agent knows that it is risky to hold higher denomination of currency. The risk associated with higher denomination currency is the sudden withdrawal of it from the market. However, he keeps on holding the risky higher denomination currency along with the non-risky lower denomination currency when expected pay-off of holding higher denomination currency dominates the same of lower denomination currency. This happens when the withdrawal risk of higher denomination currency is low enough and the constraint is positively sloped,
\[
(1 - \gamma)^{-1} \left[ \frac{x_1}{x_2} - \gamma \right] < p < 1.
\]  
**Figure 1** shows when the constraint is positively sloped, we have interior solution and the individual holds both lower and higher denomination currency notes. The optimal solution is, \(r_m^* > x_1\) and \(\sigma_m^* > 0, \alpha^* > 0\).
Case 2: \( 0 < p \leq (1 - \gamma)^{-1} \left( \frac{x_1}{x_2} - \gamma \right) \)

The slope of the constraint is zero when expected pay-off of higher denomination currency equals to the expected pay-off of lower denomination currency. The slope of the constraint is zero when \( p = (1 - \gamma)^{-1} \left( \frac{x_1}{x_2} - \gamma \right) \). Figure 2 shows that when the slope of the constraint is zero, we have a corner solution and the individual holds only the non-risky lower denomination currency notes. The optimal solution in this case is, \( r_m^* = (1 - \beta) x_1 \) and \( \sigma_m^* = 0, \alpha^* = 0 \).

Let us note that an individual should ideally be indifferent between holding the lower and the higher denomination currency notes when their expected pay-offs are identical. However, our analysis shows that the individual would hold only the non-risky currency notes due to the risk of withdrawal associated with higher denomination currency.

Again the slope of the constraint is negative when expected pay-off of the lower denomination currency dominates the same of higher denomination currency. In this case the withdrawal risk of higher denomination currency is high enough with, \( 0 < p < (1 - \gamma)^{-1} \left( \frac{x_1}{x_2} - \gamma \right) \) which makes the constraint negatively sloped. Figure 3 shows when the slope of the constraint is negative, we again have a corner solution and the individual holds only the non-risky lower denomination currency notes. The optimal solution in this case is, \( r_m^* = (1 - \beta) x_1 \) and \( \sigma_m^* = 0, \alpha^* = 0 \).

3. Conclusions

The above simple model of optimal cash holding in the wake of retracting higher currency notes shows that if the fear of currency withdrawal is quite high, then an agent would rather hold all his cash in lower denomination currency and
none in higher denomination currency at all. However, if an agent believes that this kind of currency withdrawal would not happen frequently and attaches a low probability to such an event, then he would continue to hold his cash reserves in both lower and higher denomination currency notes.

Thus, from the policy point of view, if the government can raise the perceived probability of higher denomination currency withdrawal by the general public, it can achieve one of the fundamental targets of demonetization, that is, less holding of higher currency notes by agents, without actually having to execute demonetization to begin with.

It is important to note here that, besides reducing tax evasion, corruption, black economy and terrorism funding etc., another major objective behind the
policy of demonetization was to promote a cashless Indian economy. Recent data published by RBI shows a significant increase in Point of Sale (POS) transaction through debit card and transactions using mobile phones and online banking in India after the policy was implemented. However, the data on the other hand also shows that currency in circulation in India has already came back almost to level when the demonetization was announced two years ago. As it is too early to assess full impact, a full blown empirical analysis is required in future to analyze the success of the demonetization policy undertaken by Government of India on November 8, 2016.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References


