

A Note Regarding Prioritization on Congestible Networks

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Abstract

This paper examines the effect of allowing prioritization of traffic on a congestible network under three demand scenarios. First, it is shown that when prioritization does not affect usage, average congestion on the network increases. Second, this result is shown to hold when there is demand shifting toward the prioritized network service but no impact on total network traffic. Finally, it is proven that for prioritization to reduce average congestion on a network it must reduce the total traffic on the network.

Keywords

Congestible Networks, Net Neutrality, Regulation

1. Introduction

On February 26, 2015, the US Federal Communications Commission (FCC) revised their Open Internet rules. These rules outline three main sets of regulations. First, Internet service providers (ISPs) cannot block access to legal content. Second, they cannot throttle access on the basis of content. Finally, and subject to much debate, the FCC disallows paid prioritization. Paid prioritization would allow ISPs to divert capacity to some content over non-prioritized content in exchange for compensation from the content provider.

A number of recent papers debate that network neutrality and paid prioritization would have an effect on consumer welfare and capacity investment. Some argue that allowing service providers to monetize capacity though paid prioritization will cause them to invest in additional capacity and use their capacity more efficiently. Proponents of allowing paid prioritization include Chen and Nalebu [1], Hermalin and Katz [2], Krämer and Wiewiorra [3], Sidak and Teece [4], and Singer and Litan [5]. Others argue that paid prioritization would alter the incentives for the service providers such that they could increase profits by slowing non-prioritized traffic rather than increasing speeds for prioritized traffic. This would create a “fast” and “slow” lane for traffic. Cheng *et*

al. [6], Economides and Tåg [7], Chettiar and Holladay [8] and Pil Choi and Kim [9] espouse variations of this view.

In this note, I examine how network congestion would be affected by prioritized traffic for a given level of capacity. Three cases are compared. First, it is shown that when consumer usage of all network services is perfectly inelastic, prioritization of some traffic increases average network congestion. Second, this result is expanded to the case where total network traffic is inelastic but traffic shifts toward the prioritized service. Finally, it is proven that for prioritization to reduce average congestion on a network it must reduce the total traffic on the network. In other words, less congestion for the prioritized traffic is insufficient to reduce average congestion unless the corresponding increase in congestion on the non-prioritized traffic reduces demand for the network overall.

2. Prioritization and Network Congestion

To model network congestion, I expand upon the seminal MacKie-Mason and Varian [10] framework of a congestible network by allowing for multiple services to run on a single network. Content providers supply excludable services on the network while the access provider runs the network itself. Additionally there may be a number of non-excludable, default network services available. Let the set of services offered be indexed $s = 0, 1, \dots, |S|$ where $s = 0$ are the non-excludable services and $s \neq 0$ are the excludable services. Index consumers $i = 1, \dots, N$, then x_{is} denotes person- i 's usage of service- s on the network. Person- i has network consumption vector $\mathbf{x}_i = (x_{i0}, \dots, x_{iS})$. Demand for the network by person- i is $X_i = \sum_{s=0}^S x_{is}$, total use of service- s is $X_s = \sum_{i=1}^N x_{is}$, and total use for the network is $X = \sum_{i=1}^N \sum_{s=0}^S x_{is}$. Each user gains utility from the consumption of each service but is negatively impacted by congestion on the network. For a network with bandwidth capacity K , the level of congestion can be defined as $Y = \frac{X}{K}$.

To include prioritization into the model, let there be one content provider, $s = 1$, who can pay the access provider a fee F for a guaranteed congestion factor of $Y_1 < Y$, where Y_{-1} is the congestion of all other traffic. Denoting the capacity dedicated to the prioritized content provider as K_1 it holds that

$$Y_{-1} = \frac{\sum_{s \neq 1} X_s}{K - K_1}, \quad Y_1 = \frac{X_1}{K_1}.$$

Thus, to maintain congestion Y_1 for prioritized traffic, a network with capacity K must allocate more of that capacity to the prioritized usage as traffic to that use increases. In other words, K_1 is endogenous. Solving for K_1 and substituting into the equation for Y_{-1} yields

$$Y_{-1} = \frac{\sum_{s \neq 1} X_s}{K - \frac{X_1}{Y_1}}. \quad (1)$$

From this equation come two key observations. First, the capacity is separable and constant-sum. Solving (1) for K shows

$$K = \frac{\sum_{s \neq 1} X_s}{Y_{-1}} + \frac{X_1}{Y_1}.$$

Unsurprisingly, capacity diverted to the prioritized traffic as X_1 increase must be lost, one-for-one, from the rest of the network. Second, congestion for users on the non-prioritized portion of the network increases more for an additional unit of traffic by prioritized users than for an additional unit of traffic by non-prioritized users.

The effect of increasing traffic for a non-prioritized service, $s \neq 1$, is $\frac{\partial Y_{-1}}{\partial X_s} = \frac{1}{K - K_1} = \frac{Y_{-1}}{\sum_{s \neq 1} X_s}$. The effect of

increasing prioritized traffic on congestion for non-prioritized traffic is $\frac{\partial Y_{-1}}{\partial X_1} = \frac{Y_{-1}}{Y_1 (K - K_1)^2} = \frac{Y_{-1}^2}{Y_1 \sum_{s \neq 1} X_s}$. As

long as the prioritized traffic is guaranteed lower congestion than the non-prioritized traffic, prioritized traffic causes more congestion for non-prioritized traffic than does other non-prioritized traffic, $\frac{\partial Y_{-1}}{\partial X_1} > \frac{\partial Y_{-1}}{\partial X_s}$. As a direct consequence, prioritization leads to an increase in the average congestion that a unit of traffic faces.

Theorem 1. *Given average congestion without prioritization $Y^0 = \frac{\sum_{s=0}^S X_s}{K}$, usage of services X_s for $s = 0, \dots, S$, capacity K , and prioritized traffic congestion of $Y_1 < Y^0$, then average congestion with prioritized traffic is larger than average congestion without prioritization:*

$$\frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} Y_{-1} + \frac{X_1}{\sum_{s \in S} X_s} Y_1 > Y^0.$$

This result presumes that demand for each of the services X_s is perfectly inelastic. It is unrealistic to assume that as service $s = 1$ is guaranteed lower congestion that it will not receive increased traffic. People will switch from other services to use the more of the prioritized service. As traffic flows to the prioritized service the prioritized traffic faces lower congestion and the non-priority traffic faces higher congestion. As demand for the prioritized service increases, the prioritized traffic will require more of the total capacity in order to maintain a constant level of congestion. This reduces the capacity available for the non-prioritized traffic increasing its congestion. Because the level of congestion guaranteed to the prioritized traffic is lower than the level of congestion of the non-prioritized traffic, the required capacity diversion is such that the average traffic faces increased average congestion.

Now, to simplify the notation, let a negative index on demand denote demand for all services except the magnitude of the index, e.g. $X_{-1} = \sum_{s \neq 1} X_s$.

Theorem 2. *For a given level of capacity $K > 0$, total demand $X > 0$, and prioritized congestion Y_1 ,*

$$\frac{X_{-1}}{X} Y_{-1} + \frac{X_1}{X} Y_1 \geq Y_0$$

for all $X_1 \in [0, X]$ where $Y_0 = \frac{X}{K}$, $X_{-1} = X - X_1$, $K_1 = \frac{X_1}{Y_1}$, $K_{-1} = K - K_1$, and $Y_{-1} = \frac{X_{-1}}{K_{-1}}$. Additionally, the result only holds with equality when $Y_1 = Y_0$ or $X_1 = 0$.

While Theorem 2 states that when total demand is inelastic, prioritizing traffic increases the average level of congestion, the proof does not require that the guaranteed level of congestion for prioritized traffic be less than before prioritization (see [Appendix](#)). In fact even deprioritized traffic will increase the average level of congestion as long as it does not eliminate usage of the deprioritized service. As such, **Theorem 2** states that to minimize the average congestion traffic on a network faces, Net Neutrality is the required policy.

When total demand for the congestible resource is not perfectly inelastic with regard to prioritized traffic, demand can either increase or decrease as prioritized congestion is reduced. Total demand may increase if prioritization of a given service attracts new users to the network. On the other hand, total demand may be reduced by prioritization if the corresponding increase in congestion for non-prioritized traffic sufficiently discourages network usage. **Theorem 3** states that average congestion can be reduced only in the case where prioritization of some traffic causes total demand for the network to decline.

Theorem 3. *Defining the average network congestion as $Y^{avg} = \frac{X_{-1}}{X} Y_{-1} + \frac{X_1}{X} Y_1$ and the absence of prioritization as $Y_1 = Y_{-1}$ then*

$$\left. \frac{\partial Y^{avg}}{\partial Y_1} \right|_{Y_1=Y_{-1}} > 0$$

if and only if $\frac{\partial X}{\partial Y_1} > 0$.

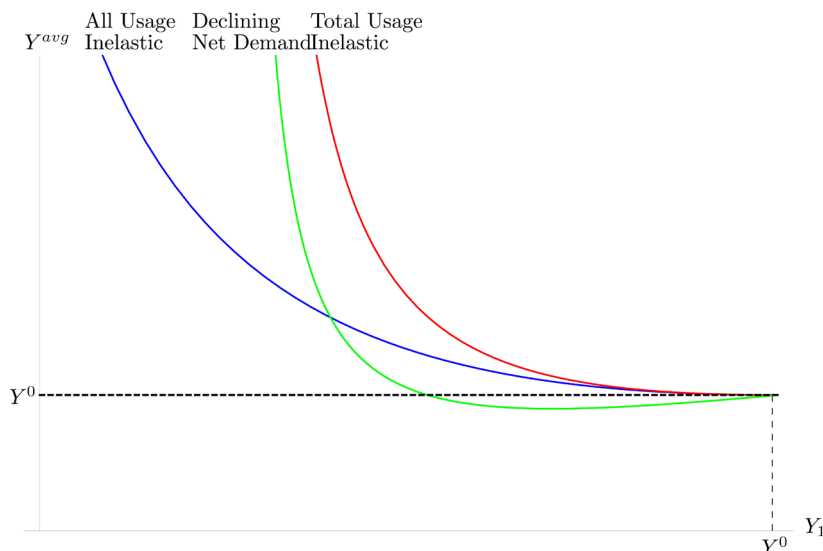


Figure 1. Prioritized traffic and average congestion.

Figure 1 summarizes these results by showing how guaranteeing lower congestion for prioritized traffic affects the average level of congestion faced by traffic on the network. When demand for all services is constant with respect to congestion, average congestion increases slowly as some traffic is prioritized. If total network usage is constant but traffic shifts towards the prioritized services then average congestion increases quickly as some traffic is prioritized. When prioritizing some traffic leads to a net decline in network traffic, prioritization initially decreases the average congestion on the network. This effect, however, is limited and guaranteeing lower and lower congestion for the prioritized service will eventually increase the average level of congestion on the network.¹

3. Concluding Remarks

While this paper is agnostic on the efficiency of paid prioritization and network neutrality, it is strongly suggestive that any prioritization scheme will lead to higher network congestion. When the cost of changing network capacity is large, firms will alter their capacity infrequently. As such, capacity can be taken as a sunk investment and congestion will depend primarily on usage. As firms are unlikely to use prioritization to reduce demand for their network, the above results can be taken to show that in the absence of a net neutrality rule average congestion on the network will increase.

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¹For readability, the case of increased demand for the network due to prioritization is omitted from the graph. This relationship is bounded below by the *total usage inelastic* curve.

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Proof Appendix

Proof of Theorem 1

We want to show that combining all traffic to use capacity K results in a lower average congestion Y^0 than prioritizing traffic ensuring traffic X_1 receives congestion Y_1 and the remaining traffic has congestion determined by the remaining capacity. Formally stated this says, given $Y^0 = \frac{\sum_{s \in S} X_s}{K}$, $Y_1 = \frac{X_1}{K_1}$, and $Y_{-1} = \frac{\sum_{s \neq 1} X_s}{K - K_1}$

such that $Y_1 < Y^0$, then

$$\frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} Y_{-1} + \frac{X_1}{\sum_{s \in S} X_s} Y_1 > Y^0.$$

Proof. For temporary simplification of notation, let $a = \sum_{s \neq 1} X_s$, $b = X_1$, and $c = \frac{Y_1}{Y^0}$. Given that $X_s > 0$ for all s and $0 < Y_1 < Y^0$ we are assured that $a > 0$, $b > 0$, $0 < c < 1$. Additionally, $a + b = \sum_{s \in S} X_s$. We proceed by algebraic manipulation from

$$1 > c.$$

Since b and c are strictly positive,

$$b > bc$$

$$\frac{b}{c} > b$$

$$a > a + b - \frac{b}{c}$$

and because $a + b > 0$

$$\begin{aligned} \frac{a}{a+b} &> \frac{a+b-\frac{b}{c}}{a+b} \\ \frac{a}{a+b} &> \frac{\left(a+b-\frac{b}{c}\right)(1-c)}{(a+b)(1-c)} \\ \frac{a}{a+b} &> \frac{\left(a+b-\frac{b}{c}\right)(1-c)}{a+b-ac-bc} \\ \frac{a}{a+b} &> \frac{\left(a+b-\frac{b}{c}\right)(1-c)}{a-(ac+bc-b)} \\ \frac{a}{a+b} &> \frac{\left(a+b-\frac{b}{c}\right)(1-c)}{a-\left(a+b-\frac{b}{c}\right)c} \\ \frac{a}{a+b} &> \frac{1-c}{\frac{a}{\left(a+b-\frac{b}{c}\right)}-c} \end{aligned}$$

Algebraic manipulation verifies that $\frac{a}{\left(a+b-\frac{b}{c}\right)} > 1 > c$, thus

$$\frac{a}{a+b} \left(\frac{a}{\left(a+b-\frac{b}{c}\right)} - c \right) > 1 - c$$

$$\frac{a}{a+b} \frac{a}{\left(a+b-\frac{b}{c}\right)} - \frac{a}{a+b} c + c > 1$$

$$\frac{a}{a+b} \frac{a}{\left(a+b-\frac{b}{c}\right)} + \frac{b}{a+b} c > 1$$

As $Y^0 > 0$ we continue multiplying by Y^0 .

$$\frac{a}{a+b} \frac{aY^0}{\left(a+b-\frac{b}{c}\right)} + \frac{b}{a+b} Y^0 c > Y^0$$

$$\frac{a}{a+b} \frac{a}{\frac{1}{Y^0} \left(a+b-\frac{b}{c}\right)} + \frac{b}{a+b} Y^0 c > Y^0$$

$$\frac{a}{a+b} \frac{a}{\frac{a+b}{Y^0} - \frac{b}{Y^0 c}} + \frac{b}{a+b} Y^0 c > Y^0$$

Substituting for a, b , and c gives

$$\frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} \frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} + \frac{X_1}{\sum_{s \in S} X_s} Y^0 \frac{Y_1}{Y^0} > Y^0$$

$$\frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} \frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} + \frac{X_1}{\sum_{s \in S} X_s} Y_1 > Y^0$$

Rearranging the definitions of Y^0 and Y_1 to solve for capacity yields $K = \frac{\sum_{s \in S} X_s}{Y^0}$ and $K_1 = \frac{X_1}{Y_1}$.

Substituting in for capacities K and K_1 and then substituting for Y_1 yields our result.

$$\frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} \frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} + \frac{X_1}{\sum_{s \in S} X_s} Y_1 > Y^0$$

$$\frac{\sum_{s \neq 1} X_s}{\sum_{s \in S} X_s} Y_1 + \frac{X_1}{\sum_{s \in S} X_s} Y_1 > Y^0$$

□

Proof of Theorem 2

Proof. Given that $X_1 \geq 0$ then for all Y_0 and Y_1 ,

$$X_1(Y_0 - Y_1)^2 \geq 0.$$

Additionally, this is a strict inequality whenever $X_1 > 0$ and $Y_0 \neq Y_1$. Expanding and substituting for $Y_0 = \frac{X}{K}$ yields

$$X_1 \left(\frac{X^2}{K^2} + Y_1^2 - 2 \frac{X}{K} Y_1 \right) \geq 0.$$

Multiplying through by K^2 and expanding gives

$$X^2 X_1 + K^2 X_1 Y_1^2 - 2KXX_1 Y_1 \geq 0.$$

From here adding terms $KX^2 Y_1 + KX_1^2 Y_1$ to each side of the inequality and algebraically rearranging using the appropriate substitutions yields the result.

$$\begin{aligned} KX^2 Y_1 + KX_1^2 Y_1 + X^2 X_1 + K^2 X_1 Y_1^2 - 2KXX_1 Y_1 &\geq KX^2 Y_1 + KX_1^2 Y_1 \\ (KX^2 Y_1 + KX_1^2 Y_1 - 2KXX_1 Y_1) + K^2 X_1 Y_1^2 - KX_1^2 Y_1 &\geq KX^2 Y_1 - X^2 X_1 \\ KY_1(X - X_1)^2 + K^2 X_1 Y_1^2 - KX_1^2 Y_1 &\geq KX^2 Y_1 - X^2 X_1 \\ (X - X_1)^2 + KX_1 Y_1 - X_1^2 &\geq X^2 - \frac{X^2 X_1}{KY_1} \\ (X - X_1)^2 + X_1 Y_1 \left(K - \frac{X_1}{Y_1} \right) &\geq \frac{X^2}{K} \left(K - \frac{X_1}{Y_1} \right) \\ X_{-1}^2 + X_1 Y_1 (K - K_1) &\geq \frac{X^2}{K} (K - K_1) \\ X_{-1}^2 + X_1 Y_1 K_{-1} &\geq XY_0 K_{-1} \\ \frac{X_{-1}}{X} \frac{X_{-1}}{K_{-1}} + \frac{X_1}{X} Y_1 &\geq Y_0 \\ \frac{X_{-1}}{X} Y_{-1} + \frac{X_1}{X} Y_1 &\geq Y_0 \end{aligned}$$

□

Proof of Theorem 3

Proof. Each of the following implications is bidirectional and follows from $Y_0 = Y_1 = Y_{-1}$, $X = X_1 + X_{-1}$ and the definitions of Y_1, Y_{-1} , and \bar{Y} .

Given that $\frac{\partial X}{\partial Y_1} > 0$ it follows

$$Y_0 \frac{\partial X}{\partial Y_1} > 0$$

$$Y_0 \left(\frac{\partial X_{-1}}{\partial Y_1} + \frac{\partial X_1}{\partial Y_1} \right) > 0$$

$$\begin{aligned}
& Y_0 \frac{\partial X_{-1}}{\partial Y_1} + Y_0 \frac{\partial X_1}{\partial Y_1} - X_1 > -X_1 \\
& Y_1 \frac{\partial X_{-1}}{\partial Y_1} + Y_{-1} \frac{\partial X_1}{\partial Y_1} - X_1 > -X_1 \\
& Y_1 \frac{\partial X_{-1}}{\partial Y_1} + Y_{-1} \left(\frac{\partial X_1}{\partial Y_1} - \frac{X_1}{Y_1} \right) > -\frac{Y_0}{Y_0} X_1 \\
& Y_1 \frac{\partial X_{-1}}{\partial Y_1} + \frac{X_{-1}}{K - \frac{X_1}{Y_1}} \left(\frac{\partial X_1}{\partial Y_1} - \frac{X_1}{Y_1} \right) > -\frac{Y_1}{Y_{-1}} X_1 \\
& Y_1 \frac{\partial X_{-1}}{\partial Y_1} + \frac{X_{-1}}{K - \frac{X_1}{Y_1}} \left(\frac{\partial X_1}{\partial Y_1} - \frac{X_1}{Y_1} \right) > -\frac{Y_1 \left(K - \frac{X_1}{Y_1} \right) X_1}{X_{-1}} \\
& Y_1 \frac{\partial X_{-1}}{\partial Y_1} + \frac{Y_1 X_{-1}}{K - \frac{X_1}{Y_1}} \left(\frac{1}{Y_1} \frac{\partial X_1}{\partial Y_1} - \frac{X_1}{Y_1^2} \right) > -\frac{Y_1 \left(K - \frac{X_1}{Y_1} \right) X_1}{X_{-1}} \\
& \frac{1}{K - \frac{X_1}{Y_1}} \frac{\partial X_{-1}}{\partial Y_1} + \frac{X_{-1}}{\left(K - \frac{X_1}{Y_1} \right)^2} \left(\frac{1}{Y_1} \frac{\partial X_1}{\partial Y_1} - \frac{X_1}{Y_1^2} \right) > -\frac{X_1}{X_{-1}} \\
& \frac{\partial \left(\frac{X_{-1}}{K - \frac{X_1}{Y_1}} \right)}{\partial Y_1} > -\frac{X_1}{X_{-1}} \\
& \frac{\partial Y_{-1}}{\partial Y_1} > -\frac{X_1}{X_{-1}} \\
& X_{-1} \frac{\partial Y_{-1}}{\partial Y_1} + X_1 > 0 \\
& Y_0 \frac{\partial X}{\partial Y_1} + X_{-1} \frac{\partial Y_{-1}}{\partial Y_1} + X_1 > Y_0 \frac{\partial X}{\partial Y_1} \\
& Y_0 \left(\frac{\partial X_{-1}}{\partial Y_1} + \frac{\partial X_1}{\partial Y_1} \right) + \left(X_{-1} \frac{\partial Y_{-1}}{\partial Y_1} + X_1 \right) > \left(\frac{X_{-1}}{X} Y_0 + \frac{X_1}{X} Y_0 \right) \frac{\partial X}{\partial Y_1} \\
& \left(\frac{\partial X_{-1}}{\partial Y_1} Y_{-1} + \frac{\partial X_1}{\partial Y_1} Y_1 \right) + \left(X_{-1} \frac{\partial Y_{-1}}{\partial Y_1} + X_1 \right) > \left(\frac{X_{-1}}{X} Y_{-1} + \frac{X_1}{X} Y_1 \right) \frac{\partial X}{\partial Y_1}
\end{aligned}$$

Dividing by X and algebraically rearranging yields

$$\frac{\partial X_{-1}}{\partial Y_1} \frac{Y_{-1}}{X} + \frac{X_{-1}}{X} \frac{\partial Y_{-1}}{\partial Y_1} - \frac{X_{-1}}{X^2} Y_{-1} \frac{\partial X}{\partial Y_1} + \frac{Y_1}{X} \frac{\partial X_1}{\partial Y_1} + \frac{X_1}{X} - \frac{X_1}{X^2} Y_1 \frac{\partial X}{\partial Y_1} > 0$$

$$\frac{\partial \left(\frac{X_{-1}}{X} Y_{-1} + \frac{X_1}{X} Y_1 \right)}{\partial Y_1} > 0$$

$$\frac{\partial \bar{Y}}{\partial Y_1} > 0$$

□