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The Benefits Distribution of Tri-Networks Convergence Chain

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Abstract

In this paper, we analyze the characteristic function of all the coalitions of the Tri-networks Convergence chain and research on the benefit distribution of Tri-networks Convergence chain based on Shapley value. We find that the broadcasting and the telecommunication operator can achieve cooperative and turn a win-lose situation into a win-win situation of reduced costs and increased revenues for the Tri-networks Convergence chain.

Keywords: Tri-Networks Convergence Chain, Benefit Distribute, Shapley Value, Mini-Max Theorem

1. Introduction

Communication networks have become a key economic and social infrastructure in economies. The network infrastructure, supporting all economic sectors, is crucial to the national and international exchange of goods and services, and acts as a main catalyst in changing economic interrelationships through rapid technological change and the proliferation of a range of new services.

The term "Tri-networks Convergence" refers to the convergence of the telecommunication network, the broadcasting network and the Internet, with an ultimate goal of sharing information resources and forming a high-speed broadband basic information platform [1].

Tri-networks Convergence draws us a picture in which we could play interactive Internet games on the TV set as well as watches TV programs and order high-definition videos on the computer and mobile phone. On January 13, 2010, the State Council decided to accelerate the integration of the telecommunication networks operators, the cable TV networks and the Internet, popularize two-way broadcast and telecommunication networks service to a pilot before 2012, and fully realize the Tri-networks Convergence in 2015. It will be another significant opportunity to China's Optical Communication industry.

China's State Council has announced its decision to promote the advancement of a three network convergence (telecom, broadcast TV and internet) to introduce innovative new services, drive consumption, etc. Two phases are planned: 2010-2012 for launching trial sites for convergent services and firming up the related regulatory policies framework; then 2013-2015 for launch of commercial convergent services. Key work areas involved include accelerating network upgrades-digitalization for cable operators and bandwidth upgrade fiberiztion for telcos, facilitating further cable industry consolidation, and adopting tax and other supportive policies to encourage value chain R & D and product development [2].

To review the successful business case from the Western developed countries, there are some experience in the Tri-networks Convergence chain, including the integrality and exact location of industry chain structure, benefit sharing mechanism, and rich content to meet customer demand [3].

Tri-networks Convergence chain is a cooperative mode by companies with the conflict of interests. It can be accepted by participants on the basis of its benefit maximization. As a manager, there is an integral vested interest among telecommunication operator, broadcasting & TV operator and its affiliated group. Many agencies are not only the maker of industry policy makers, but also the operator in enterprise. And departmental and regional barriers and the protection of business interests conflicts become a great obstacle to the smooth implementation of Tri-networks Convergence [4]. The important factors caused benefit conflicts of Tri-networks Convergence are organizational power, policy control, and resource control and benefits distribution [5]. Therefore, whether the enterprises in tri-networks convergence chain succeed depend on the reasonable benefits distribution. If the benefit distribution is unreasonable, there will probably be opportunism tendency which leads to fail in enterprises.

How to balance and distribution the interests among telecommunication operator, broadcasting operator and its affiliated business reasonably is not only the key to implement Tri-networks Convergence successfully, but also the most important strategy for Tri-networks Convergence chain. However, there is little research about benefits distribution of Tri-networks Convergence currently. The problem of profits distribution is inescapable in virtually every organization and consequently pervades every facet of accounting. Shapley value, whose essence is that profits are distributed according to each participant's contribution, is the main method of analyzing cooperative game (Shapley, 1953) [6].

What is especially surprising in Shapley's result is that nothing in the axioms (with the possible exception of the dummy axiom) hints at the idea of marginal contributions, so marginality in general is the outcome of all the axioms, including additivity or linearity. Among the axioms utilized by Shapley, additivity is the one with a lower normative content: it is simply a mathematical property to justify simplicity in the computation of the solution. Young (1985, 1994) provides a beautiful counterpart to Shapley's theorem. He drops additivity (as well as the dummy player axiom), and instead, uses an axiom of marginality) [7,8]. Apart from these two, Hart and Mas-Colell (1996) provide further acclimatization of the Shapley value using the idea of potential and the concept of consistency [9]. There are three main extensions that have been proposed: the Shapley λ-transfer value (Shapley (1969)), the Harsanyi value (Harsanyi (1963), and the Maschler-Owen consistent value (Maschler and Owen (1992)) [10]. They were axiomatized in Aumann (1985), Hart (1985), and de Clippel, Peters and Zank (2004), respectively. As for the core, there is a value equivalence theorem. The result holds for the TU domain (see Shapley (1964), Aumann (1975), Aumann and Shapley (1974)). It can be shown that the Shapley value payoffs can be supported by competitive prices. Furthermore, in large enough economies, the set of competitive payoffs "shrinks" to approximate the Shapley value. However, the result cannot be easily extended to the NTU domain. While it holds for the λ-transfer value, it need not obtain for the other extensions. For further details, the interested reader is referred to Hart (2008) and the references therein. Gul (1989) was the first to propose a procedure that provided some non-cooperative foundations of the Shapley value [11]. Later, other authors have provided alternative procedures and tech- niques to the same end, including Winter (1994), Krishna and Serrano (1995), Hart and Mas-Colell (1996), and Perez-Castrillo and Wettstein (2001) [12,13].

Hart (1995) first definitions a firm were economic models that considered the firm as a black box where physical inputs and labor came out in an output, at minimum cost and maximum profit [9]. O'Neill (1987) provides a test of the maxmin prediction in a zero-sum game and claimed that in the finitely repeated game the equilibrium prediction is confirmed [14]. The claim was challenged by Brown and Rosenthal (1990) who observed that the data at the individual level reveals significant serial correlations in each player's choices [15]. Cooper *et al.* (1993) have shown that in coordination games players tend to coordinate on the equilibrium preferred by the first mover [16]. Weber *et al.* (2004) test the notion of "virtual observability," according to which players behave as if moves were observable, and hence, tend to coordinate on Nash equilibrium, which is also the subgame-perfect equilibrium of the sequential game with observable actions. This game has a unique Nash equilibrium and thus, the effect of timing in our experiment is completely different than its effect in those previous studies [17]. These studies have important academic value to build effective the benefit distribution mechanisms of Tri-networks Convergence chain. However, these studies with Shapley Value considered how to allocate the revenue only. They didn't consider how to get revenue.

We analyze characteristic functions of the Tri-networks Convergence chain interactions firm with the Maximin and the Minimax. We research on benefit distribution of Tri-networks Convergence chain based on Shapley Value. The contribution of our approach is twofold: first, a mathematical component, where we show how to compute the characteristic functions of the model with the Maximin; and second, an economic element, where we reconsider the construction under a cooperative framework and no integration that such a more narrowly focused regulatory policy may have on the future development of Tri-networks Convergence and its alliance revenue function model to study the sources of Tri-networks Convergence revenue chain in this paper. We find that the broadcasting and the telecommunication operators can achieve cooperation and turn a win-lose situation into a win-win situation. It can reduce costs and increase revenues for the whole Tri-networks Convergence chain.

This paper includes four parts as follows. In first part we have computed the characteristic function for all the coalitions of the Tri-networks Convergence chain using the Maximin and the Minimax Theorem. The second part is divided into two subsections. In Section 2.1, we will define the term "cooperative game theory", "the solution of Core", and "Shapley value". In Section 2.2, the Maximin and the Minimax Theorem are applied to compute the characteristic functions. Section 3 is the main part of this paper. We introduce our approach of a combinatorial calculation benefit distribute of the set of minimal winning coalitions of the Tri-networks Convergence chain based on Shapley value. Section 4 contains concluding remarks.

2. Analyze on the Characteristic Functions of Tri-Networks Convergence Chain

2.1. Basic Definitions on the Cooperative Game Theory, Core and Shapley Value

Cooperative game theory is one of the two counterparts of game theory. It studies the interactions among coalitions of players. There are its main questions. Given the sets of feasible payoffs for each coalition, what pay- off will be awarded to each player? One can take a positive or normative approach to answering this question, and different solution concepts in the theory lean towards one or the other [18].

Core is a solution concept that assigns the set of payoffs, which no coalition can improve upon or block, to each cooperative game. In a context in which there is unfettered coalitional interaction, the core arises as a good positive answer to the question posed in cooperative game theory. In other words, if a payoff does not belong to the core, one should not expect to see it as the prediction of the theory [19-27].

Shapley value is a solution that prescribes a single payoff for each player, which is the average of all marginal contributions of that player to each coalition he or she is a member of. It is usually viewed as a good normative answer to the question posed in cooperative game theory [28-31]. That is, we should pay more for those who contribute more to the groups.

The actors in cooperative game theory are coalitions, that is, groups of players. For the most part, two facts, that a coalition has formed and that it has a feasible set of payoffs available to its members, are taken as given. Given the coalitions and their sets of feasible payoffs as primitives, the question tackled is the identification of final payoffs awarded to each player. That is, given a collection of feasible sets of payoffs, one for each coalition, can one predict or recommend a payoff (or set of payoffs) to be awarded to each player? Such predictions or recommendations are embodied in different solution concepts.

2.2. The Computation for Characteristic Function

The telecommunication operators buys q_1 and the Internet operators buys q_2, so that the broadcasting supplier the media and program resources traffic is $q = q_1 + q_2$ ($q_1, q_2 \rhd 0$). For simplicity, we assume linear demand functions of the telecommunication operators and the Internet operators is equal to $q_1 = f_1(p_1) = a - bp_1$, and $q_2 = f_2(p_2) = c - dp_2$ .The telecommunication operator expected benefits is $\Pi_1 = p_1 f(p_1) = (a - bp_1)p_1$, and the Internet operator's expected benefits is $\Pi_2 = p_2 f(p_2) = (c - dp_2)p_2$.

The broadcasting supplier has to supply the media and program resources quantities at procurement cost $q_1 c_{B1} + c_1 + q_2 c_{B2} + c_2$, and the broadcasting's expected benefits as following [20]:

$$\Pi_3 = q_1 w_1 + q_2 w_2 - (q_1 c_{B1} + c_1 + q_2 c_{B2} + c_2) \quad (1)$$

where c_{B1}, c_{B2} are the variable costs; c_1, c_2 are the fixed cost to supply the media and program resources traffic q. The broadcasting supplier to supply telecommunication and Internet operators at prices

$$w_1, w_2 \quad (w_1, w_2 \geq 0, w_1 \in \left[0, \bar{w}_1\right], w_2 \in \left[0, \bar{w}_2\right])$$

The telecommunication and Internet operators buy the traffic of the media and program resources at prices w_1, w_2 and sell them at p_1, p_2, respectively. The prices p_1, p_2 are positive and $p_1 \in \left[0, \bar{p}_1\right]$, $p_2 \in \left[0, \bar{p}_2\right]$ and they are decided by each firm.

The maxi-min strategy is a strategy to be adopted in game theory, where the player follows the policy which gives the best result of all the bad results possible, i.e. the maximum of the minimum. Potential-maximizing strategy profiles are pure-strategy equilibria, but the converse is not necessarily true (Monderer and Shapley, 1996).

The maxi-min strategy is a strategy to be adopted in game theory, the set of strategies $s_{-i}^{i^*}$ is a set of $(n-1)$ mini-max strategies chosen by all the players except i to keep i's payoff as low as possible, no matter how he responds. $s_{-i}^{i^*}$ Solves as follows:

$$\underset{s-i}{\text{Minimize}}\ \underset{s_i-i}{\text{Maximum}}\, \pi_i(s_i, s_{-i}) \quad (2)$$

Player i's mini-max payoff, mini-max value, or security value is his payoff from the solution of (2).

Shapley (1953) is interested in solving in a fair and unique way the problem of distribution of surplus among the players, when taking into account the worth of each coalition. To do this, he restricts attention to single-valued solutions and resorts to the axiomatic method. He proposes the following axioms on a single-valued solution: 1) Efficiency: The payoffs must add up to v (N), which means that the entire grand coalition surplus is allocated; 2) Symmetry: If two players are substitutes because they contribute the same to each coalition, the solution should treat them equally; 3) Additivity: The solution to the sum of two TU games must be the sum of what it awards to each of the two games; 4) Dummy player: If a player contributes nothing to every coalition, the solution should pay him nothing.

The computation for the characteristic function for the telecommunication operators becomes following [19-27]:

$$\begin{aligned} v(1) &= \max_{p_1} \min_{p_2,w_1,w_2} \Pi_1(p_1,p_2,w_1,w_2) \\ &= \max_{p_1} \min_{p_2,w_1,w_2} \left[p_1(a-bp_1) - w_1(a-bp_1) \right] \end{aligned} \quad (3)$$

We must find the minimum values of p_2, w_1, w_2 that minimize this function for a given p_1, and then we will maximize that function with respect to p_1. In our context, the minimum value of w_1 that minimizes this function given p_1 is $p_1 w_1 = \frac{a}{b}$, so we replace this value in the function and look for p_1 maximizes the function.

$$\begin{aligned} v(1) &= \max_{p_1} \left[p_1(a-bp_1) - \frac{a}{b}(a-bp_1) \right] \\ &= \max_{p_1} \Pi_1(p_1,p_2,w_1,w_2) \end{aligned}$$

$$\frac{\partial \Pi_1}{\partial p_1} = a - 2bp_1 + a = 0$$

To ensure the existence of a maximum, we check the second derivative $\frac{\partial^2 \Pi_1}{\partial p_1{}^2} = -2b$. It is negative and, therefore, we have got a maximum. Thus, $p_1 = \frac{a}{b}$ and $q_1 = 0$, and replacing these values in the function (3) the characteristic function for the telecommunication operators becomes:

$$v(\{1\}) = v(1) = 0 \quad (4)$$

This is the payoff that the telecommunication operators will have in his worst scenario. It means that if the broadcasting charges him the highest price $w_1 = \frac{a}{b}$, which is the highest cost that the telecommunication operators can face, the telecommunication operator best response is to set the highest price of $p_1 = \frac{a}{b}$ where the demand quantity is equal to 0.

The steps to compute the characteristic function to the Internet operators are similar to the ones already computed for telecommunication operators. We proceed to simply write down the results and present the equation

$$\begin{aligned} v(2) &= \max_{p_2} \min_{p_1,w_1,w_2} \Pi_2(p_1,p_2,w_1,w_2) \\ &= \max_{p_2} \min_{p_1,w_1,w_2} \left[p_2(c-dp_2) - w_2(c-dp_2) \right] \end{aligned} \quad (5)$$

where $p_2 = \frac{c}{d}$ and $q_2 = 0$. The characteristic function of Internet operators will receive in case that the other Internet network join themselves against him becomes:

$$v_2(\{2\}) = v(2) = 0 \quad (6)$$

Equations (4) and (6), the characteristic functions for the telecommunication operators and the Internet operators respectively, indicate the lowest value that they are able to get under the worst scenario. Similarly, for the broadcasting supplier, the characteristic function becomes:

$$\begin{aligned} v(3) &= \max_{w_1,w_2} \min_{p_1,p_2} \Pi_3(p_1,p_2,w_1,w_2) \\ &= \max_{w_1,w_2} \min_{p_1,p_2} \begin{bmatrix} w_1(a-bp_1) + w_2(c-dp_2) \\ -c_{B1}(a-bp_1) - c_{B2}(c-dp_2) \end{bmatrix} \end{aligned} \quad (7)$$

We have to look for the minimum values of p_1, p_2 that minimize the function. These values are:

$$v(3) = \max_{w_1,w_2} \left[(w_1 - c_{B1})\left(a - b\frac{a}{b}\right) + (w_2 - c_{B2})\left(c - c\frac{d}{c}\right) \right]$$

$$v(3) = \max_{w_1,w_2} [0] \quad (8)$$

The characteristic function for the broadcasting supplier is:

$$v(\{3\}) = v(3) = 0 \quad (9)$$

The Equation (9) tells us the value that the broadcasting supplier can get in the worst case. We can imagine that this is the case when the telecommunication and Internet operators join together and argue that the quality is not expected and they do not want the TV signal. The broadcasting supplier payoff is zero because the broadcasting supplier cannot force the telecommunication and Internet operator to buy the TV signal.

The characteristic function for the coalition $v(1,2)$ is obtained considering the sum of $\Pi_1 + \Pi_2$ because the coalition $\{1,2\}$ gets the payoff of both players together.

$$\begin{aligned} v(1,2) &= \max_{p_1,p_2} \min_{w_1,w_2} \left[\Pi_1(p_1,p_2,w_1,w_2) + \Pi_2(p_1,p_2,w_1,w_2) \right] \\ &= \max_{p_1,p_2} \min_{w_1,w_2} \begin{bmatrix} p_1(a-bp_1) - w_1(a-bp_1) \\ +p_2(c-dp_2) - w_2(c-dp_2) \end{bmatrix} \end{aligned} \quad (10)$$

Looking for the minimum values of w_1, w_2 that minimize the function and replacing them, we obtain:

$$v(1,2)=\max_{p_1,p_2}\begin{bmatrix} p_1(a-bp_1)-\frac{a}{b}(a-bp_1) \\ +p_2(c-dp_2)-\frac{c}{d}(c-dp_2) \end{bmatrix} = \max_{p_1,p_2}[\Pi_1+\Pi_2] \quad (11)$$

We have to look for the values of p_1, p_2 that make the function maximum. Thus, we derivate with respect to p_1 *and* p_2. The first derivatives must be equal to 0 and the second ones must be negative to ensure the presence of a maximum.

$$\frac{\partial[\Pi_1+\Pi_2]}{\partial p_1}=2a-2bp_1=0$$

$$\frac{\partial^2[\Pi_1+\Pi_2]}{\partial p_1^{\,2}}=-2b$$

$$\frac{\partial[\Pi_1+\Pi_2]}{\partial p_2}=2c-2dp_2=0$$

$$\frac{\partial^2[\Pi_1+\Pi_2]}{\partial p_2^{\,2}}=-2d$$

These values are $p_1=\frac{a}{b}$, $q_1=0$; $p_2=\frac{c}{d}$, $q_2=0$;

Replacing in Equation (26), we get the characteristic function

$$v(\{1,2\})=v(1,2)=0 \quad (12)$$

This is the payoff that telecommunication and Internet operators will get when the coalition is between the telecommunication and Internet operators parties and in the worst condition that the broadcasting supplier against them. The broadcasting supplier will charge them the highest price and their response will be to put the highest prices and they will sell 0. The benefits they get are equal to zero in the worst condition. This equation points out the fact that if the telecommunication and Internet operators cooperate with each other, without any consideration of the third player (the broadcasting supplier), they will be able to get 0. They cannot force the telecommunication and Internet operators to sell them the TV signal. The characteristic function for the coalition $v(1,3)$ is:

$$v(1,3)=\max_{p_1,w_1,w_2}\min_{p_2}\begin{bmatrix} \Pi_1(p_1,p_2,w_1,w_2) \\ +\Pi_3(p_1,p_2,w_1,w_2) \end{bmatrix} = \max_{p_1,w_1,w_2}\min_{p_2}\begin{bmatrix} p_1(a-bp_1)+w_2(c-dp_2) \\ -c_{B1}(a-bp_1)-c_{B2}(c-dp_2) \end{bmatrix} \quad (13)$$

Looking for the minimum value of p_2 that minimizes the function, we get:

$$v(1,3)=\max_{p_1,w_1,w_2}\begin{bmatrix} p_1(a-bp_1)+w_2\left(c-d\frac{c}{d}\right) \\ -c_{B1}(a-bp_1)-c_{B2}\left(c-d\frac{c}{d}\right) \end{bmatrix}$$

$$v(1,3)=\max_{p_1,w_1,w_2}[p_1(a-bp_1)-c_{B1}(a-bp_1)] = \max_{p_1,w_1,w_2}[\Pi_1+\Pi_3] \quad (14)$$

Now, we have to find the value of p_1 maximizes from (29)

$$\frac{\partial[\Pi_1+\Pi_3]}{\partial p_1}=a-2bp_1+c_{B1}b=0$$

$$\frac{\partial^2[\Pi_1+\Pi_3]}{\partial p_1^{\,2}}=-2b$$

$$p_1=\frac{a+c_{B1}b}{2b},\quad q_1=\frac{a-c_{B1}b}{2}$$

Replacing these values in the Equation (30), we get the characteristic function:

$$v(\{1,3\})=v(1,3)=\frac{(a-c_{B1}b)^2}{4b} \quad (15)$$

This is the payoff that the coalition between the telecommunication operator and the broadcasting supplier can get if the Internet operators are assumed to oppose them. The broadcasting supplier will supply $q_1=\frac{a-c_{B1}b}{2}$ and the telecommunication operators will sell it at the price $p_1=\frac{a+c_{B1}b}{2b}$ supply characteristic function implies that, under any circumstances, the telecommunication operators and the broadcasting supplier, together, are sure to obtain the least amount given by this equation. This result is the same as if the broadcasting supplier and the telecommunication operators were vertically integrated. It is necessary to note that this coalition makes sense if the investments are specific because, if the investments were general, the telecommunication operators neither need to form coalitions nor to be vertically integrated.

Doing the same for the coalition between the Internet operators and the broadcasting supplier, we get:

$$v(2,3)=\max_{p_1,w_1,w_2}\min_{p_1}\begin{bmatrix} \Pi_2(p_1,p_2,w_1,w_2) \\ +\Pi_3(p_1,p_2,w_1,w_2) \end{bmatrix} = \max_{p_1,w_1,w_2}\min_{p_1}\begin{bmatrix} p_2(c-dp_2)+w_1(a-bp_1) \\ -c_{B1}(a-bp_1)-c_{B2}(c-dp_2) \end{bmatrix} \quad (16)$$

Looking for the minimum value of p_1 that minimizes the function, we get:

$$v(2,3)=\max_{p_2,w_1,w_2}\begin{bmatrix} p_2(c-dp_2)+w_1\left(a-b\frac{a}{b}\right) \\ -c_{B1}\left(a-b\frac{a}{b}\right)-c_{B2}(c-dp_2)\end{bmatrix} \quad (17)$$

$$\begin{aligned} v(2,3)&=\max_{p_2,w_1,w_2}\left[p_2(c-dp_2)-c_{B2}(c-dp_2)\right] \\ &=\max_{p_2,w_1,w_2}\left[\Pi_2+\Pi_3\right]\end{aligned} \quad (18)$$

Now, we have to find the value of p_2 that maximizes (18):

$$\frac{\partial[\Pi_2+\Pi_3]}{\partial p_2}=c-2dp_2+c_{B2}d=0 \quad \frac{\partial^2[\Pi_2+\Pi_3]}{\partial {p_2}^2}=-2d$$

$$p_2=\frac{c+c_{B2}d}{2d},\ q_2=\frac{c-c_{B2}d}{2}$$

Replacing these values in the Equation (13), we get the characteristic function

$$v(\{2,3\})=v(2,3)=\frac{(c-c_{B2}d)^2}{4d} \quad (19)$$

This is the payoff that the coalition between the telecommunication operators and the broadcasting supplier can get if the telecommunication operator acts against them. The broadcasting supplier will supply $q_2=\frac{c-c_{B2}d}{2}$ and the Internet operators will sell it at the price $p_2=\frac{c+c_{B2}d}{2d}$. The characteristic function implies that, under any circumstances, the Internet operators and the broadcasting supplier, together, are sure to obtain the least amount given by this equation.

As in the previous case, this result is the same as if the broadcasting supplier and the Internet operators were vertically integrated. The characteristic function for the coalition $v(1,2,3)$ is given by:

$$\begin{aligned} v(1,2,3)&=\max_{p_1,p_2,w_1,w_2}\begin{bmatrix}\Pi_1(p_1,p_2,w_1,w_2) \\ +\Pi_2(p_1,p_2,w_1,w_2) \\ +\Pi_3(p_1,p_2,w_1,w_2)\end{bmatrix} \\ &=\max_{p_1,p_2,w_1,w_2}\begin{bmatrix} p_1(\beta_1-\gamma_1p_1)+p_2(\beta_2-\gamma_2p_2) \\ -\lambda c_{s1}(\beta_1-\gamma_1p_1)-\lambda c_{s2}(\beta_2-\gamma_2p_2)\end{bmatrix}\end{aligned} \quad (20)$$

And now we must find the values p_1, p_2 that maximize the function.

$$\frac{\partial[\Pi_1+\Pi_2+\Pi_3]}{\partial p_1}=a-2bp_1+c_{B1}b=0$$

$$\frac{\partial^2[\Pi_1+\Pi_2+\Pi_3]}{\partial {p_1}^2}=-2b$$

$$p_1=\frac{a+c_{B1}b}{2b},\ q_1=\frac{a-c_{B1}b}{2}$$

$$\frac{\partial}{\partial p_2}=c-2dp_2+c_{B2}d=0,\ \frac{\partial^2}{\partial {p_2}^2}=-2d$$

$$p_2=\frac{c+c_{B2}d}{2d},\ q_2=\frac{c-c_{B2}d}{2}$$

Replacing these values in the equation we get that:

$$v(\{1,2,3\})=v(1,2,3)=\frac{(a-c_{B1}b)^2}{4b}+\frac{(c-c_{B2}d)^2}{4d} \quad (21)$$

This expression is the characteristic function of the grand coalition. This is the maximum payoff that the grand coalition or total coalition can achieve if they decide to cooperate channel with each other.

3. The Calculation Benefit Distribute of the Tri-Networks Convergence Chain

A solution concept for the game under consideration is the Shapley value (Peleg and Sudhölter, 2003) and (Branzei and Tijs, 2005). Shapley (1953) looked at what each player could reasonably get before the game has begun. He put three axioms, which he called $\varphi_i(v)$, player *i*'s expectation in a game with a characteristic function *v*, should satisfy the following:

S1: $\varphi_i(v)$ is independent of the labeling of the players. If π is a permutation of 1, 2. n and πv is the characteristic function of the game, with the players numbers permuted by π, then $\varphi_{\pi(i)}(\pi v)=\varphi_i(v)$.

S2: The sum of the expectations should equal the maximum available from the game, so:

$$\sum_{i=1}^{n}\varphi_i(v)=v(N)$$

S3: If u, v are the characteristic functions of two games, $u+v$ is the characteristic function of the game playing both games together. φ Which satisfy $\varphi_i(u+v)=\varphi_i(u)+\varphi_i(v)$. The benefit distributes method of Tri-networks Convergence based on the Shapley value as follows:

The revenue value of the grand coalition broken into simple linear combination:

$$v=a_1v_1+a_2v_2+a_3v_3+a_{12}v_{12}+a_{13}v_{13}+a_{23}v_{23}+a_{123}v_{123} \quad (22)$$

Solving the Equation (22), we get

$$\begin{cases} a_1 = v_1, \quad a_2 = v_2, \quad a_3 = v_3 \\ a_{12} = v_{12} - \left(v_1 + v_2\right) \\ a_{13} = v_{13} - \left(v_1 + v_3\right) \\ a_{23} = v_{23} - \left(v_2 + v_3\right) \\ a_{123} = v_{123} - (v_{12} + v_{13} + v_{23}) + (v_1 + v_2 + v_3) \end{cases} \tag{23}$$

where, a_1, a_2, a_3 respective the benefits of the telecommunication operators, Internet operators and broadcasting supplier; a_{12}, a_{13}, a_{23} is the revenue value of the grand coalition that the telecommunication operators, Internet operators and broadcasting supplier, a_{123} is the revenue value of Tri-networks Convergence. In the process of benefit distribution, if the weight establishment of two coalitions is 1/2, and the weight of Tri-networks Convergence is 1/3, we have:

$$\begin{aligned} \phi_1(v) &= a_1 + \frac{1}{2}(a_{12} + a_{13}) + \frac{1}{3}a_{123} \\ \phi_2(v) &= a_2 + \frac{1}{2}(a_{12} + a_{23}) + \frac{1}{3}a_{123} \\ \phi_3(v) &= a_3 + \frac{1}{2}(a_{13} + a_{23}) + \frac{1}{3}a_{123} \end{aligned} \tag{24}$$

We have verify that $V_{123} = \phi_1(V) + \phi_2(V) + \phi_3(V)$, Replacing the coefficients of the solution in the Equation (24) we get:

$$\begin{aligned} \phi_1(v) &= \frac{1}{3}(\mathrm{v}_1 + \mathrm{v}_{123} - v_{23}) + \frac{1}{6}(v_{12} + v_{13} - v_2 - v_3) \\ &= \frac{1}{2}\frac{\left(a - c_{B1}b\right)^2}{4b} \\ \phi_2(v) &= \frac{1}{3}(v_2 + v_{123} - v_{13}) + \frac{1}{6}(v_{12} + v_{23} - v_1 - v_3) \\ &= \frac{1}{2}\frac{\left(c - c_{B2}d\right)^2}{4d} \\ \phi_3(v) &= \frac{1}{3}(v_3 + v_{123} - v_{12}) + \frac{1}{6}(v_{13} + v_{23} - v_1 - v_2) \\ &= \frac{1}{2}\left(\frac{\left(a - c_{B1}b\right)^2}{4b} + \frac{\left(c - c_{B2}d\right)^2}{4d}\right) \end{aligned} \tag{25}$$

Simplified formula (25) was:

$$\begin{aligned} \phi_1(v) &= \frac{1}{2}\frac{\left(a - c_{B1}b\right)^2}{4b} \\ \phi_2(v) &= \frac{1}{2}\frac{\left(c - c_{B2}d\right)^2}{4d} \\ \phi_3(v) &= \frac{1}{2}\left(\frac{\left(a - c_{B1}b\right)^2}{4b} + \frac{\left(c - c_{B2}d\right)^2}{4d}\right) \end{aligned} \tag{26}$$

where, $\varphi_1(\mathrm{v}), \varphi_2(\mathrm{v}), \varphi_3(\mathrm{v})$ respective the benefit distribution value of the telecommunication operators, Internet operators and broadcasting supplier.

4. Conclusions

The Shapley value (1953), or simply the value, is one of the most successful solution concepts for cooperative games. Shapley constructed the value axiomatically; in such a way that each axiom justifies that the value provides an evaluation of the player's possibilities by playing the game.

We show that using a biform game model with three agents to analyze cooperative channel the telecommunication and Internet operators will invest in channel and the broadcasting supplier will share the cost of it. The broadcasting supplier uses his major bargaining power through the sharing costs in the cooperative channel to motivate sales at the telecommunication and Internet operators level and increase the total benefits for all the players, achieving the first best. So, our result is the same as the fully cooperative case in the cooperative game theory although the telecommunication and Internet operators have more bargaining power. The contribution of our approach is twofold: first, a mathematical component, where we show how to apply the maxi-min to compute the characteristic functions for all the coalitions of the Tri-networks Convergence chain, and second, an economic element, where we reconsider some of the effects relationship under a cooperative framework and no integration that such a more narrowly focused regulatory policy may have on the future development of Tri-networks Convergence.

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