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Heat Transfer with Viscous Dissipation in Couette-Poiseuille Flow under Asymmetric Wall Heat Fluxes

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ABSTRACT

Analytical solutions of temperature distributions and the Nusselt numbers in forced convection are reported for flow through infinitely long parallel plates, where the upper plate moves in the flow direction with constant velocity and the lower plate is kept stationary. The flow is assumed to be laminar, both hydro-dynamically and thermally fully developed, taking into account the effect of viscous dissipation of the flowing fluid. Both the plates being kept at specified and at different constant heat fluxes are considered as thermal boundary conditions. The solutions obtained from energy equation are in terms of Brinkman number, dimensionless velocity and heat flux ratio. These parameters greatly influence and give complete understanding on heat transfer rates that has potentials for designing and analyzing energy equipment and processes.

Keywords: Viscous Dissipation; Couette-Poiseuille Flow; Newtonian Fluid; Nusselt Number; Brinkman Number; Constant Heat-Flux

1. Introduction

Flow of Newtonian fluids through various channels is of practical importance and heat transfer is dependent on flow conditions such as flow geometry and physical properties. Investigations in heat transfer behavior through various channels showed that the effect of viscous dissipation cannot be neglected for some applications, such as flow through micro-channels, small conduits and extrusion at high speeds. The thermal development of forced convection through infinitely long fixed parallel plates, both plates having specified constant heat flux had been investigated [1-5]. For the same but filled by a saturated porous medium, heat transfer analysis was done where the walls were kept at uniform wall temperature with the effect of viscous dissipation and axial conduction taken into account [6]. In [7], it was concluded that in a porous medium, the absence of viscous dissipation effect can have great impact. For the horizontal double passage channel, uniform wall temperature with asymmetric and symmetric heating and the effect of viscous dissipation had been investigated [8].

For the pipe flow, where the walls are kept either at constant heat flux or constant wall temperature, analytical solution is obtained for both hydro-dynamically and thermally fully developed and thermally developing Newtonian fluid flow, considering the effect of viscous dissipation [9,10].

Analytical solution with the effect of viscous dissipation was derived for Couette-Poiseuille flow of nonlinear visco-elastic fluids and with the simplified Phan-Thien-Tanner fluid between parallel plates, with stationary plate subjected to constant heat flux and the other plate moving with constant velocity but insulated [11-13]. Numerical solution of fully developed laminar heat transfer of power-law non-Newtonian fluids in plane Couette flow, with constant heat flux at one wall with other wall insulated had been investigated [14] and analytical solution was derived for Newtonian fluid [15].

A numerical investigation had been done to find the heat transfer for the simultaneously developing steady laminar flow, where the fluid was considered to be viscous non-Newtonian described by a power-law model flowing between two parallel plates with several different thermal boundary conditions [16]. When a thin slab was symmetrically heated on both sides, the hyperbolic heat conduction equation was solved analytically [17]. Considering the effect of viscous dissipation and pressure stress work of the fluid, the steady laminar boundary

layer flow along a vertical stationary isothermal plate was studied. The variation of wall heat transfer and wall shear stress along the plate was discussed [18].

The Bingham fluid was assumed to be flowing in-between two porous parallel plates. With the slip effect at the porous walls, the analytical solutions were obtained for the Couette-Poiseuille flow [19]. Numerical evaluation for developing temperature profiles by a finite-difference method were carried out for non-Newtonian fluid through parallel plates and circular ducts. The effects of viscous dissipation and axial heat conduction were taken into account. Graphical representation of Nusselt numbers were noted for various parameters [20]. The thermal entrance region of a horizontal parallel plate channel, where the lower plate was heated isothermally and the upper plate was cooled isothermally was considered. Numerical results were found on the onset of instability for longitudinal vortices, with effect of viscous dissipation [21]. A numerical analysis was carried out, taking viscous dissipation into account for pseudo-plastic non-Newtonian fluids aligned with a semi-infinite plate [22].

From the literature survey, it is observed that heat transfer analysis with effect of viscous dissipation is not found for the Couette-Poiseuille flow with both the plates being kept at specified but different constant heat fluxes. The heat transfer analysis with one plate moving is a different fundamental problem worth pursuing. This study is necessary specifically in the design of special heat exchangers and other devices where the dimensions have to be kept very small. Hence, the case of lower plate being fixed and the upper plate moving with constant velocity, both being imposed to different but constant heat fluxes is considered. The energy equation is solved leading to expressions in temperature profiles and Nusselt number, that could be useful to industrial applications.

2. Statement of Problem and Mathematical Formulation

Consider two flat infinitely long parallel plates distanced W or 2 apart, where the upper plate is moving with constant velocity U and the lower plate is fixed. The coordinate system chosen is shown in **Figure 1**. The flow through the plates is considered at a sufficient distance from the entrance such that it is both hydro-dynamically and thermally fully developed. The axial heat conduction in the fluid and through the wall is assumed to be negligible. The fluid is assumed to be Newtonian and with constant properties. The thermal boundary conditions are the upper plate is kept at constant heat flux while the lower plate at different constant heat flux.
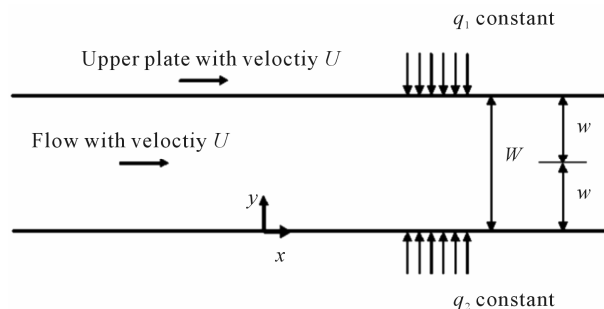
The momentum equation in the x-direction is described as

Figure 1. Notation to the problem.

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} = \frac{1}{\mu}\frac{\mathrm{d}P}{\mathrm{d}x}, \tag{1}$$

where u is the velocity of the fluid, μ is the dynamic viscosity, P is the pressure.

The velocity boundary conditions are $u = 0$ when $y = 0$ and $u = U$ when $y = W$.

Using the following dimensionless parameters:

$$u^* = u/u_m,\ U^* = U/u_m, \\ Y = y/W, \tag{2}$$

the well-known velocity-distribution is [15],

$$u^* = \left(3U^* - 6\right)\left(Y^2 - Y\right) + U^* Y, \tag{3}$$

where the mean velocity (u_m) is given by

$$u_m = \frac{1}{W}\int_0^W u\,\mathrm{d}y. \tag{4}$$

For the above equation, expression for u is obtained by solving the momentum Equation (1).

The energy equation, including the effect of viscous dissipation, is given by

$$u\frac{\partial T}{\partial x} = \frac{\gamma}{\Pr}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{5}$$

where the second term on the right-hand side is the viscous-dissipative term. In accordance to the assumption of a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is neglected in the energy equation [23]. Following this, the temperature gradient along the axial direction is independent of the transverse direction and given as

$$\frac{\partial T}{\partial x} = \frac{\mathrm{d}T_1}{\mathrm{d}x} = \frac{\mathrm{d}T_2}{\mathrm{d}x}, \tag{6}$$

where T_1 and T_2 are the upper and lower wall temperatures, respectively.

By taking $\alpha = k/\rho c_p$, introducing the non-dimensional quantity

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$$\theta = \frac{T - T_1}{q_1 W / k}, \tag{7}$$

and defining a dimensionless constant β,

$$\beta = \frac{Pr u_m kW}{\gamma q_1} \frac{dT_1}{dx}, \tag{8}$$

and modified Brinkman number Br_{q_1} as

$$Br_{q_1} = \frac{\mu u_m^2}{2W q_1}, \tag{9}$$

Equation (5) can be written as

$$\frac{d^2\theta}{dY^2} = \beta\left[\left(3U^* - 6\right)\left(Y^2 - Y\right) + U^* Y\right] \\ - 2Br_{q_1}\left[\left(3U^* - 6\right)\left(2Y - 1\right) + U^*\right]^2 \tag{10}$$

The thermal boundary conditions are

$$k\frac{\partial T}{\partial y} = q_1 \text{ at } y = W, \text{ or } \frac{\partial \theta}{\partial Y} = 1 \text{ at } Y = 1, \\ T = T_1 \text{ at } y = W, \text{ or } \theta = 0 \text{ at } Y = 1. \tag{11}$$

The solution of Equation (10) under the above thermal boundary conditions can be obtained as

$$\theta(Y) = \left(\frac{1}{4}\beta U^* - \frac{1}{2}\beta - 6U^{*2}Br_{q_1} + 24U^*Br_{q_1} - 24Br_{q_1}\right)Y^4 + \left(\frac{-1}{3}\beta U^* + \beta + 8U^{*2}Br_{q_1} - 40U^*Br_{q_1} + 48Br_{q_1}\right)Y^3 \\ + \left(-4U^{*2}Br_{q_1} + 24U^*Br_{q_1} - 36Br_{q_1}\right)Y^2 + \left(1 - \beta + 8U^{*2}Br_{q_1} - 24U^*Br_{q_1} + 24Br_{q_1}\right)Y \\ + \frac{1}{12}U^*\beta + \frac{1}{2}\beta - 6U^{*2}Br_{q_1} + 16U^*Br_{q_1} - 12Br_{q_1} - 1. \tag{12}$$

To evaluate β in the above equation, a third boundary condition is required:

$$-k\frac{\partial T}{\partial y} = q_2 \text{ at } y = 0, \text{ or } \frac{\partial \theta}{\partial Y} = -\frac{q_2}{q_1} \text{ at } Y = 0. \tag{13}$$

By substituting Equation (13) into Equation (12), β can be expressed as

$$\beta = 1 + q_2/q_1 + 8U^{*2}Br_{q_1} - 24U^*Br_{q_1} + 24Br_{q_1}. \tag{14}$$

Therefore, the solution of Equation (10) under the above thermal boundary conditions can be written in a simplified form as

$$\theta(Y) = \frac{1}{4}AU^*Y^4 - \frac{1}{3}AU^*Y^3 - \frac{1}{2}AY^4 + AY^3 - 6U^{*2}Br_{q_1}Y^4 + 8U^{*2}Br_{q_1}Y^3 + 24U^*Br_{q_1}Y^4 \\ - 40U^*Br_{q_1}Y^3 - 4U^{*2}Br_{q_1}Y^2 + 24U^*Br_{q_1}Y^2 - 24Br_{q_1}Y^4 + 48Br_{q_1}Y^3 - 36Br_{q_1}Y^2 \\ - \frac{q_2}{q_1}Y + \frac{1}{12}AU^* - \frac{1}{2} + \frac{1}{2}\frac{q_2}{q_1} - 2U^{*2}Br_{q_1} + 4U^*Br_{q_1}, \tag{15}$$

where

$$A = 1 + \frac{q_2}{q_1} + 8U^{*2}Br_{q_1} - 24U^*Br_{q_1} + 24Br_{q_1}. \tag{16}$$

In fully developed flow, it is usual to utilize the mean fluid-temperature, T_m, rather than the centerline temperature, when defining the Nusselt number. Thus mean or bulk temperature is given by

$$T_m = \frac{\int_{A_c} \rho u T dA_c}{\int_{A_c} \rho u dA_c}, \tag{17}$$

with A_c the cross-sectional area of the channel and the denominator on the right-hand side of Equation (17) can be written as

$$\rho\int_0^1\left(\left(3U^* - 6\right)\left(Y^2 - Y\right) + U^*Y\right)dA_c = \rho LW. \tag{18}$$

Using Equations (3) and (15), the numerator of Equation (17) can be found. Therefore the dimensionless mean temperature is given by

$$\theta_m = \frac{k}{q_1 W}\left(T_m - T_1\right). \tag{19}$$

At this point, the convective heat transfer coefficient can be evaluated by the equation

$$q_1 = h\left(T_1 - T_m\right). \tag{20}$$

Defining Nusselt number to be

$$Nu = \frac{hD_h}{k} = \frac{q_1 2W}{k\left(T_1 - T_m\right)} = -\frac{2}{\theta_m}, \tag{21}$$

where D_h is the hydraulic diameter defined by $D_h = 2W$, the expression for Nusselt number can be shown to be

$$Nu = \dfrac{-2}{\begin{aligned}&\Big[\big(3U^* - 6\big)\Big[9Br_{q_1}\big/35 + 5U^{*2}Br_{q_1}\big/21 - 62U^*Br_{q_1}\big/105 - 3(q_2/q_1)\big/140 + 13/210\\ &\quad - (11/1260)\Big[1 + 8U^{*2}Br_{q_1} - 24U^*Br_{q_1} + 24Br_{q_1} + (q_2/q_1)\Big]U^*\Big]\\ &\quad + U^*\Big[(1/60)\Big[1 + 8U^{*2}Br_{q_1} - 24U^*Br_{q_1} + 24Br_{q_1} + (q_2/q_1)\Big]U^*\\ &\quad + 6U^*Br_{q_1}\big/5 - 2/15 - 7U^{*2}Br_{q_1}\big/15 - 3Br_{q_1}\big/5 + (q_2/q_1)\big/30\Big]\Big]\end{aligned}} \tag{22}$$

When $q_2 = 0$,

$$Nu = \dfrac{210}{-522U^*Br_{q_1} - 94U^{*3}Br_{q_1} + 366U^{*2}Br_{q_1} - 11U^* + U^{*2} + 8U^{*4}Br_{q_1} + 162Br_{q_1} + 39}, \tag{23}$$

agreeing with reference [15].

Explicit expressions for Nusselt number for various values of U^*, Br_{q_1} and q_2/q_1 are given in the following discussions.

3. Graphical Results and Discussions

For the purpose of discussion on the behavior of the Couette-Poiseuille flow, two types of graphs based on the analytical solutions are made. The temperature profile in the channel is plotted with variations of various parameters to indicate the heated region, and the Nusselt number is plotted to reveal the heat transfer characteristics of the flow.

3.1. Temperature Profiles against the Channel Width for Various Parameters

3.1.1. Temperature Profiles for the Case of Insulated Lower Plate

Figure 2 shows the dimensionless temperature profiles of θ versus Y, where the lower plate is insulated at five dimensionless velocities $U^* = -1.0,\ -0.5,\ 0.0,\ 0.5$ and 1.0, and at six selected Br_{q_1} values from -0.01 to 0.5, as shown in (a) to (f). The temperature distributions have similar pattern but different shapes, and all the curves converge at $Y = 1$, θ equal to 0, by definition. At $Y = 0$, the curves are vertical to satisfy the insulated condition. As expected, generally the motion of the upper plate tends to impart more heat into the fluid layers that are dragged along, unless off-set by the viscous dissipation effects. It is observed that when $Br_{q_1} = -0.01,\ 0.0,\ 0.01$ and 0.1, the temperature distribution is negative which implies there is decrease in heat transfer, whereas when $Br_{q_1} = -0.1$ and 0.5, θ manifests in a different way such that θ takes both negative and positive values.

3.1.2. Temperature Profiles for a Fixed Brinkman Number for Various Heat Flux Ratios

The effect of viscous dissipation is seen in the value of modified Brinkman number. It is interesting to observe the behavior of the temperature profiles for various heat flux ratios for a fixed modified Brinkman number and hence to note the effect of viscous dissipation. In **Figure 3**, for a Br_{q_1} value of 0.01, the temperature distribution is investigated at $U^* = -1.0,\ -0.5,\ 0.0,\ 0.5$ and 1.0 for various heat flux ratios. When $q_2/q_1 = 0$ and 0.5, the values of theta are all negative. For the equal heat fluxes, for $U^* = -1.0,\ -0.5$ and 0.0, theta takes only negative values, but for $U^* = 0.5$ and 1, theta takes both positive as well as negative values. When $q_2/q_1 = 2.0$ and 10.0, theta takes both positive as well as negative values. For $q_2/q_1 = 50$, when the upper plate moves in the negative direction with values $U^* = -1.0,\ -0.5$, theta takes both positive as well as negative values and when the upper plate is fixed and moves in the positive direction with values $U^* = 0.5$ and 1.0, theta takes positive values. As expected again, all the curves converge at $Y = 1$.

3.2. Nusselt Number Variations

Figure 4 shows the plots of Nusselt number versus the heat flux ratio q_2/q_1 at $U^* = -1.0,\ -0.5,\ 0.0,\ 0.5$ and 1.0 at various Br_{q_1} values. The hyperbolic curves have asymptotes occurring at different q_2/q_1 values. It is observed that, for the specified values of U^*, when $Br_{q_1} = -0.01,\ 0.0,\ 0.01$ and 0.1, the asymptotes fall to the positive direction of q_2/q_1, whereas when $Br_{q_1} = -0.1$ at $U^* = -0.5$, the asymptote falls at $q_2/q_1 = -0.6536$ and when $Br_{q_1} = 0.5$ at $U^* = 1.0$, the asymptote falls at $q_2/q_1 = -0.7458$, as given in **Table 1**.

4. Conclusion

Heat transfer with the effect of viscous dissipation has been analysed. Analytical expressions for Nusselt number have been obtained for fully developed Newtonian fluid flow between infinitely long parallel plates, where the lower plate is fixed and the upper plate is moving with constant velocity. When both plates are kept at
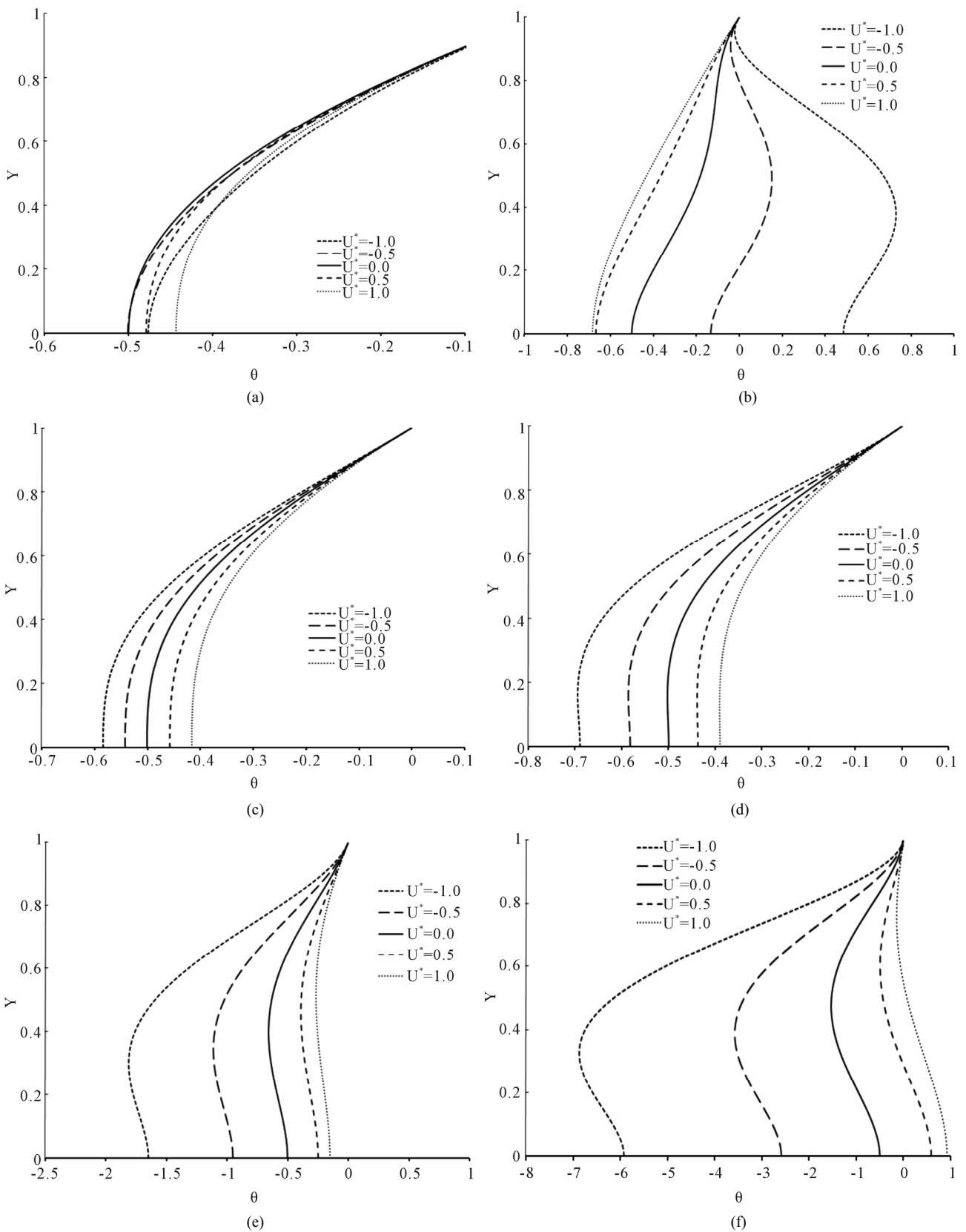
Figure 2. Temperature profiles at $U^* = -1.0, -0.5, 0.0, 0.5$ and 1.0 at various for the case of $q2/q1 = 0$. (a) $Brq1 = -0.01$; (b) $Brq1 = -0.10$; (c) $Brq1 = 0.00$; (d) $Brq1 = 0.01$; (e) $Brq1 = 0.10$; (f) $Brq1 = 0.50$.
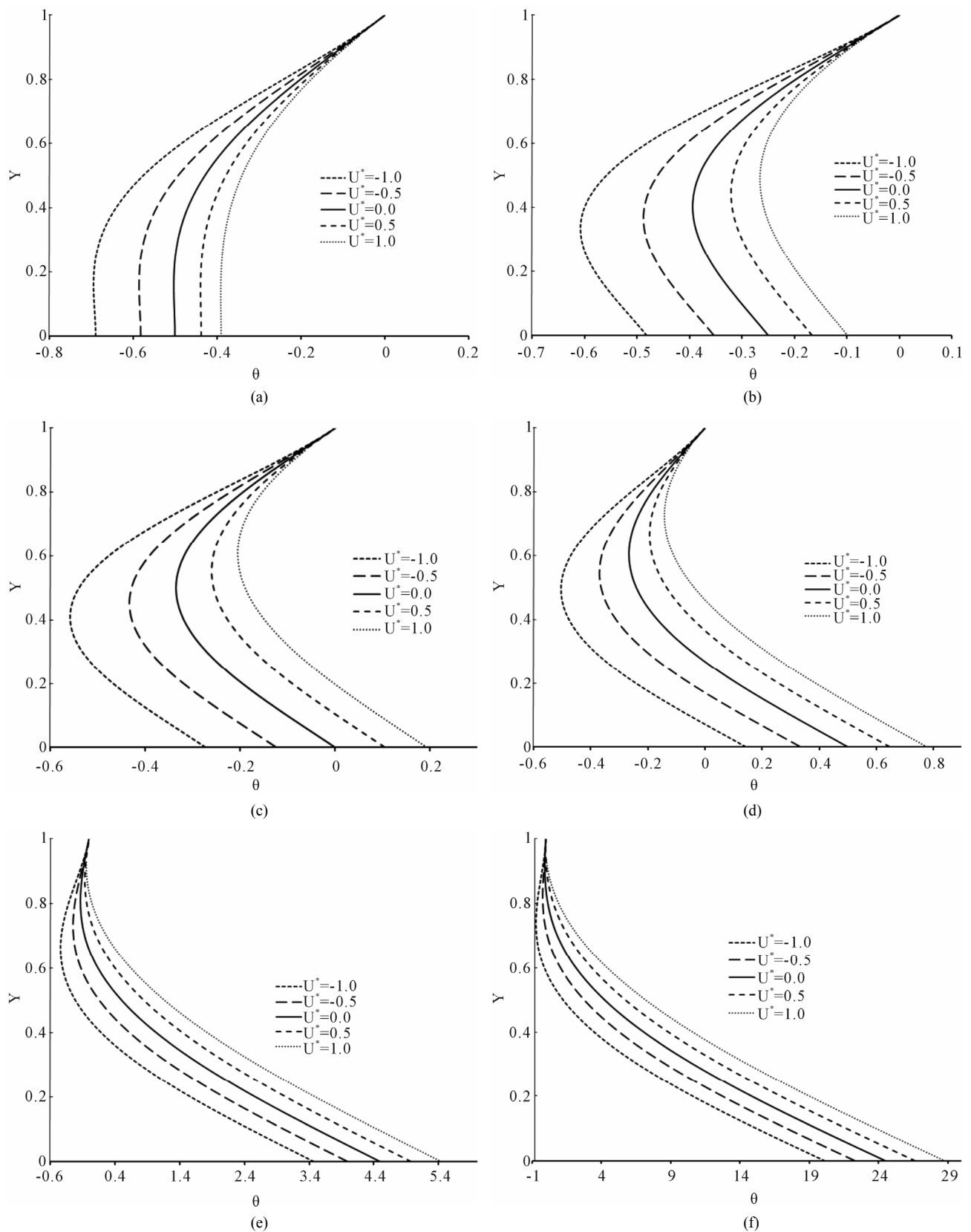
Figure 3. Temperature profiles at $U^* = -1.0, -0.5, 0.0, 0.5$ and 1.0 at various q_2/q_1, for the case of $Br_{q1} = 0.01$. (a) $q_2/q_1 = 0.0$; (b) $q_2/q_1 = 0.50$; (c) $q_2/q_1 = 1.0$; (d) $q_2/q_1 = 2.0$; (e) $q_2/q_1 = 10.0$; (f) $q_2/q_1 = 50.0$.
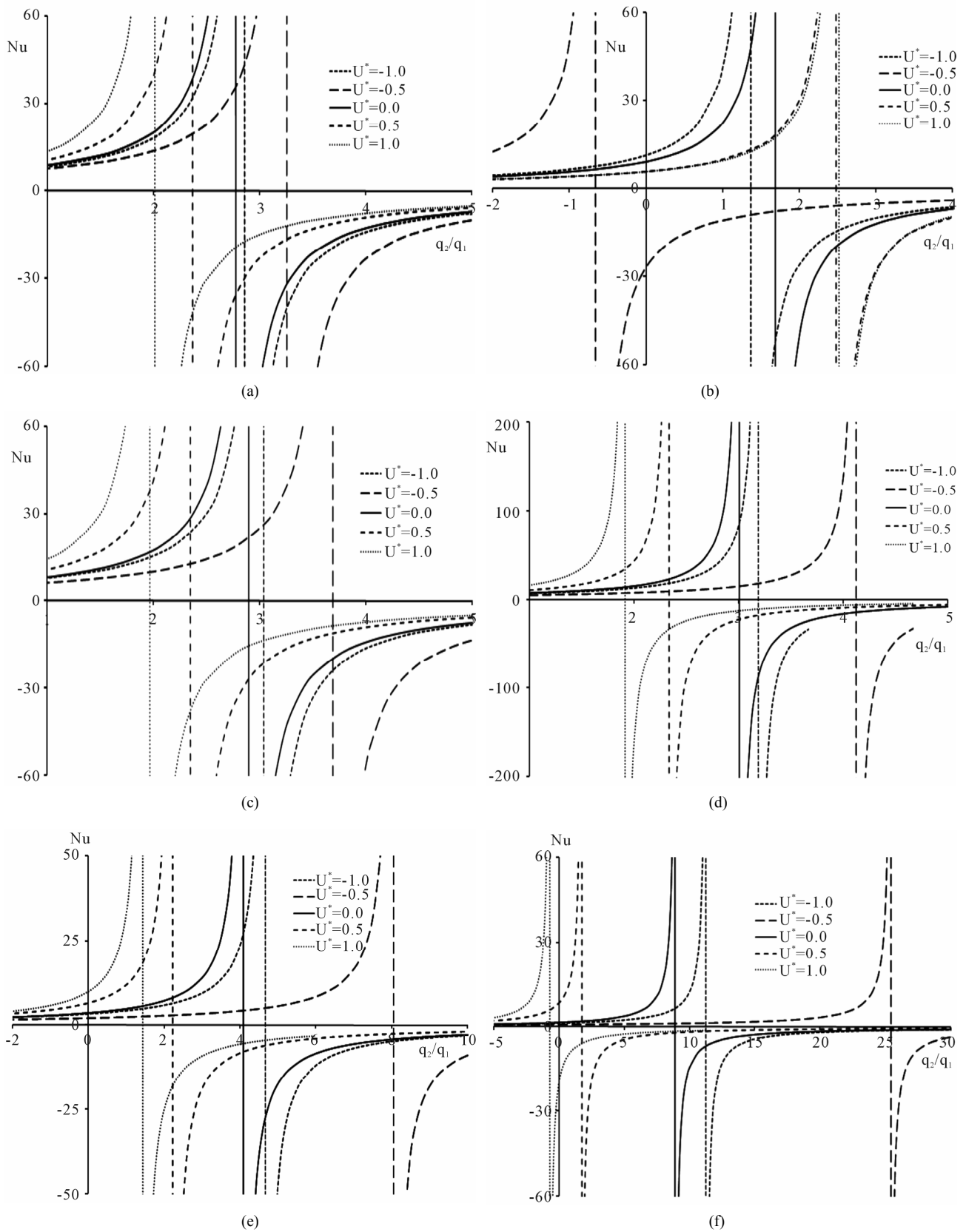
Figure 4. Nusselt number versus q_2/q_1, at $U^* = -1.0$, -0.5, 0.0, 0.5, and 1.0, at various Br_{q1}. Vertical lines are asymptotes. (a) $Br_{q1} = -0.01$; (b) $Br_{q1} = -0.10$; (c) $Br_{q1} = 0.00$; (d) $Br_{q1} = 0.01$; (e) $Br_{q1} = 0.10$; (f) $Br_{q1} = 0.50$.

Table 1. Values of q_2/q_1 at various Br_{q1} and U^*.

U^*	Br_{q1}					
	−0.01	−0.1	0.0	0.01	0.1	0.5
−1.0	2.8594	1.3807	3.0237	3.1881	4.6668	11.2392
−0.5	3.2563	−0.6536	3.6907	4.1252	8.0351	25.4124
0.0	2.7689	1.6889	2.8889	3.0089	4.0889	8.8889
0.5	2.3609	2.4783	2.3478	2.3348	2.2174	1.6957
1.0	2.0203	2.5085	1.9661	1.9119	1.4237	−0.7458

Nusselt number versus constant heat flux ratio.

different constant heat fluxes, the dimensionless temperature distribution is given by Equation (15), and the Nusselt number by Equation (22) and they are in terms of U^*, q_2/q_1 and Various dimensionless velocity values such as −1.0, −0.5, 0.0, 0.5, and 1.0, constant heat flux ratios 0, 0.5, 1.0, 2.0, 10.0 and 50.0 and modified Brinkaman numbers −0.01, −0.1, 0.0, 0.01, and 1.0 are considered in the analysis. The behaviour of the temperature distribution and the Nusselt number against these parameters are discussed. The Brinkman Number, the speed of the moving plate and different values of heat fluxes at both the plates have significant impact in the thermal development.

5. Acknowledgements

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