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On the Self-Similarity in Biological Processes

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Abstract

We show that the processes described by Avrami functions are self-similar. A comparative function characterizes a self-similar process by a certain Avrami exponent. We define the self-similar categories of some well-known biological processes. The method to determine the Avrami exponent by choosing the comparative function is demonstrated on the diffusion model of the growth of nuclei. We generalize the results.

Keywords

Self-Similar Process, Comparative Function, Avrami Exponent

1. Introduction

The self-organizing behavior of the materials is a well-known and widely investigated topic in research [1] and especially in biology [2]. The description of the self-organized, self-controlled (self-limited) processes was firstly in focus in the solid-state reactions (precipitations, phase-transitions, aggregations, nucleation, growth, etc.) pioneering by Kolmogorov [3], Johnson, Mehl [4] and Avrami [5] [6] [7], and later modified by others [8] [9].

The Avrami equation could serve as a mathematical model of different biological processes [10] [11]. Experimental data collected by Cope, [12] [13], and others [14] [15], show a certain universality of the Avrami equation to describe the real processes. The universality makes it possible to study different processes without knowing the exact structure and dynamics of a given system. The situation is similar to the description of the critical phenomena [16], where the physical laws connect to broad, general-function categories.

Our objective in this present work is to derive the universality of the Avrami-equation.

2. Method

Self-similarity means the scaling possibility of all the measurable quantities of the studied object. Let us assume that r is the linear size of the scale and L is a scale-dependent property: $L(r)$. By lengthening the scale by a times, the property transforms as

$$L(ar) = kL(r) \tag{1}$$

where the factor k depends only on a. The power type functional relationship is valid in the biology and the fractal theory:

$$L(r) = Ar^{\alpha} \tag{2}$$

which changes in the form of

$$L(ar) = A(ar)^{\alpha} = a^{\alpha}L(r) \tag{3}$$

When Equation (2) is converted to the log-log scale, the result is a straight line with a gradient equal to the slope.

$$\lg L(r) = \lg A + \alpha \lg r \tag{4}$$

The slope usually verifies the self-similarity of fractals.

Definition of the self-similarity in (1) means that the relationship is valid for any a and k independent from r. Hence k is the general function of a, expressing (1) in the form of

$$L(ar) = k(a)L(r) \tag{5}$$

From this, we get a new and more general definition of self-similarity, namely

$$L : r \in I_r \mapsto L(r) \in R^1 \tag{6}$$

the mapping is self-similar, if $L(ar)$ may be expressed in a more general form where the scale-transformed function can be separated in the form of a product:

$$L(ar) = k(a)G(r) \tag{7}$$

Solving the Equation (7), we suppose that $a = 1$. Then,

$$L(r) = k(1)G(r) = cG(r) \tag{8}$$

where $c = k(1)$ is a constant. Consequently, $L(r)$ and $G(r)$ differ only in the fixed multiplicator. By the last property the Equation (7) can be rewritten:

$$L(ar) = \frac{k(a)}{k(1)}L(r) = \eta(a)L(r) \quad \left[\eta(a) = \frac{k(a)}{k(1)}\right] \tag{9}$$

Repeating the same procedure for the case of $r = 1$, the following relationship is valid for any u, v variables:

$$L(uvr) = \eta(uv)L(r) = \eta(u)\eta(v)L(r) \tag{10}$$

Therefore, the function $\eta(uv)$ in Equation (10) is also a self-similar function:

$$\eta(uv) = \eta(u)\eta(v) \tag{11}$$

The general solution of this function-equation is difficult. We may choose $a = r, r = 1$ for the common part of variables from the relationship Equation (9):

$$L(r) = \frac{k(a)}{k(1)} L(r) = \eta(r) L(1) = C\eta(r) \tag{12}$$

where $C = L(1)$ is a constant. Consequently, when $L(r)$ is a continuous function on this common set, then it is true also for $\eta(r)$. In this case, the solution of the function Equation (12) is known for positive r values [17]:

$$\eta(r) = Kr^{\alpha} \tag{13}$$

where K and α are constants. Hence, we get from Equation (12):

$$L(r) = Ar^{\alpha} \tag{14}$$

where A is another constant. Consequently, the power-law principle shown in Equation (1), can be applied indeed in general case for the description of self-similarity.

The concept of self-similarity may also be extended from the structures to the processes [18]. Let us take a process described by the

$$f : t \in I = [0, \infty) \mapsto f(t) \in R \tag{15}$$

function. Let us choose the time t expressed in the form of

$$t = \tau t' \tag{16}$$

with the aid of an earlier t' time. Our statement is that the process described by the function f is self-similar, if

$$f(t) = f(\tau t') = g(\tau) h(t') \quad \forall t, t' \in I = [0, \infty) \tag{17}$$

Since we have proved earlier that the solution of the equation of the above function has the form of

$$f(t) = abt^{n},\ g(\tau) = a\tau^{n}, \quad h(t') = bt'^{n} \tag{18}$$

thus, self-similarity can be written in a simpler form of:

$$f(\tau t') = f(\tau) f(t') \quad \forall t, t' \in I = [0, \infty) \tag{19}$$

Equation (19) means that the value of the physical quantity of the process can be established at a later point in time by the extension or shrinking of an earlier value, where the amount of extension or shrinking depends only on the ratio of the two time points. Therefore, the ratio of function values is the function of the ratio of time points. A solution of Equation (19) (Cauchy-type function-equation) is [17]:

$$f(t) = t^{\alpha} \tag{20}$$

where α is an arbitrary number. Of course, in nature, not every process is self-similar. Those deserve attention, which can be transformed them into self-similar processes. When f is not self-similar, we may apply a suitably chosen u transformation, when the composition of $u \circ f$ will be self-similar, namely:

$$u\left[f\left(\tau t'\right)\right]=g\left(\tau\right)h\left(t'\right)\quad\forall t,t'\in I=\left[0,\infty\right)\tag{21}$$

The u function is named: comparative function. It is proven [19] [20], that the comparison functions of the most common processes scarcely differ from each other when the appropriate parameters are chosen.

A wide group of processes differ regarding mathematics; however, a process can be transformed into self-similar one with the help of a comparative function scarcely dissimilar in approximation capability.

We are going to show that the power type equation is the consequence of some simple and very general assumptions.

3. Results

The Avrami equation [5] [6] [7] has the following form:

$$x\left(t\right)=1-\mathrm{e}^{-\left(ct\right)^{\alpha}}\tag{22}$$

where t is the time of the reference system, and α and c are constants. (The function is identical to the well-known Weibull function [21], which is widely used to describe the reliability/aging processes [22].) Let us rewrite Equation (22) into another form:

$$g\left(t\right)=\ln\frac{1}{1-x}=\left(ct\right)^{\alpha}\tag{23}$$

Moreover, it is transformed to a new time-scale by the

$$t\mapsto\upsilon=ct\tag{24}$$

linear transformation. Following these changes, the Equation (22) Avrami-equation takes its final form as:

$$f\left(\upsilon\right)=\ln\frac{1}{1-x}=\left(\upsilon\right)^{\alpha}\tag{25}$$

The function $\ln\frac{1}{1-x}$ is the comparative function for the Avrami processes, (Avrami comparative function). Consequently, the Equation (24) linear transformation could be chosen when the Equation (25) Avrami comparative function is self-similar.

In the case when f is a continuously differentiable function, the function Equation (19) is identical to the following differential equation:

$$\frac{\mathrm{d}f}{\mathrm{d}\upsilon}=\alpha\frac{f}{\upsilon}\tag{26}$$

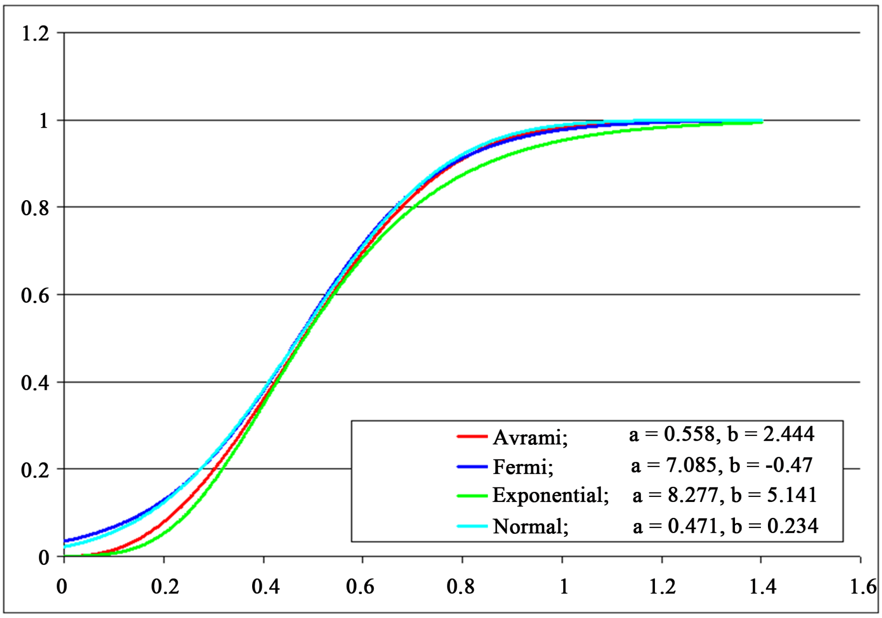
Substituting function Equation (25) into Equation (26), we get the differential equation:

$$\frac{\mathrm{d}x}{\mathrm{d}\upsilon}=\alpha\upsilon^{\alpha-1}\left(1-x\right)\tag{27}$$

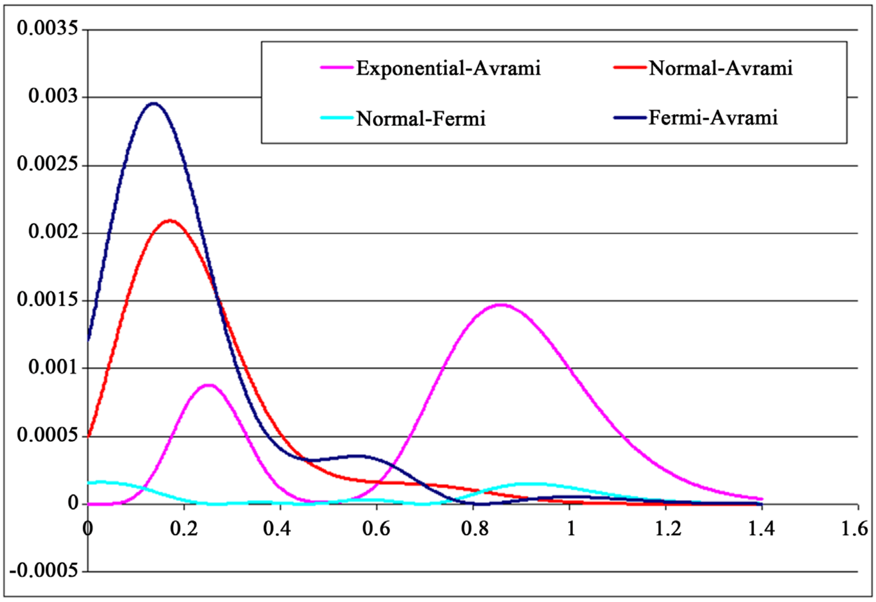
Transforming Equation (27) back to the reference time-scale, we obtain the differential equation describing the dynamic features of the process.

$$\frac{\mathrm{d}x}{\mathrm{d}t}=c\alpha\left(ct\right)^{\alpha-1}\left(1-x\right)\tag{28}$$

It is proven [19] [20] that the comparison functions of the most common processes scarcely differ from each other when the appropriate parameters are chosen (**Figure 1**).



(a)



(b)

Figure 1. A particular set of parameters allows the transformed functions to be identical within 0.3% accuracy. (R^2 Avrami-Fermi = 0.99981, R^2 Avrami-Exponent. = 0.99915, R^2 Avrami-Normal = 0.99986, R^2 Exponent.-Fermi = 0.99894, R^2 Exponent.-Normal = 0.99846, R^2 Normal-Fermi = 0.99984). Upper panel: the given transformed functions, lower panel: the square of their deviation. $\text{Fermi} := \frac{1}{\exp\left(-a\left(x+b\right)\right)+1}$;

$\text{Exponential} := \exp\left(-a\exp\left(-bx\right)\right)$; $\text{Avrami} := \exp\left(-ax^b\right)$.

A wide group of processes differs regarding mathematics. However, it can be transformed into a self-similar one with the help of a comparative function hardly dissimilar in approximation capability.

4. Discussion

With our results, we could explain numerous important processes in biology. Let us describe first the growth of nuclei in a homogeneous thermodynamic system. For the sake of simplicity, consider the nuclei as a sphere, and the inside solid material has a constant number of particles by volume units n_0, as well as the concentration distribution of the particles in the neighborhood of the nuclei considered as having spherical symmetry. For the diffusion growth of these nuclei, the conservation of the particles could be formulated:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = I \tag{29}$$

where N is the number of the particles in the nuclei, I is the particle current on the surface of the nuclei. If the nuclei radius is r and it is homogenous, then

$$N = n_0 \frac{4\pi}{3} r^3 \tag{30}$$

moreover, the particle current

$$I = 4\pi r r^2 j \tag{31}$$

where j is the current density of particles. If j = const. then

$$r = \frac{j}{n_0} t \tag{32}$$

The exponent, in this case, is trivially $n = 1$. Do not forget: this is a particular instance when both the concentration gradient and the thickness of the boundary of the nuclei are constant. Let us study when only the current density kept constant on the nuclei boundary. Then, based on (25):

$$n_0 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = I = 4\pi r^2 D \frac{\partial n}{\partial r} = \text{all.} \tag{33}$$

where D is the diffusion constant. If the concentration distribution is stationary, then

$$n_0 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = I = 4\pi D r \left[n_\infty - n(r) \right] \tag{34}$$

where n_∞ is the particle concentration far from the nuclei. The particle concentration of the nuclei could be written [23]:

$$n(r) = n_{\text{plane}} \left(1 + \frac{2\alpha}{n_0 k T r} \right) \tag{35}$$

where n_{plane} is the concentration at a plane, α is the surface tension, and T is the temperature. If $(\alpha/n_0) \ll 1$, then

$$\frac{2\alpha}{n_0 kTr} \ll 1 \tag{36}$$

Hence from the condition Equation (36) and Equation (35):

$$r \approx t^{\frac{1}{2}} \tag{37}$$

so the exponent is $n = 0.5$. Contrarily, if $\left(\alpha/n_0\right) \gg 1$, then

$$r \approx t^{\frac{1}{3}} \tag{38}$$

so the exponent is $n = 1/3$. Consequently, the volume of the nuclei depends on the following:

$$V_{\text{grain}} \approx t^{3n} \tag{39}$$

which is the Avrami exponent.

Let us calculate the Avrami comparative function for the same conditions. Denote the part of the old phase transforming into the new one by x. According to the Kolmogorov-Avrami assumptions [3] [5] [6] [7], the speed of the phase transformation is proportional to the growth rate of the nuclei volume ($\mathrm{d}V_{nucl}/\mathrm{d}t$) and the non-transformed part of the phase:

$$\frac{\mathrm{d}x}{\mathrm{d}t} \approx \frac{\mathrm{d}V_{nucl}}{\mathrm{d}t}(1-x) \tag{40}$$

Hence, by integration of (40), the V_{nucl}:

$$\ln\frac{1}{1-x} \approx V_{nucl} \tag{41}$$

Using the k = const. factor, we get:

$$\ln\frac{1}{1-x} = kt^{3n} \tag{42}$$

Consequently, the nuclei growth process by diffusion with the

$$\ln\frac{1}{1-x} \tag{43}$$

Avrami comparative function and on the

$$k^{\frac{1}{3n}}t \tag{44}$$

time-scale is a self-similar process and could be described by the Avrami exponent:

$$\alpha = 3n \tag{45}$$

A summary of these approximations is shown in **Table 1**.

To generalize the above, let us denote a cluster size with arbitrary shape by ξ. In this case, we can assume its volume is $V_{nucl} \approx \xi^k$. Hence the Avrami-exponent would be

$$\alpha = \beta n$$

The actual value of β depends on the form of the cluster, e.g. in the needle

Table 1. Avrami exponent at growth of nuclei by diffusion.

Character of the process	Avrami exponent (α)
Constant-current density of the diffusion	3
Constant flux of the diffusion, with $(\alpha/n_0) \ll 1$	3/2
Constant flux of the diffusion, with $(\alpha/n_0) \gg 1$	1

form; it is $\beta = 1$; in the planar form, $\beta = 2$. The above considerations could be extended even for fractal dimensions: $\beta = \text{dfr}$, where dfr is the measured fractal dimension.

Various processes of class $\alpha = 1.0$ are summarized in **Table 2** to refer to the time-scale $\upsilon = ct$.

The process character is the Avrami exponent α. Based on the extensive works of FW Cope [10] [11], we had collected the Avrami exponents for various solid-states (**Table 3**) and biological processes (**Table 4**).

The method could also be generalized on vectorial processes. Assuming that the processes are disjointed and the individual modes have the same comparative functions, then all the $g_i(t)$ comparative functions of the modes could be transformed into the form:

$$g_i(t) = (c_i t)^\alpha, (i = 1, 2, \cdots, N) \tag{47}$$

where c_i is the scale factor of the time-scale of the i-th mode and N is the number of modes. The example to study could be a bioprocess, chemical catabolism, exchanging materials with the actual neighborhood. There could be two reasons to change the number of particles in the system: a breakdown of a molecule or a molecular exchange at the surface of the system. We must consider not only one but numerous chemical components participating in this process. It could happen even in the simplest double-component system, not only because of the chemical potential of the breaking but also because the final component differs from its environmental values. Under these circumstances, the diffusion of both compounds starts and becomes coupled according to Onsager's non-equilibrium [25]. The particle numbers of the components (1 and 2 denote the original and the decomposed components, respectively) could be described by:

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t} &= J_{k1} - \lambda x_1 \\ \frac{\mathrm{d}x_2}{\mathrm{d}t} &= J_{k2} + \lambda x_1 \end{aligned} \tag{48}$$

where λ is the constant of the decomposition. On the other hand, for J_{k1} and J_{k2}, the Onsager equations determine particle currents:

$$\begin{aligned} J_{k1} &= -D_{11}x_1 - D_{12}x_2 \\ J_{k2} &= -D_{21}x_1 - D_{22}x_2 \end{aligned} \tag{49}$$

where the matrix of D_{11}, D_{12}, D_{21} and D_{22} conduction constants is a positive definite structure due to the second (entropy) law of thermodynamics. Furthermore,

Table 2. Processes with $\alpha = 1.0$ (time-scale $\upsilon = ct$).

Process	Descriptive function	Comparative function
Decomposition, breakdown	$x = x_\infty e^{-ct}$	$\ln \frac{x_\infty}{x}$
Infinite proliferation	$x = x_\infty e^{ct}$	$-\ln \frac{x_\infty}{x}$
Logistic process by Fermi distribution	$x = 1 - \frac{x_\infty}{1 + e^{-ct}}$	$\ln \frac{1-x}{x_\infty + x - 1}$
Logistic process by Gompertz distribution	$x = x_{1\infty} e^{-x_{2\infty} e^{-ct}}$	$\ln \frac{x_{2\infty}}{\ln \frac{x_{1\infty}}{x}}$
Demographic explosion	$x = \frac{x_\infty}{\pi} \operatorname{arccot}(ct)$	$\cot \frac{x}{x_\infty} \pi$

Table 3. Avrami exponents of some solid-state processes [24].

Processes in solids [24]	Avrami exponent (α)
Beginning of nucleation	3
Nucleation with constant speed	4
Nucleation with growing speed	<4
Staring nucleation and surface growth	2
Diffusional growth from nuclei	1.5
Diffusional growth from growing nuclei	2.5
Diffusional growth in 1 or 2 dimensions	1
Limited diffusional growth in 1 or 2 dimensions	1.5

Table 4. Avrami exponents of some biological processes [19].

Biological processes [10], [11]	Avrami exponent (α)
Growth of bacteria	4
K+ conductance decay in nerve axon	1.9
K+ leakage from poisoned muscle	1.7
Growth (weight) of rat	2.0
Growth (length) of regular leg of salamander	2.3
Growth (height) of sunflower plant	1.9
Synthesis of chlorophyll in maize plant	2.2
Muscle tension during tetanic contraction	1.21
Myosin splitting of ATP	1.24
Fresh green leaf IR phosphorescence decay	1.0
Dried green leaf IR phosphorescence decay	1.28
Melanin IR phosphorescence decay	1.08
Cytochrome c IR phosphorescence decay	1.16

the set of the equation is symmetrical ($D_{12} = D_{21}$) according to the Onsager's theory, [25]. Combining Equation (48) and Equation (49), we get the following vectorial differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} D_{11} + \dot{\lambda} & D_{12} \\ D_{21} - \dot{\lambda} & D_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{50}$$

describing the time variation of the particle currents. The modes correspond with the eigenvalues of the coefficient matrix. The time dependence of the modes:

$$\mathrm{e}^{\eta t} \tag{51}$$

moreover, the characteristic equation to determine the eigenvalues:

$$\det\begin{bmatrix} D_{11} + \lambda - \eta & D_{12} \\ D_{21} - \lambda & D_{22} - \eta \end{bmatrix} = 0 \tag{52}$$

Hence, we get:

$$\eta_{1,2} = \frac{-\left(D_{11} - D_{22} + \lambda\right) \pm \sqrt{\left(D_{22} - D_{11} - \lambda\right)^2 + 4D^2_{12} - 4D_{12}\lambda}}{2} \tag{53}$$

Both the modes of the vectorial process belong to the class $\alpha = 1.0$, with the $\ln\frac{x_{i\infty}}{x_i}, \left(i = 1, 2\right)$ comparative function and with $\upsilon_1 = \eta_1 t$ and $\upsilon_2 = \eta_2 t$ time scale.

Description of the self-similar traveling waves is also possible in above format. Let us denote the variable for the traveling wave:

$$\xi := vt + y \tag{54}$$

which belongs to a wave traveling in the y-direction at velocity v. According to our previous results, the comparative function is:

$$g\left(y, t\right) = \left(c\xi\right)^{\alpha} \tag{55}$$

The comparative functions are similar by an acceptable accuracy with appropriately chosen parameters. Let us choose the Avrami-like comparative function. By this, the traveling wave could be described by the following wave function:

$$x\left(y, t\right) = 1 - \mathrm{e}^{-\left[c\left(vt + y\right)\right]^{\alpha}} \tag{56}$$

This wave is not a solution because the corresponding

$$\frac{\mathrm{d}x}{\mathrm{d}\xi} = c\alpha\left(c\xi\right)^{\alpha - 1}\left(1 - x\right) \tag{57}$$

Avrami-equation leads to a linear equation with non-constant coefficients:

$$\frac{1}{v}\frac{\partial x}{\partial t} + \frac{\partial x}{\partial y} - 2c\alpha\left(c\left(vt + y\right)\right)^{\alpha - 1}\left(1 - x\right) = 0 \tag{58}$$

The two-component autocatalytic reactions could be described by the following vector equation [26] [27] [28]:

$$\begin{aligned}\frac{\mathrm{d}x_1}{\mathrm{d}\upsilon} &= x_1\left(1-x_2\right)\\ \frac{\mathrm{d}x_2}{\mathrm{d}\upsilon} &= -x_2\left(1-x_1\right)\end{aligned} \tag{59}$$

Introducing the variable of the traveling wave by Equation (54) and regarding the process as self-similar by Equation (55), then we receive the following coupled vectorial wave-equations:

$$\begin{aligned}\left(\frac{\partial}{\partial t}+v_1\frac{\partial}{\partial y}\right)x_1 &= x_1\left(1-x_2\right)\\ \left(\frac{\partial}{\partial t}+v_2\frac{\partial}{\partial y}\right)x_2 &= -x_2\left(1-x_1\right)\end{aligned} \tag{60}$$

A solution of Equation (60) was made with bilinear transformation by Hirota [29] and with Malfliet [30], obtaining a shock-wave:

$$x_1 = x_2 = \frac{1}{2}\left\{1+\tanh\left[\frac{v_1+v_2}{2}\left(y+\left(v_1-v_2\right)t\right)\right]\right\} \tag{61}$$

This equation is very similar to the various assumptions (e.g. [31] [32] [33] [34] [35]) of the energy-pack-like information transfer. Consequently, these biological processes presumably could be described by such self-similar function classes, based on the dynamical observations for the long-range correlation lengths ([36] [37] [38]). The long-range correlation could be derived from self-similar time structures establishing a new discipline (Fractal Physiology, [39] [40] [41] [42]). The fundamental role of the self-similarity in biology is well recognized in the scaling behavior of living objects [43] [44] [45].

5. Conclusion

In this paper, we show the possible transformation of the mathematical models of biological processes to the self-similar processes with the help of an appropriate comparative function. The mathematical expressions of the applied comparative functions are different, but by the suitable choice of their parameters, they approach each other well with little error. Consequently, in the practical solutions, to use one of them for the various tasks is satisfactory. The Avrami equation will describe the mathematical model of the processes with Avrami-like comparative function. We believe this is the reason for the unusual universality of the Avrami equation in biology. Consequently, the self-organizing conditions and the analogy of the self-organized processes could be a fruitful heuristic method in biological model calculations.

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