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The Lump Solutions of the (1 + 1)-Dimensional Ito-Equation

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Abstract

In this paper, several kinds of lump solutions for the (1 + 1)-dimensional Ito-equation are introduced. The proposed method in this work is based on a Hirota bilinear differential equation. The form of the solutions to the equation is constructed and the solutions are improved through analysis and symbolic computations with Maple. Finally, figure of the solution is made for specific examples for the lump solutions.

Keywords

Ito-Equation, Lump Solution, Solitons, Hirota's Bilinear Method

1. Introduction

In recent years, the study to the exact solutions of nonlinear equation is one of the hot topics in nonlinear science. A variety of nonlinear complex physical phenomena appear in many fields, such as chemistry, biology, engineering and social sciences. So, seeking exact solutions of nonlinear partial differential equations (NLPDEs) has become more and more attractive. In order to solve the exact solutions of NLPDEs, sciences have come up with lots of ways, for example Backlund transformation [1] [2], Darboux transformation [3] and Hirota bilinear methods [4]. Among these ways, Hirota bilinear method plays an important role in presenting lump solutions owing to its simplicity and directness. In these solutions, lump solutions are a kind of regular and rationally function solutions, localized in all directions in the space [5]. Lump solutions are very important in fluid dynamic, propagation of surface waves, and many other fields of physics and some engineering fields. Some lump solutions to many integrable equation, for example the Benjamin-Ono equation [6], the KP equation [7] and (2 + 1)-dimensional Ito-equation [8], have been found. The lump solution which is called the vortex and anti-vortex solution for Ito-equation was firstly put forward by Zakharov [9] and later by Craik [10]. The aim of this study is using the bilinear equations to search the solutions of (1 + 1)-dimensional Ito-equation. The Ito-equation is usually written as

$$u_{2t} + u_{3x,t} + 6u_x u_t + 3uu_{x,t} + 3u_{2x}\int_{-\infty}^{x} u_t \mathrm{d}x' = 0. \tag{1.1}$$

which is an extension of the K-dV(mK-dV) type to higher orders. Ito-equation is usually used to predict the rolling behavior of ships in regular sea. It also can be used to describe the interaction process of two internal long waves where $u(x,t)$ is an analytic function.

In this paper, we would like to present the lump solutions of (1 + 1)-dimensional Ito-equation. In Section 2, we mainly introduce the bilinear form of the Ito-equation through tedious transformation. In Section 3, based on the bilinear forms, we will get the lump solutions of the Equation (1.1) via analysis and symbolic computations in Section 3. And plots with different parameters will be made to show the change of the equation. In section 4, we will give the conclusions.

2. The Bilinear Equation for Ito-Equation

Hirota bilinear forms are one of the integrability characteristics of nonlinear partial differential equations and the bilinear equation can be solved by the Wronskian technique [11]. Hirota bilinear equation plays an important role in generating the lump solutions. By Painleve analysis [12], we assume that the lump solution of Equation (1.1) as

$$u = 2\partial_x^2 \ln f. \tag{2.1}$$

where $f(x,t)$ is unknown real function. Through transformation Equation (2.1), the bilinear equation of Equation (1.1) can be presented

$$\left[D_t^2 + D_x^3 D_t\right] f \cdot f = 0. \tag{2.2}$$

where D_t, D_x are all the bilinear derivative operators and D-operator [13] is defined by

$$D_x^m D_t^n a(x,t)\cdot b(x,t) = \left(\partial_t - \partial_{t'}\right)^n \left(\partial_x - \partial_{x'}\right)^m a(x,t) b(x',t')\Big|_{x'=x,t'=t}. \tag{2.3}$$

where m and n are the positive integers, $a(x,t)$ is the function of x and t, and $b(x,t)$ is the function of the formal variables x and t.

3. Lump Solutions for Ito-Equation

Based on analysis and symbolic computations with Maple, we can show that the (1 + 1)-dimensional bilinear Ito-equation has a class of solutions determined by $f(x,t)$. Inthis section, we make $f(x,t)$ as a combination of positive quadratic function, that is

$$f(x,t) = a_1 + \left(a_2 x + a_3 t + a_4\right)^2 + \left(a_5 x + a_6 t + a_7\right)^2. \tag{3.1}$$

where $a_i, i = 1, 2, \cdots, 7$ are some constants to be determined. Substituting Equation (3.1) into Equation (2.1) yields the exact solution of Equation (1.1)

$$u(x,t) = \frac{2\left(2a_2^2 + 2a_5^2\right)}{a_1 + \left(a_2 x + a_3 t + a_4\right)^2 + \left(a_5 x + a_6 t + a_7\right)^2} - \frac{2\left(2\left(a_2 x + a_3 t + a_4\right)a_2 + 2\left(a_5 x + a_6 t + a_7\right)a_5\right)^2}{\left(a_1 + \left(a_2 x + a_3 t + a_4\right)^2 + \left(a_5 x + a_6 t + a_7\right)^2\right)^2}. \tag{3.2}$$

Substituting Equation (3.1) into Equation (2.2) leads to

$$\begin{aligned}&\left(-4a_3^4 - 8a_3^2 a_6^2 - 4a_6^4\right)t^2 + \left(\left(-8a_2 a_3^3 - 8a_2 a_3 a_6^2 - 8a_3^2 a_5 a_6 - 8a_5 a_6^3\right)x - 8a_3^3 a_4 \right.\\ &\left. -8a_3^2 a_6 a_7 - 8a_3 a_4 a_6^2 - 8a_6^3 a_7\right)t + \left(-4a_2^2 a_3^2 + 4a_2^2 a_6^2 - 16a_2 a_3 a_5 a_6 + 4a_3^2 a_5^2 - 4a_5^2 a_6^2\right)x^2\\ &+\left(-8a_2 a_3^2 a_4 - 16a_2 a_3 a_6 a_7 + 8a_2 a_4 a_6^2 + 8a_3^2 a_5 a_7 - 16a_3 a_4 a_5 a_6 - 8a_5 a_6^2 a_7\right)x\\ &+24a_2^2 a_3 + 24a_2^2 a_5 a_6 + 24a_2 a_3 a_5^2 - 4a_3^2 a_4^2 + 4a_3^2 a_7^2 - 16a_3 a_4 a_6 a_7 + 4a_4^2 a_6^2 + 24a_5^3 a_6\\ &-4a_6^2 a_7^2 + 4a_1 a_3^2 + 4a_1 a_6^2 = 0\end{aligned} \tag{3.3}$$

Next equating all coefficients of different powers of x^2, x, t^2, t to zero, through the coefficients of the Equation (3.3), we can get the following relations between the parameters:

$$a_3 = 0, a_6 = 0. \tag{3.4}$$

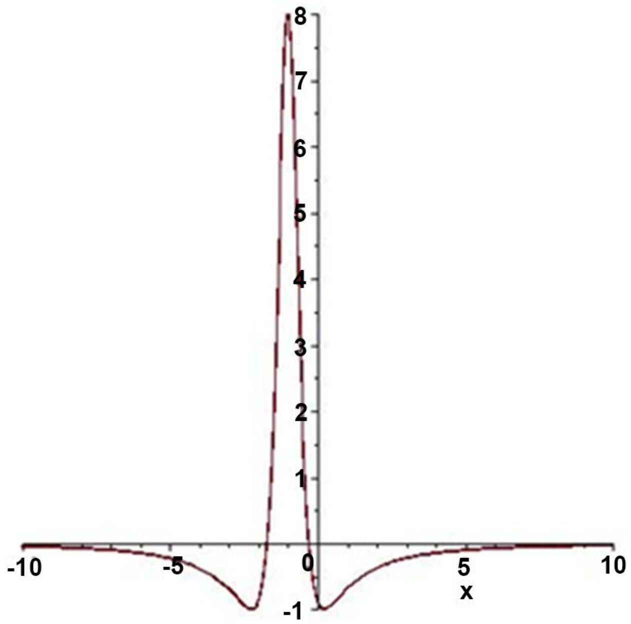
where a_1, a_2, a_4, a_5 and a_7 are some free real numbers. Substituting Equation (3.4) into Equation (3.2), we have

$$u(x,t) = \frac{2\left(2a_2^2 + 2a_5^2\right)}{a_1 + \left(a_2 x + a_4\right)^2 + \left(a_5 x + a_7\right)^2} - \frac{2\left(2\left(a_2 x + a_4\right)a_2 + 2\left(a_5 + a_7\right)a_5\right)^2}{\left(a_1 + \left(a_2 x + a_4\right)^2 + \left(a_5 x + a_7\right)^2\right)^2} \tag{3.5}$$

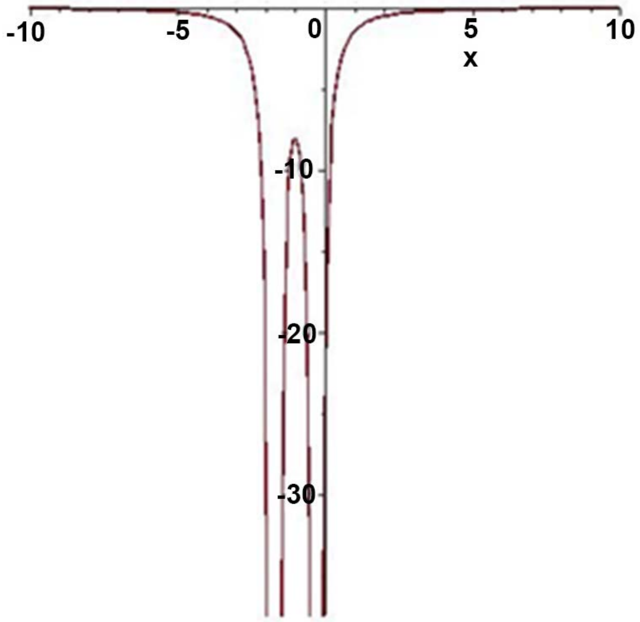
Similarly, solution Equation (3.5) is also a lump solution, which contains five free parameters a_1, a_2, a_4, a_5 and a_7. When the values of these parameters are changed, the structure of the lump solutions will also change accordingly. Here, we give two plots with particular choice of the involved parameters for lump solutions (**Figure 1**).

4. Conclusions

Recently, a lot of work has been done to learn the lump solutions of the Ito-equation and Ito-like equations. It is natural and interesting to search for lump solutions to nonlinear partial differential equations. Based on the advantage of Hirota bilinear forms, lump solutions for Ito-equation have been solved with symbolic computations in this paper. At the beginning of the paper, we give the Hirota bilinear equation of Ito-equation via tedious calculation and analysis. And then, we verify the two lump solutions for the equation. Meanwhile, we found that the lump solutions contain many parameters and relations between these parameters which can affect the structure of solutions. Finally, we get different plots with particular choices of the involved parameters which have been made to show the lump solutions and their energy distributions.



(a)



(b)

Figure 1. When the Equation (3.5) selects the following parameters, the equation will be rendered with the lump solutions of a and b. $(a): a_1 = 1, a_2 = 1, a_4 = 1, a_5 = 1, a_7 = 1$; $(b): a_1 = -1, a_2 = -1, a_4 = -1, a_5 = -1, a_7 = -1$.

Next, we would like to discuss that there is any other solutions to the Ito-equation. Furthermore, we can extend this method to solve other equations and learn their lump solutions. Those problems may be also interesting and be worthy to study.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Miura, R.M. (1978) Backlund Transformation. Springer-Verlag, Berlin.

[2] Rogers, C. and Shadwick, W.F. (1982) Backlund Transformations and Their Applications.

[3] Matveev, V.B. and Salle, M.A. (1991) Darboux Transformation and Solitons. *Journal of Neurochemistry*, **42**, 1667-1676.

[4] Hirota, R. (2004) The Direct Method in Soliton Theory. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511543043

[5] Ma, W.X. (2015) Lump Solutions to the Kadomtsev-Petviashvili Equation. *Physics Letters A*, **379**, 1975-1978.

[6] Tan, W. and Dai, Z. (2017) Spatiotemporal Dynamics of Lump Solution to the (1 + 1)-Dimensional Benjamin-Ono Equation. *Nonlinear Dynamics*, **89**, 2723–2728. https://doi.org/10.1007/s11071-017-3620-0

[7] Yu, J.P. and Sun, Y.L. (2016) Study of Lump Solutions to Dimensionally Reduced Generalized KP Equations. *Nonlinear Dynamics*, **87**, 1-9.

[8] Yang, J.Y., Ma, W.X. and Qin, Z. (2017) Lump and Lump-Soliton Solutions to the

(2 + 1)-Dimensional Ito Equation. Analysis. *Mathematical Physics*, No. 1, 1-10.

[9] Zaharov, V.E. (1976) Exact Solutions in the Problem of Parametric Interaction of Three-Dimensional Wave Packets. *Doklady Akademii Nauk Sssr*, **228**, 1314-1316.

[10] Craik, A.D.D. (1978) Evolution in Space and Time of Resonant Wave Triads. II. A Class of Exact Solutions. *Proceedings of the Royal Society A*, **363**, 257-269. https://doi.org/10.1098/rspa.1978.0167

[11] Ma, W.X. and You, Y. (2005) Solving the Korteweg-de Vries Equation by Its Bilinear Form: Wronskian Solutions. *Transactions of the American Mathematical Society*, **357**, 1753-1778. https://doi.org/10.1090/S0002-9947-04-03726-2

[12] Weiss, J., Tabor, M. and Carnevale, G. (1983) The Painleve Property for Partial Differential Equations. *Journal of Mathematical Physics*, **24**, 522-526. https://doi.org/10.1063/1.525721

[13] Liu, H., Li, J. and Liu, L. (2013) The Recursion Operator Method for Generalized Symmetries and Bäcklund Transformations of the Burgers' Equations. *Journal of Applied Mathematics and Computing*, **42**, 159-170. https://doi.org/10.1007/s12190-012-0633-1