Open Access Library Journal
2019, Volume 6, e5273
ISSN Online: 2333-9721
ISSN Print: 2333-9705

ϕ-Pseudo Symmetric ϵ-Para Sasakian Manifolds

P. Somashekhara, Venkatesha

Department of Mathematics, Kuvempu University, Shimoga, India
Email: somumathrishi@gmail.com, vensmath@gmail.com

How to cite this paper: Somashekhara, P. and Venkatesha (2019) ϕ-Pseudo Symmetric ϵ-Para Sasakian Manifolds. *Open Access Library Journal*, **6**: e5273.
https://doi.org/10.4236/oalib.1105273

Received: February 20, 2019
Accepted: May 28, 2019
Published: May 31, 2019

Abstract

The present paper focuses on the study of ϕ-pseudo symmetric, ϕ-pseudo concircularly symmetric and ϕ-pseudo Ricci symmetric on ϵ-Para Sasakian Manifolds. Also interesting results are obtained.

Subject Areas

Geometry

Keywords

ϵ-Para Sasakian Manifolds, ϕ-Pseudo Symmetric and ϕ-Pseudo Ricci Symmetric

1. Introduction

Majority of present approaches to mathematical general relativity launch with the concept of a manifold. The standpoint of physics and relativity is to the investigation of manifolds with indefinite metrics. Several authors have studied manifold with indefinite matrices. Bejancu and Duggal [1] originated the concept of ϵ-Sasakian manifolds in 1993. De and Sarkar [2] pioneered (ϵ)-Kenmotsu manifolds and investigated some curvature conditions on it. Pandey and Tiwari [3] constructed the relation between semi-symmetric metric connection and Riemannian connection of (ϵ)-Kenmotsu manifolds and have studied several curvature conditions. The notion of (ϵ)-Para Sasakian Manifolds was pioneered by Tripathi *et al.* [4] in 2009.

The Riemannian symmetric spaces were introduced by French mathematician Carton during the nineteenth century and play a main tool in differential geometry. A Riemannian manifold is locally symmetric [5], if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M, g). During the last five decades the notion of locally symmetric manifolds has been studied by many authors in several ways to a different extent such as recurrent manifold by Walker [6], semi-symmetric manifold by Szabó [7], pseudosymmetric manifold in the sense of Deszcz [8], a non-flat Riemannian manifold $(M^n, g)(n > 2)$ is said to be pseudosymmetric in the sense of Chaki [9] if it satisfies the relation

$$\begin{aligned}&(\nabla_W R)(X,Y,Z,U)\\&=2A(W)R(X,Y,Z,U)+A(X)R(W,Y,Z,U)+A(Y)R(X,W,Z,U)\\&\quad+A(Z)R(X,Y,W,U)+A(U)R(X,Y,Z,W),\end{aligned} \tag{1.1}$$

i.e.,

$$\begin{aligned}&(\nabla_W R)(X,Y)Z\\&=2A(W)R(X,Y)Z+A(X)R(W,Y)Z+A(Y)R(X,W)Z\\&\quad+A(Z)R(X,Y)W+g(R(X,Y)Z,W)\rho,\end{aligned} \tag{1.2}$$

for any $X,Y,Z,U,W \in T_P(M)$ and where R is the Riemannian curvature tensor of the manifold, A is a non-zero 1-form such that $g(X,\rho)=A(X)$ for every vector field X. Every recurrent manifold is pseudosymmetric in the sense of Chaki [9] but not conversely. The pseudosymmetry in the sense of Chaki is not equivalent to that in the sense of Deszcz [8]. However, the pseudosymmetry by Chaki will be the pseudosymmetry by Deszcz if and only if the non-zero 1-form associated with n-dimensional pseudosymmetry is closed. Pseudosymmetric manifolds also have been studied by Chaki and De [10], Özen and Altay [11], Tarafder [12], De, Murathan and Özgür [13], Tarafder and De [14] and others. Many authors have been weakened by Ricci symmetry that has been differently extended such as a Ricci recurrent, Ricci symmetric and pseudo Ricci symmetric for past two decades.

A non-flat Riemannian manifold (M^n, g) is said to be pseudo-Ricci symmetric [15] if its Ricci tensor S of type $(0,2)$ is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z)=2A(X)S(Y,Z)+A(Y)S(X,Z)+A(Z)S(Y,X), \tag{1.3}$$

for any $X,Y,Z \in T_P(M)$ where A is a nowhere vanishing 1-form and ∇ refers the operator of covariant differentiation with respect to the metric tensor g. Such a n-dimensional manifold is denoted by $(PRS)_n$. The pseudo-Ricci symmetric manifolds have also been studied by Arslan *et al.* [16], De and Mazumder [17] and many others. The notion of locally ϕ-symmetric Sasakian manifold was introduced by Takahashi [18] due to a weaker version of locally symmetry. Generating the notion of locally ϕ-symmetric Sasakian manifolds, De *et al.* [19], introduce the notion of ϕ-recurrent Sasakian manifolds also Shukla *et al.* [20] studied ϕ-symmetric and ϕ Ricci symmetric para Sasakian manifolds.

Inspired by above studies this paper makes an attempt to study of ϕ-pseudo symmetric and ϕ-pseudo Ricci symmetric ϵ-para Sasakian manifolds. It is organized as follows. Section 2 is related with ϵ-para Sasakian manifolds. Section 3 is dealt with the study of ϕ pseudo symmetric ϵ-para Sasakian manifolds. In Section 4, we study of ϕ-pseudo Concircularly symmetric ϵ-para Sasakian manifold.

In Section 5, we study ϕ-pseudo Ricci symmetric ϵ-para Sasakian manifold. The relation (1.3) can be written as

$$(\nabla_X Q)(Y) = 2A(X)Q(Y) + A(Y)Q(X) + S(Y,X)\rho, \tag{1.4}$$

where ρ is the vector field associated to the 1-form A such that $A(X) = g(X,\rho)$ and Q is the Ricci operator, *i.e.*, $g(QX,Y) = S(X,Y)$.

2. Preliminaries

Let (M^n, g) be an almost paracontact manifold is equipped with an almost paracontact structure (ϕ,ξ,η) consisting of a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying

$$\phi^2 X = X - \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = -1,\ \phi\xi = 0,\ \eta \circ \phi = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) - \epsilon\eta(X)\eta(Y), \tag{2.3}$$

where $\epsilon = \pm 1$, in this case (M^n, g) is called an (ϵ)-almost paracontact metric manifold equipped with an (ϵ)-almost paracontact structure (ϕ,ξ,η,g) [21]. In particular, $\text{index}(g) = 1$, then (ϵ)-almost paracontact metric manifold will be called a Lorentzian almost paracontact metric manifold. In view of equation [22] [23], we have

$$g(\phi X, Y) = g(X, \phi Y), \tag{2.4}$$

$$g(x,\xi) = \epsilon\eta(X), \tag{2.5}$$

for any $X, Y \in T_pM$, the structure of a vector field ξ is a never light like. An (ϵ)-almost paracontact metric manifold (respectively a Lorentzian almost paracontact manifold $(M^n,\phi,\xi,g,\epsilon)$ is said to be space-like (ϵ)-almost paracontact metric manifold (respectively a space-like Lorentzian almost paracontact manifold), if $\epsilon = 1$ and (M^n, g) is said to be a time-like (ϵ)-almost paracontact metric manifold (respectively a Lorentzian almost paracontact manifold), if $\epsilon = -1$. An (ϵ)-almost paracontact metric structure is called an (ϵ)-Para Sasakian structure if

$$(\nabla_X \phi)(Y) = -g(X,\phi Y)\xi - \epsilon\eta(Y)\phi^2 X, \tag{2.6}$$

where ∇ is the Levi-Civita connection. A manifold (M^n, g) endowed with an (ϵ)-para Sasakian structure is called an (ϵ)-para Sasakian manifold. For $\epsilon = 1$ and g is a Riemannian, (M^n, g) is the usual para Sasakian manifold [24]. For $\epsilon = -1$, g Lorentzian and ξ replaced by $-\xi$, (M^n, g) becomes a Lorentzian para Sasakian manifold [23]. In an (ϵ)-para Sasakian manifold, we have

$$\nabla_X \xi = \epsilon\phi X, \tag{2.7}$$

$$g(\xi,\xi) = \pm 1 = \epsilon, \tag{2.8}$$

$$(\nabla_X \eta)(Y) = \epsilon g(\phi X, Y) = \Omega(X,Y), \tag{2.9}$$

for any $X, Y \in T_pM$, where Ω is the fundamental 2-form. In an (ϵ)-almost para Sasakian manifold (M^n, g), the following relations are hold.

$$\eta(R(X,Y)Z) = \epsilon[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \tag{2.10}$$

$$R(\xi,X)Y = -\epsilon g(X,Y)\xi - \epsilon\eta(Y)X, \tag{2.11}$$

$$R(X,Y)\xi = -\epsilon\eta(Y)X + \epsilon\eta(X)Y, \tag{2.12}$$

$$(\nabla_X R)(Y,Z)\xi = \epsilon^2[g(\phi X,Y)Z - g(\phi X,Z)Y]. \tag{2.13}$$

In an n-dimensional (ϵ)-para Sasakian manifold (M^n, g), the Ricci tensor satisfies

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.14}$$

$$S(X,\xi) = S(\xi,X) = -(n-1)\eta(X). \tag{2.15}$$

3. ϕ-Pseudo Symmetric on ϵ-Para Sasakian Manifold

Definition 3.1. *A ϵ-Para Sasakian manifold $(M^n)(\phi,\xi,\eta,g)$ is said to be a ϕ-pseudo symmetric if the curvature tensor R satisfies*

$$\begin{aligned} &\phi^2((\nabla_W R)(X,Y)Z) \\ &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z + A(Y)R(X,W)Z \\ &\quad + A(Z)R(X,Y)W + g(R(X,Y)Z,W)\rho, \end{aligned} \tag{3.1}$$

for any $X,Y,Z,W \in T_P M$. If $A = 0$ the manifold is said to be ϕ-symmetric.

By virtue of (2.1), it follows that

$$\begin{aligned} &(\nabla_W R)(X,Y)Z - \eta((\nabla_W R)(X,Y)Z)\xi \\ &= 2A(W)R(X,Y)Z + A(X)R(W,Y)Z + A(Y)R(X,W)Z \\ &\quad + A(Z)R(X,Y)W + g(R(X,Y)Z,W)\rho, \end{aligned} \tag{3.2}$$

from which it follows that

$$\begin{aligned} &g((\nabla_W R)(X,Y)Z,U) - \eta((\nabla_W R)(X,Y)Z)\eta(U) \\ &= 2A(W)g(R(X,Y)Z,U) + A(X)g(R(W,Y)Z,U) \\ &\quad + A(Y)g(R(X,W)Z,U) + A(Z)g(R(X,Y)W,U) \\ &\quad + g(R(X,Y)Z,W)A(U). \end{aligned} \tag{3.3}$$

Taking an orthonormal frame field and contracting (3.3) over X and U, then by using (2.2) and (2.5), we get

$$\begin{aligned} &(\nabla_W S)(Y,Z) - g((\nabla_W R)(\xi,Y)Z,\xi) \\ &= 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W) \\ &\quad + A(R(W,Y)Z) + A(R(W,Z)Y). \end{aligned} \tag{3.4}$$

Using (2.11) and (2.13), we have

$$g((\nabla_W R)(\xi,Y)Z,\xi) = 0, \tag{3.5}$$

by virtue of (3.5), it follows from (3.4) that

DOI: 10.4236/oalib.1105273

$$(\nabla_W S)(Y,Z) = 2A(W)S(Y,Z) + A(Y)S(W,Z) + A(Z)S(Y,W) + A(R(W,Y)Z) + A(R(W,Z)Y), \tag{3.6}$$

This leads to the following:

Theorem 3.1. *A ϕ-pseudo symmetric on a ϵ-para Sasakian manifold is Pseudo-Ricci symmetric if and only if* $A(R(W,Y)Z) + A(R(W,Z)Y) = 0$.

Putting $Z = \xi$ in (3.2), by using (2.10), (2.12) and (2.13), we have

$$\begin{aligned} &A(\xi)R(X,Y)W \\ &= \epsilon^2\left[g(\phi W,Y)X - g(\phi W,X)Y\right] + \epsilon\{2A(W)[\eta(Y)X - \eta(X)Y] \\ &\quad + A(X)[\eta(Y)W - \eta(W)Y] + A(Y)[\eta(X)W - \eta(W)X] \\ &\quad + [\eta(Y)g(X,W) - \eta(X)g(Y,W)]\rho\}. \end{aligned} \tag{3.7}$$

This leads to the following:

Theorem 3.2. *A ϕ-pseudo symmetric on a ϵ-para Sasakian manifold, the curvature tensor satisfies the relation* (3.7).

From (3.7) follows that

$$\begin{aligned} A(\xi)S(Y,W) = \epsilon^2(n-1)\Omega(W,Y) + \epsilon\{(n-1)\eta(Y)A(W) \\ + (n-1)\eta(W)A(Y) + \eta(Y)\eta(W) + g(Y,W)\}, \end{aligned} \tag{3.8}$$

replacing Y by ϕY and W by ϕW and using (2.3), (2.14), we have

$$S(Y,W) = \frac{1}{A(\xi)}\left\{\epsilon g(Y,W) - \left[\epsilon^2 + (n-1)A(\xi)\right]\eta(Y)\eta(W) + \epsilon^2(n-1)\Omega(Y,W)\right\}. \tag{3.9}$$

Hence we can state the following:

Theorem 3.3. *A ϕ-pseudo symmetric on a ϵ-para Sasakian manifold, the curvature tensor satisfies the relation* (3.9), *provided* $A(\xi) \neq 0$.

4. ϕ-Pseudo Concircularly Symmetric ϵ-Para Sasakian Manifold

Definition 4.2. *A n-dimensional ϵ-para Sasakian manifold is said to be ϕ-pseudo Concircularly symmetric, if its Concircular curvature tensor $\tilde{C}$ is given by* [25]

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\left[g(Y,Z)X - g(X,Z)Y\right]. \tag{4.1}$$

Satisfies the relation

$$\begin{aligned} &\phi^2\left(\left(\nabla_W \tilde{C}\right)(X,Y)Z\right) \\ &= 2A(W)\tilde{C}(X,Y)Z + A(X)\tilde{C}(W,Y)Z + A(Y)\tilde{C}(X,W)Z \\ &\quad + A(Z)\tilde{C}(X,Y)W + g\left(\tilde{C}(X,Y)Z,W\right)\rho, \end{aligned} \tag{4.2}$$

for any $X,Y,Z,W \in T_P M$, *where A is a non-zero* 1-*forms, such that* $A(X) = g(X,\rho)$.

by virtue of (2.1), it follows from (4.2)

$$\begin{aligned}&\left(\nabla_W \tilde{C}\right)(X,Y)Z-\eta\left(\left(\nabla_W \tilde{C}\right)(X,Y)Z\right)\xi\\&=2A(W)\tilde{C}(X,Y)Z+A(X)\tilde{C}(W,Y)Z+A(Y)\tilde{C}(X,W)Z\\&\quad+A(Z)\tilde{C}(X,Y)W+g\left(\tilde{C}(X,Y)Z,W\right)\rho,\end{aligned}\tag{4.3}$$

which follows that

$$\begin{aligned}&g\left(\left(\nabla_W \tilde{C}\right)(X,Y)Z,U\right)-\eta\left(\left(\nabla_W \tilde{C}\right)(X,Y)Z\right)\eta(U)\\&=2A(W)g\left(\tilde{C}(X,Y)Z,U\right)+A(X)g\left(\tilde{C}(W,Y)Z,U\right)\\&\quad+A(Y)g\left(\tilde{C}(X,W)Z,U\right)+A(Z)g\left(\tilde{C}(X,Y)W,U\right)\\&\quad+g\left(\tilde{C}(X,Y)Z,W\right)A(U).\end{aligned}\tag{4.4}$$

Taking an orthonormal frame field and contracting (4.4) over X and U, by using (2.1) and (4.1), we get

$$\begin{aligned}&(\nabla_X S)(Y,Z)-\frac{dr(W)}{n}g(Y,Z)+g\left(\left(\nabla_W \tilde{C}\right)(\xi,Y)Z,\xi\right)\\&=2A(W)S(Y,Z)+A(Y)S(W,Z)+A(Z)S(Y,W)-\frac{r}{n}\left[2A(W)g(Y,Z)\right.\\&\quad\left.+A(Y)g(W,Z)+A(Z)g(Y,W)\right]+A\left(\tilde{C}(W,Y)Z\right)+A\left(\tilde{C}(W,Z)Y\right),\end{aligned}\tag{4.5}$$

by virtue of (3.5) and from (4.1), yields

$$g\left(\left(\nabla_W \tilde{C}\right)(\xi,Y)Z,\xi\right)=-\frac{dr(W)}{n(n-1)}\left[g(Y,Z)-\eta(Y)\eta(Z)\right].\tag{4.6}$$

In view of (4.6) from (4.5), we have

$$\begin{aligned}&(\nabla_X S)(Y,Z)-\frac{dr(W)}{n}g(Y,Z)-\frac{dr(W)}{n(n-1)}\left[g(Y,Z)-\eta(Y)\eta(Z)\right]\\&=2A(W)S(Y,Z)+A(Y)S(W,Z)+A(Z)S(Y,W)-\frac{r}{n}\left[2A(W)g(Y,Z)\right.\\&\quad\left.+A(Y)g(W,Z)+A(Z)g(Y,W)\right]+A\left(\tilde{C}(W,Y)Z\right)+A\left(\tilde{C}(W,Z)Y\right).\end{aligned}\tag{4.7}$$

This leads to the following:

Theorem 4.4. *A ϕ-pseudo Concircularly symmetric ϵ-para Sasakian manifold is pseudo-Ricci symmetric if and only if*

$$\begin{aligned}&\frac{dr(W)}{n}g(Y,Z)+\frac{dr(W)}{n(n-1)}\left[g(Y,Z)-\eta(Y)\eta(Z)\right]\\&-\frac{r}{n}\left[2A(W)g(Y,Z)+A(Y)g(W,Z)+A(Z)g(Y,W)\right]\\&+A\left(\tilde{C}(W,Y)Z\right)+A\left(\tilde{C}(W,Z)Y\right)=0.\end{aligned}\tag{4.8}$$

Putting $Z=\xi$ in (4.3) and using (2.10), (2.12), (2.15) and (4.1), we obtain

$$\begin{aligned}
&\epsilon^2\left[\Omega(W,X)Y-\Omega(W,Y)X\right]-\epsilon\frac{dr(W)}{n(n-1)}\left[\eta(Y)X-\eta(X)Y\right]\\
&+\epsilon\left[1-\frac{r}{n(n-1)}\right]\left[\eta(Y)g(X,W)-\eta(X)g(Y,W)\right]\rho\\
&\frac{r}{n(n-1)}A(\xi)\left[g(Y,W)X-g(X,W)Y\right]\\
&+\epsilon\left[1-\frac{r}{n(n-1)}\right]\left\{2A(W)\left[\eta(Y)X-\eta(X)Y\right]\right.\\
&\left.+A(X)\left[\eta(Y)W-\eta(W)Y\right]+A(Y)\left[\eta(W)X-\eta(X)W\right]\right\}\\
&=A(\xi)R(X,Y)W.
\end{aligned} \tag{4.9}$$

Hence we can state the following:

Theorem 4.5. *In a ϕ-pseudo Concircularly symmetric ϵ-para Sasakian manifold, the curvature tensor satisfies the relation* (4.9).

Next, we take inner product of (4.9) with U and taking an orthonormal frame field and contracting (4.9) over X and U, yields

$$\begin{aligned}
A(\xi)S(Y,W)=&\epsilon^2(1-n)\Omega(Y,W)-\epsilon\frac{dr(W)}{n}\eta(Y)\\
&+\epsilon\left[1-\frac{r}{n(n-1)}\right]\left[\eta(Y)\eta(W)-g(Y,W)\right]+\frac{r}{n}A(\xi)g(Y,W)\\
&+\epsilon\left[1-\frac{r}{n(n-1)}\right](n-1)\left[2A(W)\eta(Y)+A(Y)\eta(W)\right].
\end{aligned} \tag{4.10}$$

Replacing Y by ϕY and W by ϕW, we obtain

$$\begin{aligned}
S(Y,W)=&\frac{\epsilon^2(1-n)}{A(\xi)}\Omega(Y,W)+\left[\frac{r}{n}-\frac{\epsilon}{A(\xi)}\left[1-\frac{r}{n(n-1)}\right]g(Y,W)\right]\\
&+\left[\frac{\epsilon^2}{A(\xi)}\left[1-\frac{r}{n(n-1)}\right]-\frac{\epsilon}{A(\xi)}-(n-1)\right]\eta(Y)\eta(W).
\end{aligned} \tag{4.11}$$

This leads to the following:

Theorem 4.6. *A ϕ-pseudo Concircularly symmetric ϵ-para Sasakian manifold, the curvature tensor satisfies the relation* (4.11).

5. ϕ-Pseudo Ricci Symmetric ϵ-Para Sasakian Manifold

Definition 5.3. *A n-dimensional ϵ-para Sasakian manifold is said to be ϕ-pseudo Ricci symmetric, if the Ricci operator Q satisfies*

$$\phi^2\left((\nabla_W Q)(Y)\right)=2A(X)QY+A(Y)QX+S(Y,X)\rho, \tag{5.1}$$

for any $X,Y\in T_PM$, where A is a non zero 1-form.

In particular if $A=0$, then (5.1) turns into ϕ-Ricci symmetric ϵ-para Sasakian manifold.

In view of (2.1), then (5.1) becomes

$$(\nabla_W Q)(Y) - \eta\left((\nabla_W Q)(Y)\right)\xi = 2A(X)QY + A(Y)QX + S(Y,X)\rho, \tag{5.2}$$

which follows

$$\begin{aligned} &g((\nabla_W Q)(Y), Z) - S(\nabla_W Y, Z) - \eta((\nabla_W Q)(Y))\eta(Z) \\ &= 2A(X)S(Y,Z) + A(Y)S(X,Z) + S(Y,X)A(Z\eta), \end{aligned} \tag{5.3}$$

putting $Y = \xi$ in (5.3), using (2.7) and (2.15), we get

$$\begin{aligned} &A(\xi)S(X,Z) + \epsilon S(\phi X, Z) \\ &= (n-1)\left[\epsilon g(\phi X, Z) + 2A(X)\eta(Z) + \eta(X)A(Z)\right]. \end{aligned} \tag{5.4}$$

Replacing X by ϕX, Z by ϕZ in (5.4) and using (2.14), we get

$$S(X,Z) = \frac{\epsilon}{A(\xi)}\left[(n-1)\Omega(X,Z) - S(\phi X, Z)\right] - (n-1)\eta(X)\eta(Z). \tag{5.5}$$

This leads to the following:

Theorem 5.7. *A ϕ-pseudo Ricci symmetric ϵ-para Sasakian manifold, the curvature tensor satisfies the relation* (5.5).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Bejancu, A. and Duggal, K.L. (1993) Real Hypersurface of Infinite Kähler Manifolds. *International Journal of Mathematics and Mathematical Sciences*, **16**, 545-556. https://doi.org/10.1155/S0161171293000675

[2] De, U.C. and Sarkar, A. (2009) On (ε)-Kenmotsu Manifolds. *Hadronic Journal*, **32**, 231-242.

[3] Pandey, G. and Tiwari, K. (2014) On a Semi-Symmetric Metric Connection in an (ε)-Kenmotsu Manifold. *Communications of the Korean Mathematical Society*, **29**, 331-343. https://doi.org/10.4134/CKMS.2014.29.2.331

[4] Tripathi, M.M., Kilic, E., Perktas, S.Y. and Keles, S. (2009) On ε-para Sasakian 3-Manifolds.

[5] Cartan, E. (1926) Surune classes remarquable d'espaces de Riemannian. *Bulletin de la Société Mathématique de France*, **54**, 214-264. https://doi.org/10.24033/bsmf.1105

[6] Walker, A.G. (1950) On Ruse's Space of Recurrent Curvature. *Proceedings of the London Mathematical Society*, **10**, 36-64. https://doi.org/10.1112/plms/s2-52.1.36

[7] Szabó, Z.I. (1982) Structure Theorems on Riemannian Spaces Satisfying $R(X,Y)\cdot R = 0$, the Local Version. *Journal of Differential Geometry*, **17**, 531-582. https://doi.org/10.4310/jdg/1214437486

[8] Deszcz, R. (1992) On Pseudosymmetric Spaces. *Bulletin de la Société mathématique de Belgique. Série A*, **44**, 1-34.

[9] Chaki, M.C. (1987) On Pseudo Symmetric Maniflod, An. Sti. Ale Univ., "AL. I. CUZA". *Din Iasi*, **33**, 53-58.

[10] Chaki, M.C. and De, U.C. (1989) On Pseudosymmetric Spaces. *Acta Mathematica Hungarica*, **54**, 185-190. https://doi.org/10.1007/BF01952047

[11] Özen, F. and Altay, S. (2001) On Weakly and Pseudo Symmetric Riemannian Spaces. *Indian Journal of Pure and Applied Mathematics*, **33**, 1477-1488.

[12] Tarafdar, M. (1991) On Pseudo Symmetric and Pseudo Ricci Symmetric Sasakian Manifolds. *Periodica Mathematica Hungarica*, **22**, 125-129. https://doi.org/10.1007/BF02327868

[13] De, U.C., Murathan, C. and Özgür, C. (2010) Pseudo Symmetric and Pseudo Ricci Symmetric Warped Product Manifolds. *Communications of the Korean Mathematical Society*, **25**, 615-621. https://doi.org/10.4134/CKMS.2010.25.4.615

[14] Tarafdar, M. and De, U.C. (1995) On Pseudo Symmetric and Pseudo Ricci Symmetric K-Contact Manifolds. *Periodica Mathematica Hungarica*, **31**, 21-25. https://doi.org/10.1007/BF01876349

[15] Chaki, M.C. (1998) On Pseudo Ricci Symmetric Maniflod. *Bulgarian Journal of Physics*, **15**, 526-531.

[16] Arslan, K., *et al*. (2001) On Pseudo Ricci Symmetric Manifolds. *Balkan Journal of Geometry and Its Applications*, **6**, 1-5.

[17] De, U.C. and Mazumder, B.K. (1998) On Pseudo Ricci Symetric Spaces. *Tensor*, **60**, 135-138.

[18] Takahashi, T. (1977) Sasakian ϕ-Symmetric Spaces. *Tohoku Mathematical Journal*, **29**, 91-113. https://doi.org/10.2748/tmj/1178240699

[19] De, U.C. and Biswas, H.A. (1993) On Pseudo-Conformally Symmetric Manifolds. *Bulletin of the Calcutta Mathematical Society*, **85**, 479-486.

[20] Shukla, S.S. and Shukla, M.K. (2010) On ϕ-Symmetric Para Sasakian Manifolds. *International Journal of Mathematical Analysis*, **4**, 761-769.

[21] Xu, X. and Chao, X. (1998) Two Theorems on (ε)-Sasakian Manifolds. *International Journal of Mathematics and Mathematical Sciences*, **21**, 249-254. https://doi.org/10.1155/S0161171298000350

[22] Deshmukh, S., Al-Sodais, H. and Alodan, H. (2011) A Note on Ricci Solitons. *Balkan Journal of Geometry and Its Applications*, **16**, 48-55.

[23] Duggal, K.L. (1990) Space Time Manifolds and Contact Structure. *International Journal of Mathematics and Mathematical Sciences*, **13**, 545-553. https://doi.org/10.1155/S0161171290000783

[24] Sato, I. (1976) On a Structure Similar to Almost Contact Structure. *Tensor*, **30**, 219-224.

[25] Yano, K. (1940) Concircular Geometry. *Proceedings of the Imperial Academy, Tokyo*, **16**, 195-200. https://doi.org/10.3792/pia/1195579139