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Determination of the geopotential and orthometric height based on frequency shift equation

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ABSTRACT

The orthometric height (OH) system plays a key role in geodesy, and it has broad applications in various fields and activities. Based on general relativity theory (GRT), on an arbitrary equi-geopotential surface, there does not exist the gravity frequency shift of an electromagnetic wave signal. However, between arbitrary two different equi-geopotential surfaces, there exists the gravity frequency shift of the signal. The relationship between the geopotential difference and the gravity frequency shift between arbitrary two points P and Q is referred to as the gravity frequency shift equation. Based on this equation, one can determine the geopotential difference as well as the OH difference between two separated points P and Q either by using electromagnetic wave signals propagated between P and Q, or by using the Global Positioning System (GPS) satellite signals received simultaneously by receivers at P and Q. Suppose an emitter at P emits a signal with frequency f towards a receiver at Q, and the received frequency of the signal at Q is f', or suppose an emitter on board a flying GPS satellite emits signals with frequency f towards two receivers at P and Q on ground, and the received frequencies of the signals at P and Q are f'_P and f'_Q, respectively, then, the geopotential dif- ference between these two points can be determined based on the geopotential frequency shift equation, using either the gravity frequency shift $f' - f$ or $f'_Q - f'_P$, and the corresponding OH difference is further determined based on the Bruns' formula. Besides, using this approach a unified world height datum system might be realized, because P and Q could be chosen quite arbitrarily, e.g., they are located on two separated continents or islands.

Keywords: Equi-Frequency Geoid; Gravity Frequency Shift Equation; GPS Signal; Geopotential; Orthometric Height; World Height Datum System Unification

1. INTRODUCTION

The orthometric height (OH), the height above the geoid along the gravity plumb line, plays an important role in geodesy, and has broad applications in various fields. Conventionally, the OH is determined by leveling with additional gravimetry [1], due to the fact that the leveling goes along the equigeopotential surface, and the non-parallel influences of different equigeopotential surfaces should be considered based on the measured gravity data. The conventional approach has at least three drawbacks: 1) the error is accumulated (becomes larger and larger) with the increase of the length of the measurement line; 2) it is difficult to connect two separated points which are located on two continents or islands separated by sea; 3) the leveling is a very laborious work requiring a lot of manpower and equipments, especially in mountainous areas.

To conquer the mentioned drawbacks in conventional approach, Bjerhammar (1985) put forward an idea to determine the OH based on the general relativity theory (GRT) [2]: the OH might be determined by precise clocks. This approach is referred to as the clock approach for convenience. Since the clock approach is based on the comparisons between precise atomic clocks between two stations by clock transportation approach [3], it is seriously constrained in practical applications due to the fact that atomic clocks are very expensive for general use and very difficult to control the normal work condition during their transportation. Just due to this reason, Shen *et al.* (1993) suggested that the OH could be determined by gravity frequency shift, which is referred to as the frequency shift approach. Both the clock approach and the frequency shift approach are referred to as the relativistic approach [4]. Using the relativistic approach, the above mentioned drawbacks existed in the conventional approach could be overcome. Especially, the Global Positioning System (GPS) technique provides a good opportunity to determine the OH by using the GPS signals based on the frequency shift approach [4-7], which is referred to as the GPS frequency approach.

Though GPS leveling provides an approach in determining the OH [8], to determine the OH with high precision, e.g., at the centimeter-level accuracy, it requires the condition that a global or local geoid with the corresponding precision (e.g., centimeter-level accuracy) has been a priori established. This condition can not be satisfied in many cases, e.g., in mountainous areas. Especially, since a precise global geoid is not yet established, the GPS leveling approach is seriously constrained in connecting the height datum marks located in different continents.

In this paper, after introducing the definition of the relativistic geoid by precise clocks in Section 2, the definition of the equi-frequency geoid and the derivation of the gravity frequency shift equation are provided in Section 3. Then, in Section 4, based on the gravity frequency shift equation, we provide the approach to determine the geopotential and OH using electromagnetic wave signals propagated between two points on ground, especially using GPS signals received by two separated receivers on ground. In the following section, we discuss some problems related to the unification of the world height system, and in the last section, we discuss the problems related to the stability of the atomic clocks, and conclude that the frequency shift approach for determining the geopotential and OH is prospective. This paper is an extension of Shen *et al.* (2008b) [7].

2. DEFINITION OF THE RELATIVISTIC GEOID

2.1. Equi-Geopotential Surfaces

We point out that there does not exist essential difference between gravitation and gravity, and the only difference is due to the choices of different reference systems [9]. Similarly, we can say the same about the gravitational potential and geopotential. In fact, the metric tensor $g_{\mu\nu}$ has the character of the gravitational potential [9-11], and consequently it has the character of the geopotential. According to GRT, a precise clock runs quicker at the position with higher geopotential than a precise clock at the position with lower geopotential. To establish the relationship between the keeping time of clocks with the geopotentials with which the clocks are located at the positions, we investigate the proper time interval [11,12]

$$\mathrm{d}\tau^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = g_{00}\mathrm{d}t^2 + 2g_{0i}\mathrm{d}t\mathrm{d}x^i + g_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad (1)$$

in which, $\mathrm{d}\tau^2$ is the proper time and it is an invariant quantity, x^μ are the 4-dimensional coordinates, where x^0 is the time coordinate, $x^i\ (i = 1,2,3)$ are the space coordinates. The Einstein summation convention is applied throughout this paper: the summation will be applied if and only if there are two same indexes, one being up and another being sub. In addition, the light unit system, $c \equiv 1$, is used. In this case, the speed is a pure quantity without unit, and the length has the same unit as that of time. Since g_{00} of $g_{\mu\nu}$ corresponds to energy, the geopotential could be expressed by g_{00} [10,13,14]. Hence, set

$$C = g_{00} \quad (2)$$

where C is a constant, which defines a set of equi-geopotential surfaces. **Eq.1** can be rewritten as

$$\frac{\mathrm{d}\tau^2}{\mathrm{d}t^2} = g_{00} + 2g_{0i}v^i + g_{ij}v^iv^j \quad (3)$$

where $v^i = \mathrm{d}x^i/\mathrm{d}t$ denotes the particle's velocity. Since the geopotential surface should keep the static balance state, it holds $v^i = 0$. Then, equation (3) becomes

$$\frac{\mathrm{d}\tau^2}{\mathrm{d}t^2} = g_{00} \quad (4)$$

From **Eqs.2** and **4** one gets

$$\mathrm{d}t^2 = \frac{\mathrm{d}\tau^2}{\mathrm{C}} \quad (5)$$

Eq.5 shows that on the equi-geopotential surface precise clocks run with the same rate. Based on this equation, Bjerhammar (1985, 1986) defined the equi-geopotential surface as "a closed curve surface on which all the precise clocks run with the same rate" [2,15], which could be properly called the equi-time-rate surface [4,10,16].

The equigeopotential surface defined as above was first put forward by Bjerhammar (1985, 1986) [2,15], later redefined by Soffel *et al.* (1988b) in a more rigorous sense [14], and it can be properly called the equi-time-rate surface [16]. On the equigeopotential surface, the clock's running rate keeps the same, and consequently the vibration frequency of the clock must also keep the same [4,11]. That is to say, if there are two points A and B on the equigeopotential surface, there does not exist gravity frequency shift. In fact, as the light signal propagates on the quigeopotential surface, there does not exist the gain or loss of energy. Based on this viewpoint, we can define the equi-geopotential surface as follows [4,10,16]: the equigeopotential surface is such a closed curve surface on which there does not exist gravity frequency shift. The equigeopotential surface so defined may be properly called the equi-frequency surface [4,16].

2.2. Relativistic Geoid

In conventional geodesy, the geoid is defined as "the closed equi-geopotential surface nearest to the mean sea level" [1], which is referred to as the conventional geoid for convenience.

In relativistic geodesy, based on the definition of the equi-time-rate surface, Bjerhammar (1985, 1986) defined the relativistic geoid as "the closed curve surface nearest to the mean sea level on which precise clocks run with the same rate" [2,15], which is properly referred to as the equi-time-rate geoid [4,10]. In fact, based on the definition of the equi-time-rate surface, the relativistic geoid can be simply defined as the equi-time-rate surface nearest to the mean sea level.

According to the relativistic definition, the geoid can be determined by using precise clocks. Combining **Eqs.4** and **5** one can write down

$$\mathrm{d}t = \left(\frac{1}{C}\right)^{1/2} \mathrm{d}\tau = \left(g_{00}\right)^{-1/2} \mathrm{d}\tau \tag{6}$$

which gives rise to a clock's running rate on an arbitrary equi-time-rate surface. Suppose the equi-time-rate geoid S_0 and an arbitrary equi-time-rate surface S_H are respectively given by the following equations:

$$\begin{aligned} g_{00} &= C_0 \\ g_{00} &= C_H \end{aligned} \tag{7}$$

where C_0 and C_H are the geopotential constants on the equi-time-rate geoid S_0 and the equi-time-rate surface S_H, respectively. Then

$$\mathrm{d}t_0 = \left(C_0\right)^{-1/2} \mathrm{d}\tau,\ \mathrm{d}t_H = \left(C_H\right)^{-1/2} \mathrm{d}\tau \tag{8}$$

and consequently we have

$$\mathrm{d}t_H = \sqrt{\frac{C_0}{C_H}} \mathrm{d}t_0 \tag{9}$$

where $\mathrm{d}t_0$ and $\mathrm{d}t_H$ denote the clocks' running rates (unit seconds) on the equi-time-rate geoid and the H-equi-time-rate surface that passes the point just above the datum point on the geoid with the OH, denoted by H, respectively.

It is noted that the difference between the relativistic geoid and the conventional geoid is about 0.5 cm [10,17]. Such a difference could be neglected in general applications, but should be taken into account in high precise geoid determination.

According to **Eq.6** the geopotential value at an arbitrary point on the Earth's surface can be determined based on the clock transportation approach [3]. Though there are other approaches for time comparison between two separated clocks located at two stations, e.g., the GPS common-view approach and the approach of two-way time transfer by satellite [18], they provide the accuracy about parts of nanoseconds, and consequently they are too poor to determine a meaningful geopotential difference. In non-rela- tivistic geodesy, the measurements of the geopotentials are generally realized by combining gravimetry and leveling. The measurement procedure is very laborious, and the accumulated measurement error becomes larger and larger as the length of the measurement line increases. These drawbacks could be overcome by clock transportation approach. We note that the accuracy of determining the geopotentials by using precise clocks depends on the accuracies of the clocks. If the accuracy level of the atomic clocks is on the order of 10^{-16}, the accuracy level of the determined geopotentials corresponds to the height difference of 1 meter. In recent years, the time and frequency science develop quickly. Atomic clocks with the stability level of 10^{-16} have been created [19-21]. It is noted that there are several study groups investigating the "optical frequency standard", and significant results have been achieved [22-27]. They compared the different "optical frequency standards", and found that all the stabilities are in the level of 10^{-18} to 10^{-19} [27]. Scientists predict that, in the near future, "optical clocks" with the stability of 10^{-18} could be realized. This will provide a firm foundation for determining the geopotential or OH at the centimeter level using clock transportation approach [3] or frequency shift approach (see Section 3).

However, concerning the clock transportation approach, at present, the atomic clocks available are very expensive, very heavy, and quite difficult for normal work during the transportation. Hence, only if portable, relatively cheap and precise clocks were created, one has to pursue other approaches to determine the geopotential differences. This is the motivation that the frequency shift approach was proposed [4,7].

3. FREQUENCY SHIFT EQUATION OF ELECTROMAGNETIC SIGNALS

On the equi-geopotential surface, an atomic clock's running rate keeps the same, and consequently the frequency of an atomic clock must also keep the same [6,10-12]. Since a clock's running rate is controlled by the vibration frequency, we can conclude that for arbitrary two points A and B at rest on a same equi-geopotential surface there does not exist electromagnetic signal's frequency shift, which is referred to as the gravity (or geopotential) frequency shift. In virtue of this viewpoint, an equi-geopotential surface could be defined as "a closed curve surface on which there does not exist gravity frequency shift" [4,6,10,16], which is referred to as the equi-frequency surface.

Then, based on the definition of the equi-frequency surface the relativistic geoid could be defined as "the closed curve surface nearest to the mean sea level on which there does not exist gravity frequency shift", which is referred to as the equi-frequency geoid [4]. Or, the relativistic geoid can be simply defined as the equi-frequency surface nearest to the mean sea level [4,6,10,16]. Based on the definition of the equi-frequency geoid one can determine the relativistic geoid by measuring the gravity frequency shifts of electromagnetic signals.

Since the frequency is inversely related to the period based on which the unit second is defined (see http://en.wikipedia.org/wiki/Second_unit), according to **Eq.9** one has [6,10,12]

$$f_H = \sqrt{\frac{C_H}{C_0}} f_0 \tag{10}$$

where C_0 and C_H are the geopotential constants corresponding to the geoid and the H-equi-frequency geopotential surface that passes the point just above the datum point on the geoid with the OH, H, respectively, f_0 and f_H are the atomic clocks' frequencies on the equi-frequency geoid and the H-equi-frequency geopotential surface, respectively. By frequency shift observations, $(f_H - f_0)/f_0$ might be determined. Hence, based on **Eq.10**, if the geopotential constant C_0 on the geoid is determined, C_H can be determined.

It is noted that, at least at present or in the near future, the equi-frequency geoid is more realizable than the equi-time-rate geoid [10]. At present, it is difficult to generally realize the comparisons between two separated clocks by clock transportation approach, due to the fact that precise atomic clock are very expensive for general usage. On the contrary, it is quite easy to generally realize the frequency shift observations, e.g., the generally used GPS observations.

Suppose a light signal with frequency f is emitted from point P and it is received at point Q. Because of the geopotential difference between these two points, the frequency of the received signal is not f but f'. Based on **Eq.4**, the running rates of the atomic clocks P and Q at arbitrary two points on ground are given by the following equation

$$\frac{\mathrm{d}t_Q}{\mathrm{d}t_P} = \sqrt{\frac{(g_{00})_P}{(g_{00})_Q}} \tag{11}$$

Based on the above equation one has

$$\frac{\Delta f}{f} = \frac{f'-f}{f} = \frac{f_Q - f_P}{f_P} = \sqrt{\frac{(g_{00})_Q}{(g_{00})_P}} - 1 \tag{12}$$

where $f = f_P$, $f' = f_Q$. In **Eq.12**, f_P and f_Q are the frequencies at P and Q, respectively. Accurate to the order V^2 (V is the gravitational potential), g_{00} can be expressed as [10,11]

$$g_{00} = -1 + 2V - 2V^2 + 2\Gamma = -1 + 2W - 2V^2 \tag{13}$$

where $W = V + \Gamma$ is the classical Newtonian geopotential, Γ is the centrifugal force potential. Throughout this paper the definition of the geopotential in physical geodesy is applied: it always holds that $W \geq 0$, which is different from the definition in physics. Combining **Eqs. 12** and **13**, accurate to ΔW, one has

$$\Delta f \equiv f' - f = -f\Delta W \equiv -f\left(W_Q - W_P\right) \tag{14}$$

where W_P and W_Q are the geopotentials at point P and Q, respectively. **Eq.14** is the gravity frequency shift equation, which was confirmed by various physics experiments [28-32].

The frequency approach has special advantages compared to the clock transportation approach (Cf. Section 4). As mentioned before, concerning the OH determination, clock transportation approach is difficult for general applications (Cf. Section 2.2). However, the gravity frequency shift between arbitrary two points P and Q on ground could be directly determined using GPS signals, even these two points are located far away from each other (Cf. Section 4.2).

Suppose the geopotential at point P is given, then from **Eq.14** one can determine the geopotential at an arbitrary point Q by measuring the gravity frequency shift Δf between P and Q, in virtue of the following equation

$$W_Q = W_P - \frac{\Delta f}{f} \tag{15}$$

If the point P is chosen on the geoid, then one has (Shen *et al.*, 2008a)

$$W_Q = C_0 - \frac{\Delta f}{f} \tag{16}$$

where C_0 is the geoid geopotential constant, the determination of which could be found in e.g., Chao *et al.* (2007) [33]. Once C_0 is determined, the geopotential at an arbitrary point Q on the Earth's surface can be determined by using frequency shift observation method. The basic principle of measuring the frequency shift is stated in the sequel.

Referring to **Figure 1**, set at point P an emitter which emits a signal with frequency f and a receiver at point Q receives the emitted signal with frequency f' com-

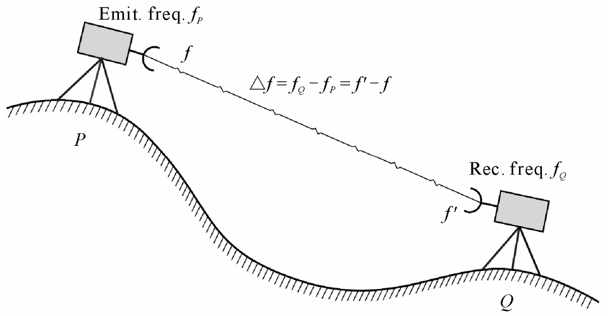

Figure 1. An emitter at point P on ground emits a light signal with frequency f towards a receiver at point Q on ground, and the received signal's frequency is not f but f', the difference between the received frequency and the emitting frequency is the gravity frequency shift $\Delta f = f' - f$.

ing from P. Then, comparing the frequency f' of the received signal with the standard frequency f itself, the frequency shift $\Delta f = f' - f$ might be determined. Consequently, according to **Eq.16** one can determine the geopotential difference $\Delta W_{PQ} = W_Q - W_P$ between P and Q. Using the same principle one can find the geopotential difference $\Delta W_{0P} = W_P - W_0$ between the geoid and the equi-frequency surface which contains the point P. If C_0 is a given constant (generally it can be determined by satellite geodesy approach, see Section 5), W_P as well as W_Q can be found. According to **Eq.16**, once the gravity frequency shift $\Delta f = f_Q - f_P$ between points P and Q is determined, the geopotential difference ΔW_{PQ} can be determined. If what it measured is the gravitational frequency shift Δf_G, it can be found the gravitational potential difference $\Delta V_{PQ} = V_Q - V_P$ between these two points by following equation:

$$\Delta V_{PQ} = V_Q - V_P = -\frac{\Delta f_G}{f} = -\frac{f' - f}{f} \tag{17}$$

where f denotes the emitting frequency, f' denotes the received frequency due to the gravitational potential difference between P and Q. Once ΔV_{PQ} is determined, the geopotential difference ΔW_{PQ} can be determined, due to the fact that, $\Delta W_{PQ} = \Delta V_{PQ} + \Delta\Gamma_{PQ}$, $\Delta\Gamma_{PQ} = \Gamma_Q - \Gamma_P$, Γ_P and Γ_Q are centrifugal force potentials at P and Q, respectively, and they are known quantities.

4. DIRECT ORTHOMETRIC HEIGHT DETERMINATION

4.1. Orthometric Height Determination between Two Points on Ground

In the sequel we consider how to determine the OH difference ΔH between two points P and Q according to the measured gravity frequency shift Δf_{PQ} between P and Q.

Without loss of generality, it is assumed that $\Delta f_{PQ} > 0$. In this case, from **Eq.15** one gets

$$W_Q = W_P - \frac{\Delta f}{f} < W_P \tag{18}$$

This means that the geopotential value at point Q is smaller than that at point P, and P and Q can be taken for granted that they are located on two different equi-frequency surfaces $W = C_P$ and $W = C_Q$, respectively. It is noted that, $W(\boldsymbol{r})$ is Newtonian geopotential at the field point $\boldsymbol{r}$, taking positive value, and the less the value of the geopotential $W(\boldsymbol{r})$, the field point is further from the center of the Earth. Let $W = W_0$ denote the equi-frequency geoid, then the geopotential differences between the equi-frequency geoid and the point P as well as Q can be respectively expressed as

$$\Delta W_{0P} = -\frac{\Delta f_{0P}}{f},\ \Delta W_{0Q} = -\frac{\Delta f_{0Q}}{f} \tag{19}$$

where Δf_{0P} and Δf_{0Q} express the gravity frequency shifts between the equi-frequency geoid and the point P as well as Q, respectively. Expanding the equi-frequency surface $W = C_P$ into Tayler series with respect to the OH H on the equi-frequency geoid $W = W_0$, one has

$$W = W_0 + \left(\frac{\partial W}{\partial H}\right)_P H_P + \cdots \tag{20}$$

where $(\partial W / \partial H)_P = -g_P$ is the gravity value on the geoid corresponding to the point P, and H_P is the OH of point P. When the height is not so large (e.g., less than 200 meters, *i.e.*, the mountainous areas are not considered), only the first two terms are kept in the right-hand side of **Eq.20**, and instead of g_P one uses the average normal gravity $\bar{\gamma}$. Hence one has

$$H_P = -\frac{\Delta W_{0P}}{\bar{\gamma}} = -\frac{W_P - W_0}{\bar{\gamma}} \tag{21}$$

Similarly

$$H_Q = -\frac{\Delta W_{0Q}}{\bar{\gamma}} = -\frac{W_Q - W_0}{\bar{\gamma}} \tag{22}$$

It is noted that the condition under which **Eqs.21** and **22** hold is that the height H is much smaller than the Earth's radius. From **Eqs.21** and **22** one can find the height difference between P and Q:

$$\Delta H \equiv H_Q - H_P = -\frac{C_Q - C_P}{\bar{\gamma}} = -\frac{\Delta W_{PQ}}{\bar{\gamma}} \tag{23}$$

Substituting **Eq.14** into **Eq.23** one gets

$$\Delta H = \frac{1}{\bar{\gamma}} \frac{\Delta f}{f} \quad (24)$$

From **Eq.24** one can see that the accuracy of ΔH depends on that of Δf, and consequently it is related to the stabilities of the frequencies of the emitter and receiver. In theory, if the stabilities of the frequencies of the emitter and receiver are better than 10^{-18} (which is possible to be achieved because of quick development of time science, Cf. Section 2), the accuracy in determining the height difference between two different points could achieve the order of centimeter. Based on the above analysis one can see that, no matter what is determined, the geopotential difference (14), the gravitational potential difference (17), or the height difference (24), the key problem is how to measure the gravity frequency shift and estimate the accuracy of Δf. We note that the gravitational frequency shift can be determined if the gravity frequency shift is determined and vice versa. It should be emphasized that **Eq.24** is only suitable to the non-rough areas. In the mountainous areas, **Eq.20** should be kept to the second order of the height *H*. The details are referred to [6,17].

4.2. OH Determination Using GPS Signals

Referring to **Figure 2**, suppose an emitter is set on board a flying satellite (e.g., GPS satellite), which can emit electromagnetic wave signals with regular intervals. Then, by receiving the signals from the emitter simultaneously at two points P and Q, one could determine the geopotential difference between P and Q, based on the gravity frequency shift **Eq.14**.

Now, suppose the signal emitter E is set on board a satellite, and two signal receivers P and Q on ground receive the signals coming from E corresponding to an emitting time t. Further suppose the received frequencies of the signals corresponding to time t are recorded

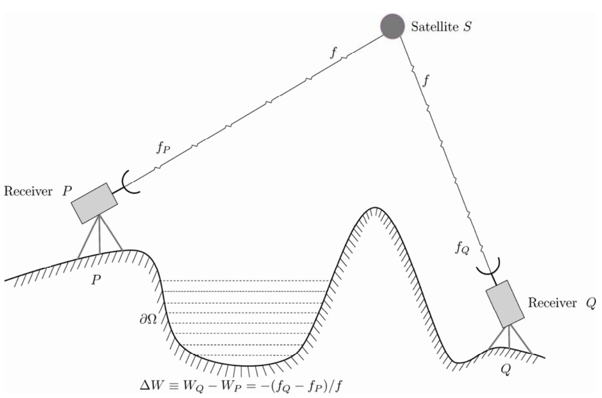

Figure 2. Two receivers at points *P* and *Q* on the Earth's surface $\partial\Omega$ receive simultaneously the light signals with frequency *f* emitted on board a flying satellite *S*.

by P and Q receivers in some way, respectively, *i.e.*, f_P and f_Q at t_P and t_Q ($t_P > t$ and $t_Q > t$, due to the delay of the signal propagation) are recorded by receivers at P and Q, respectively. Note that the time t_P at which the signal is received by P receiver is generally different from the time t_Q at which the signal is received by Q receiver. By comparing the received frequencies f_P and f_Q it could be determined the geopotential difference $\Delta W_{PQ} = W_Q - W_P$ [4], which is just given by **Eq.14**.

One of the advantages by using the geopotential frequency shift approach lies in that a unified global height datum system could be established: two receivers located at two height datum points A and B, which belong to two separated continents or islands, could simultaneously receive the signals emitted by a satellite source emitter, and consequently the frequency shift between A and B is determined; then, based on the geopotential frequency shift equation the geopotential difference as well as the OH difference between A and B is determined. By such a way, the height datum of one continent (or island) could be connected to the height datum of another continent (or island). Then, a unified global height datum system might be established.

In practical applications, however, the gravity frequency shift signals in GPS frequency observations are largely contaminated by other noise frequency shifts, which include the first-order Doppler frequency shift, ionosphere frequency shift, troposphere frequency shift, clock errors and random influences. To separate the gravity frequency shift signals from other noises is not so easy as generally imagined. Hence, to determine the geopotential or OH of an arbitrary point on ground, the key problem is how to draw the gravity frequency shift signals from the GPS frequency observations. The investigations on this problem will be provided in a separated paper.

5. UNIFICATION OF THE WORLD ORTHOMETRIC HEIGHT DATUM SYSTEM

Theoretically, the final determination of the geoid depends on the choice of constant $W_0 \equiv C_0$ on the geoid. Given different W_0 s, we get different equigeopotential surfaces. In practice, to determine the geoid, we always choose some tidal gauges' average sea level as the datum (standard) of a local geoid. Theoretically, if W_0 is not determined properly, the real geoid will deviate from the datum. The key problem lies in that at this situation we still regard the datum as in consistency with the geoid. As a result, there will be a systematic error in the height system; and furthermore, since the mean sea level does not coincides with any equigeopotential surface

[10,34-41], various datums in the world are in fact located on different equigeopotential surfaces. This will give rise to the inconsistency of the world height system. If one chooses point A as the datum of the geoid, the geopotential constant W_0 on the geoid cannot be chosen arbitrarily but determined uniquely [10]. This is because of two causes. One is in that to determine the geoid, a reference ellipsoid is needed. The normal potential U_0 on the surface of the ellipsoid is given a priori, or can be uniquely determined by the given parameters of the ellipsoid. No matter which method is chosen, W_0 is completely determined, since one should investigate gravity and normal gravity, geopotential and normal geopotential, etc., in the same system (a unified coordinate system). Another one lies in that to solve the boundary value problem we require that the gravitational part of W is regular at infinity. This condition (combining with the choice of the ellipsoid) will limit the variation of W_0. Theoretically, we may suggest different methods to determine W_0. The most basic method might be stated as follows [10]: Suppose we have chosen a reference ellipsoid E_r, e.g., WGS84 ellipsoid [42]. If a definite shape of the ellipsoid is given (*i.e.*, given E_r's semimajor axis *a* and semiminor axis *b*), the normal geopotential U_0 can be calculated theoretically, and consequently the geopotential constant W_0 on the geoid is determined (because of the condition that $U_0 = W_0$). However, a dilemma occurs: If U_0 is given, the E_r's semimajor and semiminor axises *a* and *b* are determined uniquely, but in this case we need to know W_0 a priori; inversely, if *a* and *b* are given, U_0 is determined uniquely, but different *a* and *b* will introduce different U_0 s. Without previous knowledge, it is impossible to choose *a* and *b* so that $U_0 = W_0$. The best way might be like this: one determines U_0 so that it app- roximates W_0 gradually. Hence, to precisely determine W_0 is a delicate matter [33,43]. Any error stemmed from W_0 will give rise to a systematic error to the geoid. In fact, How to precisely determine W_0 is an open problem.

Once W_0 is determined, the standard level geoid is determined. Then, using GPS frequency shift approach, we can unify the world height datum system. The basic principle is stated as follows.

Suppose W_0 is precisely determined, e.g., with the accuracy of 1 cm level. Then, with the same accuracy level one can determine the OH of a datum point A located at least in a relatively plain area with small OH. Then, the point A is taken as the datum point of world height datum system. Referring to **Figure 2**, suppose the OH H_P of a point P on ground is determined by leveling plus gravimetry approach between A and P. Then, using GPS frequency shift approach one can determine the OH H_Q of an arbitrary point Q on ground. Since the point Q on ground is quite arbitrary, one can unify the world height datum system using the frequency shift approach.

6. DISCUSSIONS AND CONCLUSIONS

According to the GRT, a precise (atomic) clock located at the position P with higher geopotential runs quicker than a precise clock located at the position Q with lower geopotential [12,44,45]. Equivalently, the vibration frequency of the clock at P is larger than that of the clock at Q. Then, the relativistic geoid can be defined based on the clock's running rate, which is referred to as the equi-time-rate geoid. In another aspect, the relativistic geoid can be defined based on the clock's vibration frequency, or can be defined by the frequency shift equation, and so defined geoid is referred to as the equi-frequency geoid. The realization of the equi-frequency geoid is based on the frequency shift approach, and the realization of the equi-time-rate geoid is based on the clock transportation approach.

With clock transportation approach [3,46], the key problem is to compare two clocks located at different places by transporting portable clocks. Hence, one needs portable clocks to complete the comparisons. At present, though portable clocks with the stability better than 1×10^{-16} are not yet available, we may determine the geopotential difference at the accuracy level of 1 $m^2 \cdot s^{-2}$ (equivalent to 0.1 m) between two separated points on ground by using clocks with the stability around 1×10^{-14} (Shen *et al.*, 2009). The problem is whether the portable clocks with the stability 1×10^{-14} [3] for the aim of the transportation comparisons are available.

With frequency shift approach, especially the GPS frequency shift approach [4,5,10,16,46], the key problem is to draw the frequency shift information from the frequency observations, which include other influences except for the gravity frequency shift. In fact, the GPS frequency observations include not only the gravity frequency shift, but also other noise frequency shifts such as the Doppler frequency shift, ionosphere frequency shift, troposphere frequency shift, etc. The noise frequency shifts should be removed from the observations. Obviously, after removing the noise frequency shifts, the accuracy in determining the OH depends on the stability of the time-keeping system. If the stability of the time-keeping system is better than 1×10^{-18}, we can determine the OH with the accuracy of 1 cm.

To directly determine the geopotential and OH using GPS signals, the GPS frequency shift approach is prospective. This is due to the fact that the time and frequency science develop very quickly, and clocks or time-keeping systems and portable clocks with the stability better than 1×10^{-18} might be available in the near future [22,27]. Then, a new era may come that the geometric position (coordinates) and the geopotential (as well as OH) could be simultaneously determined using GPS technique.

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