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Research on the nonlinear spherical percolation model with quadratic pressure gradient and its percolation characteristics

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ABSTRACT

For bottom water reservoir and the reservoir with a thick oil formation, there exists partial penetration completion well and when the well products the oil flow in the porous media takes on spherical percolation. The nonlinear spherical flow equation with the quadratic gradient term is deduced in detail based on the mass conservation principle, and then it is found that the linear percolation is the approximation and simplification of nonlinear percolation. The nonlinear spherical percolation physical and mathematical model under different external boundaries is established, considering the effect of wellbore storage. By variable substitution, the flow equation is linearized, then the Laplace space analytic solution under different external boundaries is obtained and the real space solution is also gotten by use of the numerical inversion, so the pressure and the pressure derivative bi-logarithmic nonlinear spherical percolation type curves are drawn up at last. The characteristics of the nonlinear spherical percolation are analyzed, and it is found that the new nonlinear percolation type curves are evidently different from linear percolation type curves in shape and characteristics, the pressure curve and pressure derivative curve of nonlinear percolation deviate from those of linear percolation. The theoretical offset of the pressure and the pressure derivative between the linear and the nonlinear solution are analyzed, and it is also found that the influence of the quadratic pressure gradient is very distinct, especially for the low permeability and heavy oil reservoirs. The influence of the non-linear term upon the spreading of pressure is very distinct on the process of percolation, and the nonlinear percolation law stands for the actual oil percolation law in reservoir, therefore the research on nonlinear percolation theory should be strengthened and reinforced.

Keywords: Nonlinear Spherical Percolation; Quadratic Pressure Gradient; Percolation Characteristics; Reservoir; Partial Penetration Completion Well; Mathematic Model

1. INTRODUCTION

So far, the research on nonlinear percolation has increasingly aroused widespread concern and attention. The nonlinear percolation is the modern development of a new direction [1]. Retaining the nonlinear term was proposed by Odeh A S [2]. He thought ignoring the quadratic gradient term would cause larger error in hydraulic fracturing, big pressure drop flow, DST and large pressure drop pulse testing. Bai M Q [3] considered that ignoring the quadratic gradient term in flow equation is equivalent to ignoring convection flow term in diffusion-convection equation. Wang Y [4] established the nonlinear flow model in poroelastic media. Chakrabarty C [5] derived the mathematical model with nonlinear diffusion equation and made the quantitative analysis of the quadratic term. Braeuning S [6] established the nonlinear radial flow model of the variable-rate well-test. Tong Dengke [7,8] solved the well test models of heterogeneous and dual porosity reservoir. Concerning the spherical flow, the linear spherical flow model was studied by William E. Brigham, Charles A. Kohlhaas and Mark A. Proett, *et al.* [9-11]. In their models, no nonlinear spherical flow model is found, so this paper presents the nonlinear spherical flow model and researches its percolation characteristics for partial penetration completion well in the formation.

2. DEDUCTION OF THE NONLINEAR SPHERICAL PERCOLATION EQUATION

When single-phase fluid flow through porous medium, it would conform to the mass conservation principle, so by this principle the flow equation of continuity can be expressed by

$$\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=-\frac{\partial}{\partial t}(\rho\phi) \quad (1)$$

where: v is flow velocity, cm/s; ρ is oil density, g/cm^3; t is flow time, s; φ is rock porosity, fraction; x, y, z represent the Cartesian coordinates.

If ignoring the impact of gravity and capillary forces, and the inertial resistance is not considered, it would conform to the Darcy's law, so the equation of motion is as follows

$$v=-\frac{k}{\mu}\nabla p \quad (2)$$

where: k is rock permeability, μm^2; μ is fluid viscosity, mPa·s; p is formation pressure, MPa.

The fluid flow through porous medium is a process of percolation, and is also a state of constantly changing process, in which the parameters related to percolation are constantly changing with pressure and temperature. Usually the change of temperature in reservoir is inappreciable, so the flow is taken as isothermal flow. The rock and fluid are elastic and slightly compressible, the state equation of fluid and the state equation of rock are expressed as follows respectively

$$\rho=\rho_0 e^{C_\rho(p-p_0)} \quad (3)$$

$$\phi=\phi_0 e^{C_f(p-p_0)} \quad (4)$$

where: C_ρ is oil compressibility, MPa^{-1}; C_f is rock compressibility, MPa^{-1}; the subscript "0" represents reference value, usually use the value in standard conditions.

Substitute **Eq.(2)** into **Eq.(1)**

$$\frac{\partial}{\partial x}(\rho\frac{k_x}{\mu}\frac{\partial p}{\partial x})+\frac{\partial}{\partial y}(\rho\frac{k_y}{\mu}\frac{\partial p}{\partial y})+ \frac{\partial}{\partial z}(\rho\frac{k_z}{\mu}\frac{\partial p}{\partial z})=\frac{\partial}{\partial t}(\rho\phi) \quad (5)$$

$$\frac{\partial}{\partial x}(\rho\frac{k_x}{\mu}\frac{\partial p}{\partial x})=\frac{\rho k_x}{\mu}\frac{\partial^2 p}{\partial x^2}+ \frac{\rho}{\mu}\frac{\partial p}{\partial x}\frac{\partial k_x}{\partial x}+\frac{k_x}{\mu}\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial x} \quad (6)$$

Changing the form of **Eq.(3)**

$$p=\frac{1}{C_\rho}\ln\rho-\frac{1}{C_\rho}\ln\rho_0+p_0 \quad (7)$$

$$\frac{\partial p}{\partial x}=\frac{1}{\rho C_\rho}\frac{\partial\rho}{\partial x} \quad (8)$$

$$\frac{\partial p}{\partial t}=\frac{1}{\rho C_\rho}\frac{\partial\rho}{\partial t} \quad (9)$$

Substitute **Eq.(8)** into **Eq.(6)**

$$\frac{\partial}{\partial x}(\rho\frac{k_x}{\mu}\frac{\partial p}{\partial x})=\frac{\rho k_x}{\mu}\frac{\partial^2 p}{\partial x^2}+ \frac{\rho}{\mu}\frac{\partial p}{\partial x}\frac{\partial k_x}{\partial x}+\frac{k_x\rho C_\rho}{\mu}(\frac{\partial p}{\partial x})^2 \quad (10)$$

By the same method, the following two equations can be deduced

$$\frac{\partial}{\partial y}(\rho\frac{k_y}{\mu}\frac{\partial p}{\partial y})=\frac{\rho k_y}{\mu}\frac{\partial^2 p}{\partial y^2}+ \frac{\rho}{\mu}\frac{\partial p}{\partial y}\frac{\partial k_y}{\partial y}+\frac{k_y\rho C_\rho}{\mu}(\frac{\partial p}{\partial y})^2 \quad (11)$$

$$\frac{\partial}{\partial z}(\rho\frac{k_z}{\mu}\frac{\partial p}{\partial z})=\frac{\rho k_z}{\mu}\frac{\partial^2 p}{\partial z^2}+ \frac{\rho}{\mu}\frac{\partial p}{\partial z}\frac{\partial k_z}{\partial z}+\frac{k_z\rho C_\rho}{\mu}(\frac{\partial p}{\partial z})^2 \quad (12)$$

Changing the form of **Eq.(4)**

$$p=\frac{1}{C_f}\ln\phi-\frac{1}{C_f}\ln\phi_0+p_0 \quad (13)$$

$$\frac{\partial p}{\partial t}=\frac{1}{\phi C_f}\frac{\partial\phi}{\partial t} \quad (14)$$

Substitute **Eq.(14)** and **Eq.(9)** into **Eq.(5)**, the right of **Eq.(5)** can be changed

$$\frac{\partial}{\partial t}(\rho\phi)=\phi\frac{\partial\rho}{\partial t}+\rho\frac{\partial\phi}{\partial t}=\rho\phi C_\rho+ \rho\phi C_f\frac{\partial p}{\partial t}=\rho\phi C_t\frac{\partial p}{\partial t} \quad (15)$$

$$C_t=C_\rho+C_f \quad (16)$$

where: C_t is total compressibility of rock and oil, MPa^{-1}.

Substitute **Eqs.(10)-(12)** and **Eq.(15)** into **Eq.(5)**, we have

$$(k_x\frac{\partial^2 p}{\partial x^2}+k_y\frac{\partial^2 p}{\partial y^2}+k_z\frac{\partial^2 p}{\partial z^2})+(\frac{\partial p}{\partial x}\frac{\partial k_x}{\partial x}+ \frac{\partial p}{\partial y}\frac{\partial k_x}{\partial y}+\frac{\partial p}{\partial z}\frac{\partial k_x}{\partial z})+C_\rho[k_x(\frac{\partial p}{\partial x})^2+$$

$$k_y(\frac{\partial p}{\partial y})^2 + k_z(\frac{\partial p}{\partial z})^2] = \mu\phi C_t \frac{\partial p}{\partial t} \quad (17)$$

If the permeability is isotropic and constant, $\partial k_x / \partial r = 0, \partial k_y / \partial r = 0, \partial k_y / \partial r = 0$, the **Eq.(17)** becomes

$$(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}) + C_\rho[(\frac{\partial p}{\partial x})^2 +$$

$$(\frac{\partial p}{\partial y})^2 + (\frac{\partial p}{\partial z})^2] = \frac{\mu\phi C_t}{k}\frac{\partial p}{\partial t} \quad (18)$$

Eq.(18) is the governing differential equation in Cartesian coordinates, the equation in radial spherical coordinates becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial p}{\partial r}) + C_\rho(\frac{\partial p}{\partial r})^2 = \frac{\mu\phi C_t}{k}\frac{\partial p}{\partial t} \quad (19)$$

where, r represents the radial spherical coordinates.

Eq.(19) is the nonlinear flow governing partial differential equation with quadratic pressure gradient term. We call the second power of the pressure gradient as quadratic pressure gradient.

The function exp(x) by use of Maclaurin series expansion is written by

$$\exp(x) = 1 + x + x^2/2 + \cdots + x^n/n! + \cdots \quad (20)$$

If we use Maclaurin series expansion for **Eqs.(3)** and **(4)** and neglect the second order and the above higher order item, the **Eqs.(3)** and **(4)** can be rewritten by **Eqs. (21)** and **(22)** respectively

$$\rho = \rho_0[1 + C_\rho(p - p_0)] \quad (21)$$

$$\phi = \phi_0[1 + C_f(p - p_0)] \quad (22)$$

The appearance of quadratic pressure gradient term is simply because that we didn't make any simplification for the state **Eqs. (3)** and **(4)** in the deduction of the flow governing partial differential equation. If we use **Eqs. (21)** and **(22)**, instead of **Eqs.(3)** and **(4)**, in the deduction of the flow governing partial differential equation, the quadratic pressure gradient term will not come up, and the deduced flow equation is the conventional linear flow equation, which is shown in almost any percolation mechanics books and papers, so the deduction of the linear flow equation is certainly omitted here. Owing to the existence of quadratic pressure gradient, the flow equation takes on nonlinear properties. Therefore it can be safely concluded that the conventional linear flow equation is the approximation and simplification of nonlinear flow equation with quadratic pressure gradient term, and that the nonlinear percolation law stands for the actual flow law of oil in reservoir.

3 .SPHERICAL PERCOLATION MODELS AND ITS SOLUTION

3.1. Physical Model

For bottom water reservoir, the position of drilling and completion of oil well is usually in the top of the oil formation, the flow diagram shown in **Figure 1**. For some reservoirs, the oil formation is very thick, the position of drilling and completion of well is usually in the middle of the formation, the flow diagram shown in **Figure 2**. For the two actual situations, the oil flow in the porous media is in the form of spherical percolation.

Physical model assumptions are as follows:

1) A single well with partial penetration completion in the formation like **Figure 1** or **Figure 2** products at constant rate, the external boundary may be infinite or closed or constant pressure;

2) The rock and the single-phase fluid are slightly compressible, a constant compressibility;

3) Isothermal and Darcy flow, the permeability and porosity of isotropic properties;

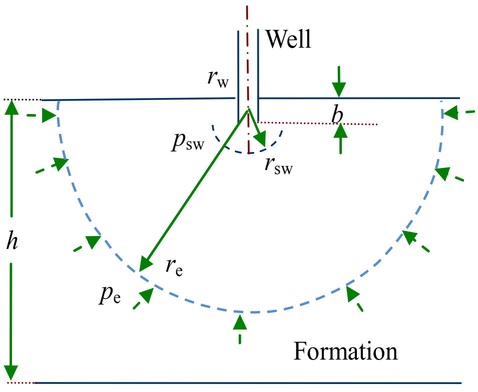

Figure 1. Spherical flow diagram for well completion position in the top of the formation.

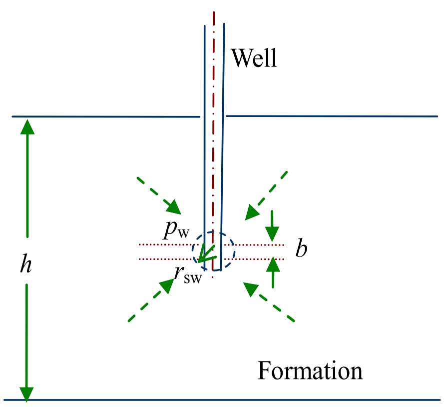

Figure 2. Spherical flow diagram for well completion position in the middle of the formation.

4) Considering wellbore storage effects (in the beginning of opening well, the fluid stored in the wellbore starts to flow, the oil in the formation does not flow);

5) At time t=0, pressure is uniformly distributed in the reservoir, equal to the initial pressure p_i;

6) Ignoring the impact of gravity and capillary forces.

3.2. Mathematic Model

The governing differential equation in radial spherical coordinate system

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial p}{\partial r})+C_\rho(\frac{\partial p}{\partial r})^2=\frac{\mu\phi C_t}{3.6k}\frac{\partial p}{\partial t} \qquad (23)$$

where: r is the radial spherical distance from well, m; the unit of well production time (t) becomes "h", so the coefficient "3.6" appears in the **Eq.(23)**.

Initial conditions

$$p\big|_{t=0}=p_i \qquad (24)$$

where, p_i is initial formation pressure, MPa.

Inner boundary condition

$$\frac{k}{\mu}(r^2\frac{\partial p}{\partial r})\Big|_{r=r_{sw}}=0.921\times10^{-3}qB+$$

$$0.022105C_s\frac{dp_w}{dt} \qquad (25)$$

where: the r_{ws} is called the pseudo well radius of radial spherical flow, and $r_{ws}=b/(2\ln(b/r_w))$ [12], b is the formation penetration thickness of well completion, m; r_w is the real well radius m; q is oil rate at wellhead, m^3/d; B is oil volume factor, dimensionless; C_s is wellbore storage coefficient, m^3/MPa; p_w is wellbore pressure, MPa.

External boundary condition

$$\lim_{r\to\infty}p=p_i \text{ (infinite)} \qquad (26)$$

$$p\big|_{r=r_e}=p_i \text{ (constant pressure)} \qquad (27)$$

$$\frac{\partial p}{\partial r}\Big|_{r=r_e}=0 \text{ (closed)} \qquad (28)$$

where, r_e is external boundary radius, m.

3.3. Solution to Mathematic Model

The dimensionless definitions are as follows:

Dimensionless pressure

$$p_D=kr_{sw}\left(p_i-p\right)/(0.921\times10^{-3}\mu qB);$$

Dimensionless radius based on pseudo spherical flow radius $r_D=r/r_{sw}$;

Dimensionless wellbore storage coefficient

$$C_D=C_s/(3.1416\phi C_t r_{sw}^3);$$

Dimensionless time $t_D=3.6kt/(\phi\mu C_t r_{sw}{}^2)$;

Dimensionless quadratic pressure gradient coefficient

$$\beta=0.921\times10^{-3}qB\mu C_\rho/(kr_{sw})$$

The dimensionless model is as follows:

The governing differential equation in radial spherical coordinate system

$$\frac{\partial^2 p_D}{\partial r_D{}^2}+\frac{2}{r_D}\frac{\partial p_D}{\partial r_D}-\beta(\frac{\partial p_D}{\partial r_D})^2=\frac{\partial p_D}{\partial t_D} \qquad (29)$$

Initial conditions

$$p_D\big|_{t_D=0}=0 \qquad (30)$$

Inner boundary condition

$$C_D\frac{dp_{wD}}{dt_D}-(r_D\frac{\partial p_D}{\partial r_D})\Big|_{r_D=1}=1 \qquad (31)$$

External boundary condition

$$\lim_{r_D\to\infty}p_D(r_D,t_D)=0 \text{ (infinite)} \qquad (32)$$

$$p_D\big|_{r_D=r_{eD}}=0 \text{ (constant pressure)} \qquad (33)$$

$$\frac{\partial p_D}{\partial r_D}\Big|_{r=r_{eD}}=0 \text{ (closed)} \qquad (34)$$

Take

$$p_D=-\frac{1}{\beta}\ln x \qquad (35)$$

where, x is substitution variable between variables.

Making the upper variable substitutions for **Eqs. (29-34)**, the model can be converted to

The governing differential equation

$$(\frac{\partial^2 x}{\partial r_D{}^2}+\frac{2}{r_D}\frac{\partial x}{\partial r_D})=\frac{\partial x}{\partial t_D} \qquad (36)$$

Initial conditions

$$x\big|_{t_D=0}=1 \qquad (37)$$

Inner boundary condition

$$(C_D\frac{\partial x}{\partial t_D}-\frac{\partial x}{\partial r_D}+\beta x)\Big|_{r_D=1}=0 \qquad (38)$$

External boundary condition

$$\lim_{r_D\to\infty}x(r_D,t_D)=1 \text{ (infinite)} \qquad (39)$$

$$x\big|_{r_D=r_{eD}}=1 \text{ (constant pressure)} \qquad (40)$$

$$\frac{\partial x}{\partial r_D}\Big|_{r_D=r_{eD}}=0 \text{ (closed)} \qquad (41)$$

Take

$$x = y / r_D + 1 \quad (42)$$

where, y is substitution variable between variables.

Making the upper variable substitutions for **Eqs. (36-41)**, the model can be converted to

The governing differential equation

$$\frac{\partial^2 y}{\partial r_D{}^2} = \frac{\partial y}{\partial t_D} \quad (43)$$

Initial conditions

$$y\big|_{t_D=0} = 0 \quad (44)$$

Inner boundary condition

$$[C_D \frac{\partial y}{\partial t_D} - \frac{\partial y}{\partial r_D} + (\beta+1)y]\big|_{r_D=1} = -\beta \quad (45)$$

External boundary condition

$$\lim_{r_D \to \infty} y = 0 \text{ (infinite)} \quad (46)$$

$$y\big|_{r_D = r_{eD}} = 0 \text{ (constant pressure)} \quad (47)$$

$$(\frac{\partial y}{\partial r_D} - y\frac{1}{r_D})\big|_{r_D=r_{eD}} = 0 \text{ (closed)} \quad (48)$$

Introduce the Laplace transform based on t_D, that is

$$L[p_D(t_D)] = \overline{p}_D(z) = \int_0^\infty p_D(t_D) e^{-zt_D} dt_D \quad (49)$$

where, z is Laplace space variable.

So, making the Laplace transform of **Eqs.(43-48)**, the model becomes:

The governing differential equation in Laplace space

$$\frac{d^2 \overline{y}}{dr_D{}^2} - z\overline{y} = 0 \quad (50)$$

Inner boundary condition in Laplace space

$$[(C_D z + \beta + 1)\overline{y} - \frac{d\overline{y}}{dr_D}]_{r_D=1} = -\frac{\beta}{z} \quad (51)$$

External boundary condition in Laplace space

$$\lim_{r_D \to \infty} \overline{y} = 0 \text{ (infinite)} \quad (52)$$

$$\overline{y}\big|_{r_D=r_{eD}} = 0 \text{ (constant pressure)} \quad (53)$$

$$(\frac{\partial \overline{y}}{\partial r_D} - \overline{y}\frac{1}{r_D})\big|_{r_D=r_{eD}} = 0 \text{ (closed)} \quad (54)$$

The general solution of **Eq.(50)** can be expressed by

$$\overline{y} = A e^{\sqrt{z} r_D} + B e^{-\sqrt{z} r_D} \quad (55)$$

For infinite boundary:

Substitute **Eq.(55)** into **Eq.(52)**, have

$$A = 0 \quad (56)$$

So the general solution of **Eq.(50)** becomes

$$\overline{y} = B e^{-\sqrt{z} r_D} \quad (57)$$

Substitute **Eq.(57)** into **Eq.(51)**, have

$$B = -\frac{\beta}{z(C_D z + \beta + 1 + \sqrt{z})e^{-\sqrt{z}}} \quad (58)$$

The general solution of **Eq.(50)** can be got by

$$\overline{y} = -\frac{\beta}{z(C_D z + \beta + 1 + \sqrt{z})e^{-\sqrt{z}}} e^{-\sqrt{z} r_D} \quad (59)$$

At the wellbore bottom, $r=r_w$, $r_D=1$, $p=p_w$, $p_D=p_{wD}$, $x=x_w$, $y=y_w$, therefore, the solution of the spherical percolation model with infinite external boundary in Laplace space can be got by

$$\overline{y}_w = \overline{y}\big|_{r_D=1} = -\frac{\beta}{z(C_D z + \beta + 1 + \sqrt{z})} \quad (60)$$

The real space solution y_w and the derivative (dy_w/dt_D) can be easily obtained by use of Stehfest numerical inversion [13] for **Eq.(60)**. Substitute the values of inversion into variable substitution relationships, **Eq.(35)** and **Eq.(42)**, so the real space solution p_{wD} and the derivative (dp_{wD}/dt_D) can be certainly gained. Accordingly, the pressure and the pressure derivative bi-logarithmic type curves of nonlinear spherical percolation can be drawn up (see **Figure 3**).

For constant pressure boundary:

At the wellbore bottom $r_D=1$, $y=y_w$, the **Eq.(55)** becomes

$$e^{\sqrt{z}} \cdot A + e^{-\sqrt{z}} \cdot B - \overline{y}_w = 0 \quad (61)$$

Substitute **Eq.(61)** into **Eq.(51)** and **Eq.(53)**, have respectively

$$-\sqrt{z}e^{\sqrt{z}} \cdot A + \sqrt{z}e^{-\sqrt{z}} \cdot B + (C_D z + \beta + 1)\overline{y}_w = -\frac{\beta}{z} \quad (62)$$

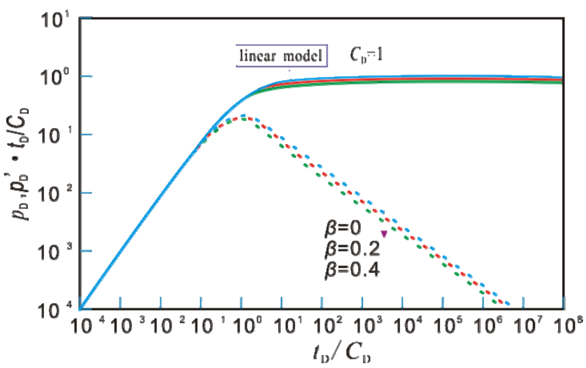

Figure 3. Type curves of nonlinear spherical percolation affected by β under infinite external boundary.

$$e^{\sqrt{z}r_{eD}} \cdot A + e^{-\sqrt{z}r_{eD}} \cdot B = 0 \quad (63)$$

For closed boundary:

Substitute **Eq.(61)** into **Eq.(54)**, have

$$(\sqrt{z}r_{eD} - 1)e^{\sqrt{z}r_{eD}} A - (\sqrt{z}r_{eD} + 1)e^{-\sqrt{z}r_{eD}} B = 0 \quad (64)$$

Combining **Eqs.(61-64)**, the coefficients A and B and the function at wellbore $\overline{y}_w$ in Laplace space can be easily obtained by use of some linear algebra method (such as Gauss-Jordan reduction, etc), then nonlinear spherical percolation type curves can also be drawn up (see **Figure 4** and **Figure 5**) by use of the same method.

4. CHARACTERISTICS OF THE NONLINEAR PERCOLATION

4.1. Parameter Sensitivity Analysis to Type Curves

Figure 3 shows the type curves of nonlinear spherical percolation affected by β under infinite external boundary. Can be seen from the figure, the curves vary with the value of the dimensionless quadratic pressure gradient coefficient β (from up to down, β=0, 0.2, 0.4), when β=0 it is just the curve of linear percolation model. It can be easily seen that the curves have the trait of unit slope in the wellbore storage stage, which shows that there is no influence of quadratic pressure gradient in this flow stage, and that the location of the pressure and the pressure derivative curves is lower than that of the conventional linear model curve in the stage of infinite-acting radial spherical flow. The bigger the β is, the greater the offset is.

Figure 4 and **Figure 5** show the type curves of nonlinear spherical percolation affected by β under constant pressure external boundary and closed external boundary respectively. Can be seen from the figures, the trait of unit slope in the wellbore storage stage still exist and there still exists a offset due to the effect of β, but in the late flow stage of boundary response the pressure derivative curves is going down until focusing on a point for constant pressure boundary and the pressure derivative curves is going up until focusing on a line together with the pressure curves for closed boundary, which is completely different from **Figure 1**.

Figure 6 shows the type curves of nonlinear spherical percolation affected by C_D under infinite external boundary. Can be seen from the figure, the curves vary with the value of the dimensionless wellbore storage coefficient C_D, and the bigger the C_D is, the lower thepressure derivative curve is. **Figure 7** shows the type curves of nonlinear spherical percolation affected by r_{eD} under different external boundaries. Can be seen from the figure, the curves vary with the value of the dimensionless radial spherical radius r_{eD}, and the bigger the r_{eD} is, the later the time of going up or going down is.

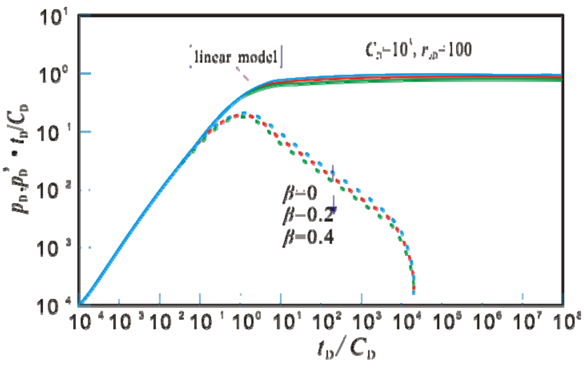

Figure 4. Type curves of nonlinear spherical percolation affected by β under constant pressure external boundary.

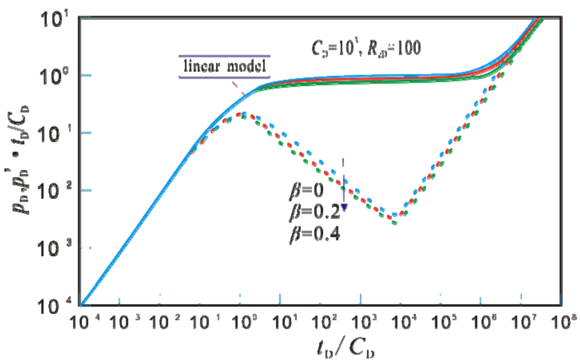

Figure 5. Type curves of nonlinear spherical percolation affected by β under closed external boundary.

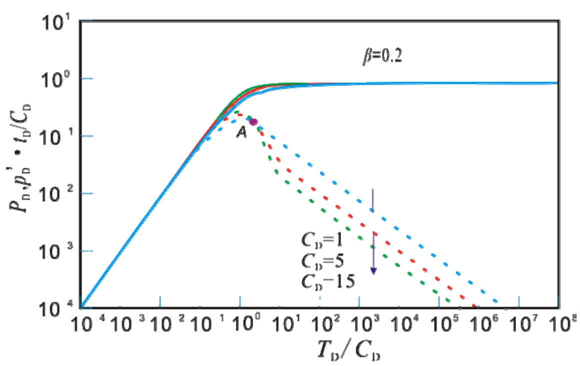

Figure 6. Type curves of nonlinear spherical percolation affected by C_D.

According to the definition of β and the probable values of β (**Table 1**), it is clearly demonstrated that β is proportional to oil viscosity μ, and inversely proportional to formation permeability k. So there is usually a bigger β for the low permeability, heavy oil reservoirs, and the influence of the quadratic pressure gradient nonlinear term is very distinct, the quadratic pressure gradient should not be neglected. For the fixed group of parameters (q, B, C_ρ), the speed of pressure wave propagation

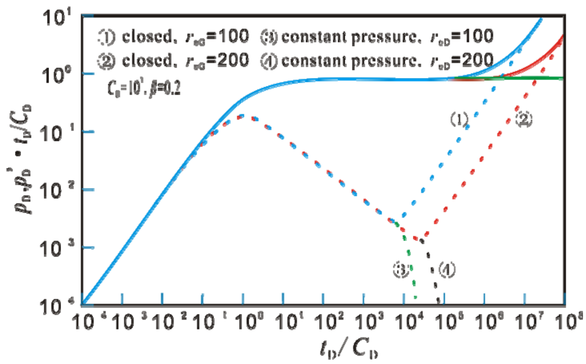

Figure 7. Type curves of nonlinear spherical percolation affected by r_{eD}.

Table 1. The probable values of β.

$k/(\times10^{-3}\mu m^2)$	$\mu/(mPa\cdot s)$	β
100	25	0.0058
10	25	0.0580
1	25	0.5800
100	100	0.0230
10	100	0.2300
1	100	2.3000

becomes slower when k/μ decreases with the increasing of β. Compared with the conventional linear model, given a fixed production time, pressure decline slows down and the speed of decline is inversely proportional to β, which is completely accordant with the theoretical curves as **Figures 3-5**. In conclusion, for a concrete reservoir, due to the effect of quadratic pressure gradient, compared with conventional linear model, the time of stable production of the nonlinear model is prolonged on condition that the same decline of reservoir pressure.

4.2. The Influence Analysis of Nonlinear Term

Table 2 and **Table 3** exhibit the results of the pressure offset and pressure derivative offset analyses vs. **Figure 3**. As shown the data in these tables, the offset increase with the increasing of time at a constant β, and pressure derivative relative offset is greater than the pressure relative offset at a fixed time. From the data in the table, it is found that the impact of the quadratic gradient is extremely intense when time is particularly long and the quadratic coefficient β is particularly large, so the quadratic pressure gradient should be retained in flow equation. After all, the nonlinear percolation law is the actual flow law of oil in porous medium, so the research on the nonlinear percolation model and its percolation law with quadratic pressure gradient should be strengthened and reinforced.

5. CONCLUSIONS

In this paper it is demonstrated that the quadratic pressure gradient has some distinct influence on the wellbore1) The linear percolation is the approximation and simplification of nonlinear percolation with quadratic pressure gradient term.

Table 2. The offset analysis of nonlinear term(β=0.2).

t_D/C_D	P_{wD}		offset	relative offset /%	$P'_{wD}\cdot t_D/C_D$		offset	relative offset /%
	linear	nonlinear			linear	nonlinear		
10^2	0.943292	0.864578	0.078714	8.34	0.029071	0.024059	0.005012	17.24
10^3	0.982148	0.896753	0.085395	8.69	0.008939	0.007390	0.001549	17.33
10^4	0.994357	0.906908	0.087449	8.79	0.002804	0.002312	0.000492	17.55
10^5	0.998216	0.910121	0.088095	8.83	0.000886	0.000728	0.000158	17.83

Table 3. The offset analysis of nonlinear term(β=0.4).

t_D/C_D	P_{wD}		offset	relative offset /%	$P'_{wD}\cdot t_D/C_D$		offset	relative offset /%
	linear	nonlinear			linear	nonlinear		
10^2	0.943292	0.801013	0.142279	15.08	0.029071	0.020499	0.008572	29.49
10^3	0.982148	0.828462	0.153686	15.65	0.008939	0.006302	0.002637	29.50
10^4	0.994357	0.837153	0.157204	15.81	0.002804	0.001970	0.000834	29.74
10^5	0.998216	0.839906	0.158310	15.86	0.000886	0.000622	0.000264	29.80

2) The new-style type curves of nonlinear spherical percolation with quadratic pressure gradient effect in shape and characteristics are obviously different from the type curves of linear model, the location of the pressure and the pressure derivative curves is lower than that of the conventional linear model curve.

3) The type curves are affected by the quadratic gradient coefficient β, the offset of pressure and pressure derivative is directly proportional to β and time.

4) For a concrete reservoir, due to the effect of quadratic pressure gradient, compared with conventional linear model, the time of stable production of the nonlinear model is prolonged on condition that the same decline of reservoir pressure.

5) The impact of the quadratic pressure gradient under certain conditions is extremely intense, especially for the low permeability and heavy oil reservoirs, and the quadratic pressure gradient term should not be neglected and should be retained in flow equation.

6) The nonlinear percolation law is the actual flow law of oil in porous medium, so the research on the nonlinear flow model and its application with quadratic pressure gradient should be strengthened and reinforced.

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