Seasonal Adjustment of the Consumer Price Index
—Based on the X-13-ARIMA-SEATS Program

Tianyi Zhang
Hebei University of Economics and Business, Shijiazhuang, China
Email: gavintyzhang@outlook.com

Abstract
This paper firstly introduces the significance of seasonal adjustments of the consumer price index (CPI). Then this paper focuses on the theory of seasonal adjustments and the ARIMA model with regression. Based on X-13 ARIMA-SEATS program, we develop a statistically robust method to conduct seasonal adjustment on China’s monthly CPI with respect to moving holidays, especially, Chinese Spring Festival. It is demonstrated that seasonally adjusted CPI time series are more sensitive and conducive to monitor the macro economy.

Keywords
CPI, Seasonal Adjustment, Spring Festival, Mobile Holiday, X-13-ARIMA-SEATS

1. Introduction
The consumer price index (CPI) mainly includes the year-on-year index, which is calculated by using the same month of previous year as the base period (previous year = 100), and the chain index by taking the previous month as the base period (previous month = 100). Although the year-on-year index could weaken the influence of seasonal factors to a certain extent, the year-on-year index includes the carryover effects and the new price-rising factor, which cannot reflect the turning point of the macro-economy timely, thus affecting the accuracy of the fluctuation calculation and the forecast of the level of consumer price and bringing difficulty for the formulating of macro-control policy. Research shows that the inflection point of the economic cycle, which is reflected by the non-seasonally adjusted year-on-year CPI lags 6 months on average [1]. Moreover, the chain index mainly reflects the short-term trend of price changes, but it does not exclude seasonal factors, holiday, working days, trading day and other non-
market factors [2]. This incurs that different monthly chain indexes are not comparable. From 2001 onwards, China has started to take the price level as the cardinal number in 2000, and fixed base price index monthly, and took it as a main indicators of Chinese price level and inflation [3].

An economic time series can be decomposed into trend, cycle, season and irregularity. Seasonal adjustment is the process of excluding seasonal factors implied in the original monthly or quarterly time series [4]. The adjusted time series is just composed of trend, cycle, and irregularity. Seasonal adjustment of monthly CPI can eliminate seasonal effects and make the data of different years and months be comparable. It can also clearly reflect the basic trend of economic internal operation and the instantaneous changes of economic and the turning point of economic changes. Besides, it can be conducive to government decision-makers to seize the best time for macro-control, stabilize the price level and promote economic development.

At present, Chinese domestic research on seasonal adjustment of CPI time series is still relatively scarce. Zhang Mingfang and others adopted X-12-ARIMA to make seasonal adjustment and analyze on CPI series [5]. In addition, they adopted TRAM/SEATS to make seasonal adjustment of Chinese mobile holidays (Spring Festival holiday) [6]. But the TREAM/SEATS method exists some limitations in adjustment and forecast ability. Dong Yaxiu and others studied the seasonal adjustment of the chain index of CPI and established a long-term forecasting model [7]. However, the drawback was that they did not consider the Spring Festival and other mobile holidays' effects. Luan Huide, Zhang Xiaotong proposed a method to construct mobile holiday regression by introducing dummy variables and assigning variable weights to the three segments of the variables, which had a founding significance [8]. Based on the previous studies, He Fengyang and others proposed the improved X-12-ARIMA-BHG model and X-12-ARIMA-LZ model [9]. The X-12-ARIMA-BHG model assumes that the weight of the economic variables in different period remain unchanged in the Spring Festival, that is, it obeys uniform distribution. But in fact some of the economic variables affected by the Spring Festival are not subject to uniform distribution. On the other hand, the construction of X-12-ARIMA-LZ model is ingenious, but the calculation is cumbersome and the seasonal adjustment is difficult. This paper first introduces the principle of seasonal adjustment, then adopts the X-13-ARIMA-SEATS program which is developed by the US Census Bureau and the Spanish bank combined with Chinese unique mobile holidays to make seasonal adjustment on the monthly CPI [10]. Finally, we use the adjusted time series to analyze and forecast the economy.

2. Principles and Methods of Seasonal Adjustment

2.1. Principles of Seasonal Adjustment

Economic time series are usually non-stationary time series. ARIMA model is the main method on modeling non-stationary time series. The process $Y_t$ of ARIMA model is expressed as:
where $\Phi(L)$ and $\Theta(L)$ are polynomials of the order $p$ and $q$ with the lag operator $L$ as the variable. $\theta_0$ is the drift term of the $\Delta^d y_t$ process. $\Delta^d y_t$ denotes a smooth ARMA process obtained by $d$-difference of $y_t$. It includes the processes of AR, MA and ARMA as well as the processes of AR, MA and ARMA for single products.

In the CPI time series modeling, we should take into account the effects induced by mobile holidays (such as the Spring Festival, Mid-Autumn Festival, Dragon Boat Festival), outliers, fixed seasonal effects, working days, trading days and other factors. A general product seasonal ARIMA model with regression term could be established as:

$$\Phi(L)(\Delta^d y_t) = \theta_0 + \Theta(L)u_t$$

where $L$ is the lag operator, $s$ is the seasonal cycle (for the monthly CPI data, $s=12$), $\Phi_p(L)$ is a non-seasonal autoregressive (AR) operator, $\Phi_s(L)$ is a seasonal autoregressive operator. $\Theta_q(L)$ is a non-seasonal moving average (MA) operator, $\Theta_q(L')$ is the seasonal moving average operator, $P$, $Q$, $p$ and $q$ respectively represent the maximum lag order of seasonal and non-seasonal autoregressive and moving average operators. $\alpha_t$ is white noise, $d$ is non-seasonal differential times, $D$ is seasonal differential times. The regression variables $\chi_t$ mainly include all kinds of outliers, mobile holiday effect, working day effect, trading day effect and so on. The above formula is called the multiplicative seasonal model of the $(p, d, q) \times (P, D, Q)$ order.

In order to get a fully fitted sequence in the product season model, the original sequence $Y_t$ is usually used as a logarithmic transformation, that is $y_t = \ln Y_t$; then plugging this in the product ARIMA model with regression term to make model identification, determine $P$, $Q$, $p$, $q$, $d$, $D$. Finally, estimating parameters by maximum likelihood method or least square.

After making forward prediction, backward prediction and a priori adjustment of various effects by the ARIMA model with regression term, this paper uses X-12 seasonal adjustment method to decompose the components based on the moving average method based on multiple iterations and then completes seasonal adjustment. It contains the Henderson symmetrical moving average adjustment and the Musgrave asymmetric moving average adjustment when we make seasonal adjustment with X-13- ARIMA-SEATS, and we use Henderson moving average to estimate the trend-cycle component.

### 2.2. Methods of Treating Regression Variables in Seasonal Adjustment

The regression variable $\chi_t$ mainly includes various outlier values and calendar-related factors. The X-13-ARIMA-SEATS program can detect outliers automatically and carry out regression analysis following the definition of outlier regression variables for all the sample intervals. Calendar effects are various ca-
lendar-related factors such as leap years, trading day effects, mobile holiday effects, etc. They will bring difficulties on judging the economic cycle, so they need to be eliminated in the ARIMA model of regression analysis.

2.2.1. Leap Year Effect
There will be a February of 29 days for every 4-year, which will have an impact on the flow of data statistics. So we need to set a leap year variable: leapyear $t = 0.75$ for February in leap year; leapyear $t = −0.25$ for February in other years; and leapyear $t = 0$ for others.

2.2.2. Trading Day Effect
If it is considered that the economic activity is different for each day of the week, since the number of occurrences of each day within a week is different, the variables considered will also be a corresponding change in the same calendar month for different years. For example, if you think that the consumption level of Sunday and Monday is different, then the economic indicator variable should be correspondingly different between months with a higher number of Sundays and months have fewer Sundays but a higher number of Mondays. In X-13-ARIMA-SEATS, the program gives selections of regression variables in trading day, and selects TD to consider the trading day effect in ARIMA model with regressions.

2.2.3. Mobile Holiday Effect
In the holidays, people tend to consume more and make the economic variables significantly different from non-holiday. But the effects of mobile holiday are different from the holiday with fixed gregorian dates (such as the National Day, Golden Week). For example, although the Spring Festival appears regularly, but does not necessarily appear on the same date each year. The effects of a fixed holiday are already considered in the seasonal effect, so the regression variable only needs to consider moving holidays.

We may take Luan Huide, Zhang Xiaotong’s method on assigning the weights of regression variables for the Spring Festival holidays. Assuming the daily weights of the Spring Festival are different before, during and after the festival, the closer the Spring Festival, the greater the impact, hence greater weights should be given. During the festival, the variables follow the uniform distribution, therefore the daily weights are equal. The weight vector for time interval of $W_b$ day before the festival is $\left(\frac{1}{\sum_{i=1}^{1}} \cdot \frac{1}{\sum_{i=1}^{2}} \cdots \frac{1}{\sum_{i=1}^{2}}\right)$. The variable weights are the same every day during the holiday season. The weight vector for time interval of one day after the festival to $W_d$ day after the festival is $\left(\frac{1}{\sum_{i=1}^{3}} \cdot \frac{1}{\sum_{i=1}^{4}} \cdots \frac{1}{\sum_{i=1}^{5}}\right)$.

According to the specific distribution of the number of effective days in different months corresponding to before, during and after the Spring Festival, we can get the proportional variable by summing the weights of each day, and then normalize them respectively. Finally, we can get regression variables $X_{s}(\omega_{s})$, $X_{d}(\omega_{d})$, $X_{a}(\omega_{a})$, for before, during and after the Spring Festival.
3. Seasonal Adjustment of CPI Monthly Data in China

3.1. Data Description

From 2001 onwards, China began to use fixed base period calculation method to publish base CPI. Chinese first round base period is fixed in 2000, that is, making the average price level in 2000 as a comparison of fixed base period, set the base index in December 2000 was 100. Then obtained the CPI fixed base index from January 1995 to June 2014 through the chain index of forward and backward recursion. Figure 1 is a fixed base consumer price index. One may note that it has an obvious seasonality: the price index reaches the highest peak in every year about February, March, and then decreases month by month. In the middle of the year the price index reaches the bottom, then it begins to rise. There is a periodic change trend in the whole sequence, which indicates that there is a seasonal variation in the whole year.

3.2. Seasonal Adjustment with X-13-ARIMA-SEATS

The X-13-ARIMA-SEATS program combines the seasonal adjustment functions of the US Census Bureau X-12-ARIMA with the TRAMO-SEATS by Bank of Spain, which allows users to define regression variables in the ARIMA model with regressions and detects the three outliers, which are AO (Additive Outlier), LS (Level Shift), TC (Temporary Change) automatically. The program can also filter the regression variables automatically according to the t statistic and select the optimal ARIMA model according to the information criterion automatically. The effects of the Spring Festival are set to $\omega_a = 20, \omega_d = 7, \omega_s = 20$. According to the above assumptions on the Spring Festival holiday regression variables, we can obtain the regression variables for periods before, during and after the Spring Festival. Table 1 lists the regression variables of the Spring Festival after normalization (Note: The Spring Festival does no effect on the month

![Figure 1. CPI’s fixed base as the base period of 2000.](image-url)
excepting January, February, March, so the regression variable value is zero). The regression model also includes leap year, trading day, outliers and other regression variables. We do not list them one by one here.

After adjusting and diagnosing, the ARIMA model selected by the program is (0,1,1)×(0,1,1), and the Spring Festival effect variables \( X_{m}(\omega_{m}) \), \( X_{d}(\omega_{d}) \), \( X_{a}(\omega_{a}) \) are significant at the 0.05 confidence level. The regression coefficients are 0.0048, 0.0028, 0.0048. The variables of trading day and leap year should be removed from the regression model because they are not significant. Besides, the program detects six outliers automatically from the Feb. Mar. Apr. monthly data and adds them into regression model. Table 2 shows 11 M statistics and Q statistics. The range of each M statistic are between 0 to 3. If the value of the M statistic and the Q statistic are both less than 1, then seasonal adjustment is robust, else the adjustment is not robust. As shown in the table, 11 M statistics and Q statistics are both less than 1. After observing the seasonally adjusted sequence (Figure 2) and the spectrum of the irregular component sequence (Figure 3), the adjusted CPI sequence does not appear significant spikes and obvious cyclical patterns. It indicates that the overall effect of the seasonal adjustment is robust.

Figure 4 shows the seasonal factors of each month. Figure 5 shows the joint seasonal factors with the Spring Festival effect. Figure 6 shows seasonal factors for each year. It can be seen from the figures, CPI in China has a strong seasonality, which is beyond doubt. It shows that the highest rate of growth appears in the first quarter of each year, the rate falls rapidly in May and appears a significantly decline in July, and the lowest point for the year appears in August and September. Due to the Spring Festival effect, there is more significant fluctuations in January, February, March than other months. Therefore, in the seasonal adjusting of economic series, the impact of the Spring Festival cannot be ignored.

### 3.3. Economic Analysis Based on Seasonal Adjustment

We could calculate the seasonally adjusted CPI series and the month-on-month growth of trend components respectively, and calculate the seasonally adjusted fixed-base CPI through the month-on-month growth of CPI after seasonal adjustment. Figure 7 shows the year-on-year growth rate and the seasonally adjusted fixed-base CPI. Figure 8 shows the quarter-on-quarter growth before or
Figure 2. Seasonally adjusted CPI series.

Figure 3. Spectrum of irregular components.

Figure 4. Seasonal factors of each month.

<table>
<thead>
<tr>
<th>Table 2: M Statistics and Q Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
</tr>
<tr>
<td>0.057</td>
</tr>
</tbody>
</table>
Figure 5. Joint seasonal factors with the spring festival effect.

Figure 6. Seasonal factors for each year.

Figure 7. Seasonally adjusted fixed base CPI and Year-on-Year CPI.

The seasonal adjustment. Figure 9 shows CPI year-on-year and quarter-on-quarter growth trend after seasonal adjustment. As can be seen from Figure 7, the sequential volatility of seasonally adjusted fixed-base CPI is earlier than the year-on-year sequential volatility for 2 to 4 months and it fluctuates more fre-
This shows that the seasonally adjusted fixed-base CPI can be more sensitive to capture the changes in the economy, and more timely to detect the inflection point of economic operation. As can be seen from Figure 8, the trend of the chain sequence before and after seasonally adjustment is almost the same. But the fluctuation is much more stable, which shows that the seasonal adjustment of CPI is significant. The important information of economic series can be preserved after eliminating the effects of seasonality, moving holiday and outlier. As can be seen from Figure 9, the trend of quarter-on-quarter growth of the trend component after seasonal adjustment is also ahead of year on year CPI, which can provide the basis for economic early warning and macro decision-making.

The purpose of seasonal adjustment is to be able to grasp the trajectory of
T. Y. Zhang

economic operation in time, and predict the future development trend according to the economic situation in the past. The principle of seasonal adjustment is to separate the seasonal component by using the past economic data through a series of algorithmic program and forecast the next year’s seasonal factor. Then one could use the forecasted seasonal factor to conduct seasonal adjustment for the next year’s economic data. By using the X-13-ARIMA-SEATS program, we can get the predicted value of the seasonal factor, the Spring Festival effect, the fixed-base CPI and the corresponding growth rate. Table 3 shows the predicted value of the fixed-base CPI in next year and the 95% confidence level.

4. Conclusion

This paper first introduces the meaning and significance of seasonal adjustment of CPI. Then, it explains the principle of the ARIMA model with regression term for leap year, trading day and Chinese special mobile holiday (Spring Festival), and the seasonal adjustment of X12. We adopt the X-13-ARIMA-SEATS program to make seasonal adjustment for the monthly time series of CPI in China. The diagnostic results show that the model can eliminate the seasonal effect well, and the adjusted series has a good smoothness. According to Figure 10, we can get the month-on-month growth rate of the adjusted CPI series, the month-on-month growth rate of the trend component, and obtain the seasonally adjusted the chain index. The article concludes that the seasonally adjusted month-on-month CPI is more sensitive than the CPI released by the Bureau of

![Figure 10](https://via.placeholder.com/150)

**Figure 10.** Predicted fixed-base CPI series and 95% confidence intervals.

<table>
<thead>
<tr>
<th>Date</th>
<th>forecast</th>
<th>upper bound</th>
<th>lower bound</th>
<th>year-on-year ratio</th>
<th>month-on-month ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul-2014</td>
<td>138.06</td>
<td>136.92</td>
<td>139.20</td>
<td>2.30</td>
<td>-0.01</td>
</tr>
<tr>
<td>Aug-2014</td>
<td>138.76</td>
<td>136.71</td>
<td>140.84</td>
<td>2.31</td>
<td>0.51</td>
</tr>
<tr>
<td>Sep-2014</td>
<td>139.63</td>
<td>136.96</td>
<td>142.36</td>
<td>2.13</td>
<td>0.63</td>
</tr>
<tr>
<td>Oct-2014</td>
<td>139.73</td>
<td>136.56</td>
<td>142.97</td>
<td>2.10</td>
<td>0.07</td>
</tr>
<tr>
<td>Nov-2014</td>
<td>139.72</td>
<td>136.12</td>
<td>143.41</td>
<td>2.20</td>
<td>-0.01</td>
</tr>
<tr>
<td>Dec-2014</td>
<td>140.33</td>
<td>136.34</td>
<td>144.44</td>
<td>2.34</td>
<td>0.44</td>
</tr>
<tr>
<td>Jan-2015</td>
<td>141.16</td>
<td>136.79</td>
<td>145.67</td>
<td>1.92</td>
<td>0.59</td>
</tr>
<tr>
<td>Feb-2015</td>
<td>142.85</td>
<td>138.11</td>
<td>147.76</td>
<td>2.64</td>
<td>1.20</td>
</tr>
</tbody>
</table>
Statistics. Our adjusted series enables one to find the inflection point in economic operation timely and provides the basis for macro-economic decision-making. Finally, we can derive the predicted value of fixed base CPI and the 95% confidence intervals in next year. At the end of this paper, it is concluded that the price level was relatively mild in the past year, and there was no significant inflation.

References


