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What Will Happen, If Zero Spin Particle Possesses Spin Rotational Construction, With Non-Zero Eigenvalues Of Spin Angular Momentum ?

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Abstract

There is no any spin rotational construction for zero spin particle, Casimir operator and the thired component of zero spin particle are $0(0+1)\hbar^2$ and $0\hbar$ respectively. Further, there are no spin interactions between zero spin particle and other spin particles.

This paper shows: in Spin Topological Space, STS [1], the third component of zero spin particle possesses non-zero eigenvalues besides original zero value, this leads to a miraculous spin interaction phenomenon between zero spin particle and other spin particles. In STS, zero spin particle could " dissolve other spin particles ", degrade the values of their Casimir operator, and decay these spin particles into other forms of spin particle.

Keywords

zero spin particle ; non-Hermitian matrix ; non-zero eigenvalues ; Casimir operator ; the third component ; Spin Topological Space, STS ; binding energy of spin particles

1 Introduction

In quantum machenics, the measurable spin properties of well-known all bosons and fermions are demostrated by two diagonal operators which called Casimir operator and the third component of spin particles.

The values $s\hbar$ of spin of any spin particle is discribed by their Casimir operators $s(s+1)\hbar^2$. The greater the s, the greater their Casimir operators.

And for the third components of these spin particles, the maximum eigenvalue is just $s\hbar$, the rest eigenvalues of theirs are always less than the value of $s\hbar$.

In quantum machenics, zero spin particle, zsp $0\hbar$ has no rotational construction. So there is actually no any spin representation for zsp in physics and Math world. Casimir Operator and the third component of zero spin particle can be obviously and trivially depicted as statements $0(0+1)\hbar^2$ and $0\hbar$, which do not contradict angular momeutum theory.

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Paragraph**2** shows: in Spin Topological Space, STS, the case about the two operators mentioned above have a slightly different behavior: Casimir operator of zero spin particle remain to be the form of $0(0+1)I_0\hbar^2$, refer to (4).

But the third component of zero spin particle turns into an infinite dimensional matrix, refer to (3) or diagonal (10.0), which shows that besides a zero value eigenvalue $0\hbar$ lying at the center " $\underline{0}$ " of (3) or diagonal (10.0), zero spin particle could possesss non-zero eigenvalues, which even be greater or less than $0\hbar$!

Paragraph**3** describes the basics of STS. In STS, the spin space of each spin particle is no longer dependent each other as we usually fimilliar with before. Now, well-known all bosons and fermions are abtributed to one spin space, STS.

Further we can use a group of unified subscripts j and a subscript -1 to describe spin classfication about bosons and fermions. Raising oprtators $\pi_j^{\dagger}$ (8), lowering operators π_{-1}^{-} (9) and the third components (10) give detailed account of the function of subscripts j in spin classfication.

As a special example, the establishment of non-trivial spin representations (1), (2), (3), (4) of zero spin is just due to two infinite dimensional non-Hermitian matrices $\pi_j^{\dagger}$ (8.0) and π_{-1}^{-} (9).

Paragraph**2** and Paragraph**3** prepare conceptual tools to discuss Paragraph**4**.

By means of addition of spin angular momenta in the frame of STS , pragraph**4** consinders the spin coupling (13) of a single boson, or a single fermion with k zero spin particles and obtains the general formula for the coupling. For understanding the physical picture of "What Will Happen", detailed account of $k=$ 1,2 are given in Table2 and Table3.

The results are incredible, the boson, or the fermion seems like the solute. And zero spin particle seems like a solvent which has miraculous power to dilute and reduce the value of Casimir operator of boson or fermion in the process of the spin coupling (13).

When the number of zero spin particle increase, the values of Casimir operator of bosons, fermions become less and less, they are degraded by zsp, and decay into other forms of spin particle. Single boson, single fermion gradually dissolve in the solvent comprised of zero spin particles, when " the density of the solvent ", or the number k of zsp approachs to infinite.

In quantum machenics, every spin particle, besides zero spin particle, is "Something", as comes down to the spin phenomena. By contrast, zero spin particle is just "Nothing" due to conventional spin concept of $0(0+1)\hbar^2$ and $0\hbar$.

Table2 and Table3 show if matrices (1), (2), (3), (4), the figures of zero spin particle in STS, are introduced to take part in spin interactions, what " what will happen..." is " **Something Plus Nothing, equals to Something for less** "

The amount of " less ", which refer to the difference of two Casimir operators possessed by spin particles before and after their combination (13) respectively, is called losses of Casimir operator $\Delta\pi(j,k)$. Table4 gives the details for the cases k=1, 2 of the losses.

When physics dimension $\hbar^2$ of Casimir operator is connected to the rotational energy of spin particle, then losses of Casimir operator would lead to the research on so-called binding energy of spin particles, on which a glimpse of comment is given in the end of this paper.

2 Zero Spin Particle Matrix representation

Matrices (1), (2), (3) are three components of zero spin particle, in this paper we use them to discuss spin angular momentum interactions between zero spin particle and other spin particles.

$$\pi_{1;0,-1} = \frac{1}{2}\{\pi^{+}_{0} + \pi^{-}_{-1}\} = \frac{1}{2}\boxed{\begin{array}{ccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0 & 5 & & & & & & & & & \mathbf{0} & \cdot \\
\cdot & -4 & 0 & 4 & & & & & & & & & \cdot \\
\cdot & & -3 & 0 & 3 & & & & & & & & \cdot \\
\cdot & & & -2 & 0 & 2 & & & & & & & \cdot \\
\cdot & & & & -1 & 0 & 1 & & & & & & \cdot \\
\cdot & & & & & 0 & \underline{0} & 0 & & & & & \cdot \\
\cdot & & & & & & 1 & 0 & -1 & & & & \cdot \\
\cdot & & & & & & & 2 & 0 & -2 & & & \cdot \\
\cdot & & & & & & & & 3 & 0 & -3 & & \cdot \\
\cdot & & & & & & & & & 4 & 0 & -4 & \cdot \\
\cdot & \mathbf{0} & & & & & & & & & 5 & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}} \quad (1)$$

$$\pi_{2;0,-1} = \frac{1}{2i}\{\pi^{+}_{0} - \pi^{-}_{-1}\} = \frac{1}{2}\boxed{\begin{array}{ccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0 & -i5 & & & & & & & & & \mathbf{0} & \cdot \\
\cdot & -i4 & 0 & -i4 & & & & & & & & & \cdot \\
\cdot & & -i3 & 0 & -i3 & & & & & & & & \cdot \\
\cdot & & & -i2 & 0 & -i2 & & & & & & & \cdot \\
\cdot & & & & -i1 & 0 & -i1 & & & & & & \cdot \\
\cdot & & & & & 0 & \underline{0} & 0 & & & & & \cdot \\
\cdot & & & & & & i1 & 0 & i1 & & & & \cdot \\
\cdot & & & & & & & i2 & 0 & i2 & & & \cdot \\
\cdot & & & & & & & & i3 & 0 & i3 & & \cdot \\
\cdot & & & & & & & & & i4 & 0 & i4 & \cdot \\
\cdot & \mathbf{0} & & & & & & & & & i5 & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}} \quad (2)$$

$$\pi_{3;0,-1} = \boxed{\begin{array}{ccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 5 & & & & & & & & & & \mathbf{0} & \cdot \\
\cdot & & 4 & & & & & & & & & & \cdot \\
\cdot & & & 3 & & & & & & & & & \cdot \\
\cdot & & & & 2 & & & & & & & & \cdot \\
\cdot & & & & & 1 & & & & & & & \cdot \\
\cdot & & & & & & \underline{0} & & & & & & \cdot \\
\cdot & & & & & & & -1 & & & & & \cdot \\
\cdot & & & & & & & & -2 & & & & \cdot \\
\cdot & & & & & & & & & -3 & & & \cdot \\
\cdot & & & & & & & & & & -4 & & \cdot \\
\cdot & \mathbf{0} & & & & & & & & & & -5 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}} \quad (3)$$

$$\pi^{2}_{0,-1} = \boxed{\begin{array}{ccccccccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0 & & & & & & & & & & \mathbf{0} & \cdot \\
\cdot & & 0 & & & & & & & & & & \cdot \\
\cdot & & & 0 & & & & & & & & & \cdot \\
\cdot & & & & 0 & & & & & & & & \cdot \\
\cdot & & & & & 0 & & & & & & & \cdot \\
\cdot & & & & & & \underline{0} & & & & & & \cdot \\
\cdot & & & & & & & 0 & & & & & \cdot \\
\cdot & & & & & & & & 0 & & & & \cdot \\
\cdot & & & & & & & & & 0 & & & \cdot \\
\cdot & & & & & & & & & & 0 & & \cdot \\
\cdot & \mathbf{0} & & & & & & & & & & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}} \quad (4)$$

(1), (2), (3) satisfy angular momentum commutation rules (5)

$$\pi_{i;0,-1}\pi_{j;0,-1} - \pi_{j;0,-1}\pi_{i;0,-1} = i\,\pi_{k;0,-1} \qquad (5)$$

subscripts $i, j, k = 1, 2, 3$ are circulative

The construction of raising operator π^{+}_{0} and lowering operator π^{-}_{-1}, which in matrix (1) and matrix (2), refer to expressions (8.0) and (9). Matrix (4) $\pi^{2}_{0,-1}$ is Casimir operator of zero spin particle, $\pi^{2}_{0,-1} = \pi^{2}_{1;0,-1} + \pi^{2}_{2;0,-1} + \pi^{2}_{3;0,-1}$.

Note: Though Casimir operator (4) is an infinite dimensional zero matrix, that does not prevent zero spin particle, which expressed in term of martices (1), (2), (3), from its the third component (3) having diagonal Non-Zero Eigenvalues ! and even having its square $\pi^{2}_{3;0,-1} \geq$ total square $\pi^{2}_{0,-1}$, because matrices (1), (2) are infinite dimensional non-Hermitian operators in Spin Topological Space.

3 Bosons and Fermions in Spin Topological Space

Three operators in (6) below, satisfy spin angular momentum commutation relus (7)

$$\{\ \pi^+_j,\ \pi^-_{-1},\ \pi_{3;\,j,\,-1}\ \} \qquad (6)$$

$$\pi^+_j\pi^-_{-1} - \pi^-_{-1}\pi^+_j = 2\pi_{3;\,j,\,-1} \qquad (7.1)$$

$$\pi_{3;\,j,\,-1}\pi^+_j - \pi^+_j\pi_{3;\,j,\,-1} = +\pi^+_j \qquad (7.2)$$

$$\pi_{3;\,j,\,-1}\pi^-_{-1} - \pi^-_{-1}\pi_{3;\,j,\,-1} = -\pi^-_{-1} \qquad (7.3)$$

These three operators are raising operators π^+_j, lowering operators π^-_{-1} and the third component operaters $\pi_{3;\,j,\,-1}$ of different spin particles which labelled by different values of $j = -1, 0, +1, +2, +3, +4, \ldots$

Write out the explicit expressions of raising operators π^+_j (8) and lowering operator π^-_{-1} (9), which appear in (6), (7)

$$\pi^+_{+4} = \mathrm{diag}\{\ ,\ 9,\ 8,\ 7,\ 6,\ 5,\ \underline{4},\ 3,\ 2,\ 1,\ 0,\ -1,\ ,\ \}_{+1} \qquad (8.4)$$

$$\pi^+_{+3} = \mathrm{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\ \}_{+1} \qquad (8.3)$$

$$\pi^+_{+2} = \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \}_{+1} \qquad (8.2)$$

$$\pi^+_{+1} = \mathrm{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\ \}_{+1} \qquad (8.1)$$

$$\pi^+_{0} = \mathrm{diag}\{\ ,\ 5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\ \}_{+1} \qquad (8.0)$$

$$\pi^+_{-1} = \mathrm{diag}\{\ ,\ 4,\ 3,\ 2,\ 1,\ 0,\ \underline{-1},\ -2,\ -3,\ -4,\ -5,\ -6,\ ,\ \}_{+1} \qquad (8.\text{-}1)$$

$$\pi^-_{-1} = \mathrm{diag}\{\ ,\ -4,\ -3,\ -2,\ -1,\ 0,\ \underline{1},\ 2,\ 3,\ 4,\ 5,\ 6,\ ,\ \}_{-1} \qquad (9)$$

Subscripts, " +1 " in (8) $\mathrm{diag}\{,\ ,\}_{+1}$ and " -1 " in (9), $\mathrm{diag}\{,\ ,\}_{-1}$, represent the first minor top-right diangonal and the first minor down-left diangonal resepectively. (8.0) and (9) construct zero spin particle representions (1), (2), (3), which mentioned in paragraph **2**

Two diagonal matrices a) and b) of spin particles $\vec{\pi}_{j,\,-1}$

a) The third components $\pi_{3;\,j,\,-1}$ (10) are obtained by using above expressions (8), (9) and (7.1) as below

$$\pi_{3;\,j,\,-1} = \tfrac{1}{2}\{\ \pi^+_j\pi^-_{-1} - \pi^-_{-1}\pi^+_j\ \} \qquad (10)$$

$$\pi_{3;\,+4,\,-1} = \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \}_0 \qquad (10.4)$$

$$\pi_{3;\,+3,\,-1} = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ \underline{3},\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\}_0 \qquad (10.3)$$

$$\pi_{3;\,+2,\,-1} = \mathrm{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\ \}_0 \qquad (10.2)$$

$$\pi_{3;\,+1,\,-1} = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 11,\ 9,\ 7,\ 5,\ 3,\ \underline{1},\ -1,\ -3,\ -5,\ -7,\ -9,\ ,\}_0 \qquad (10.1)$$

$$\pi_{3;\,0\,,\,-1} = \mathrm{diag}\{\ ,\ 5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\ \}_0 \qquad (10.0)$$

$$\pi_{3;\,-1,\,-1} = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 9,\ 7,\ 5,\ 3,\ 1,\ \underline{-1},\ -3,\ -5,\ -7,\ -9,\ -11,\ ,\ \}_0 \qquad (10.\text{-}1)$$

subscript " 0 " in (5) represents the major diangonal, sometime, " 0 " is omitted if no confusion.

b) Casimir oprtator, the sum of square $\pi^2_{j,-1}$ of $\vec{\pi}_{j,-1}$

The total square of $\pi_{j,-1}$ is defined as

$$\pi^2_{j,-1} = \vec{\pi}_{j,-1} \cdot \vec{\pi}_{j,-1} = \pi^2_{1;j,-1} + \pi^2_{2;j,-1} + \pi^2_{3;j,-1} = \frac{(s^2-1^2)}{2^2} = \frac{j(j+2)}{4} \qquad (11)$$

here

$$\pi^2_{1;j,-1} + \pi^2_{2;j,-1} = \tfrac{1}{2}\{\ \pi^+_j\pi^-_{-1} + \pi^-_{-1}\pi^+_j\ \} \qquad (12)$$

The concrete results of **a)** (10) and **b)** (11) are given in Table1

Table1 Bosons and Fermions in **STS**

	$\pi_{3;j,-1}$	$\pi^2_{j,-1}$	$\frac{j\hbar}{2}$ *Particle Spin*		S $= j+1$	j
. .	. .	. .	. .		. .	. .
(10.4)	$\pi_{3;+4,-1}$	$\frac{24\hbar^2}{4} = 2(2+1)$	$\frac{4\hbar}{2}$	boson	5	+4
(10.3)	$\pi_{3;+3,-1}$	$\frac{15\hbar^2}{4} = \frac{3}{2}(\frac{3}{2}+1)$	$\frac{3\hbar}{2}$	fermion	4	+3
(10.2)	$\pi_{3;+2,-1}$	$\frac{8\hbar^2}{4} = 1(1+1)$	$\frac{2\hbar}{2}$	boson	3	+2
(10.1)	$\pi_{3;+1,-1}$	$\frac{3\hbar^2}{4} = \frac{1}{2}(\frac{1}{2}+1)$	$\frac{1\hbar}{2}$	fermion	2	+1
(10.0)	$\pi_{3;0,-1}$	$\frac{0\hbar^2}{4} = 0(0+1)$	$\frac{0\hbar}{2}$	boson	1	0
(10.-1)	$\pi_{3;-1,-1}$	$-\frac{1\hbar^2}{4} = -\frac{1}{2}(-\frac{1}{2}+1)$	$-\frac{1\hbar}{2}$	negative fermion	0	–1

4 What Will Happen ...

A Combination (13) of a boson, or a fermion spin particle with k zero spin particles is introduced as below

$$\vec{\pi}_{j/(k+1),\,-1} = \frac{1}{k+1}\{\quad \vec{\pi}_{j,\,-1} \quad + \quad k\,\vec{\pi}_{0,\,-1} \quad\} \ (13)$$

$\frac{1}{k+1}$\{ **One Boson** + k **Zero Spin Particles** \} (13.1)

$\frac{1}{k+1}$\{ **One Fermion** + k **Zero Spin Particles** \} (13.2)

We find the combination is a new spin particle that satisfy angular momentum rule below

$$\vec{\pi}_{j/(k+1),\,-1} \times \vec{\pi}_{j/(k+1),\,-1} = i\vec{\pi}_{j/(k+1),\,-1} \qquad (14)$$

Two diagonal matrices c) and d) of spin particles $\vec{\pi}_{j/(k+1),\,-1}$

c) The third components $\pi_{3,\,j/(k+1),\,-1}$ with clear figures are shown in Table2, by directly substituting the rusults of (10.j) into (15)

$$\pi_{3,\,j/(k+1),\,-1} = \frac{1}{k+1}\{\ \pi_{3,j,-1} + k\pi_{3,0,-1}\ \} \qquad (15)$$

then we get a general formula (16)

$$\frac{j\hbar}{2(k+1)} = \frac{1}{k+1}\{\quad \frac{j\hbar}{2} \quad + \quad k\,\frac{0\hbar}{2} \quad\} \qquad (16)$$

The influence, of the number k of zero spin particle(s) on a boson, or on a fermion in formula (16), is detailed in Table2

Table2 The Third Components of Spin Particles with diffenent j and k

$\pi_{3,\ j/(k+1),\ -1}$ $\frac{j\hbar}{2(k+1)}$ j	$\pi_{3,\ j,\ -1}$, k=0 $\frac{j\hbar}{2}$	$\pi_{3,\ 0,\ -1}$, k=0 $\frac{0\hbar}{2}$	$\pi_{3;\ j/2,\ -1}$, k=1 $\frac{j\hbar}{4}$	$\pi_{3;\ j/3,\ -1}$, k=2 $\frac{j\hbar}{6}$
+6	$\frac{6\hbar}{2}$ $3\hbar$ boson	$\frac{0\hbar}{2}$ zero spin	$\frac{6\hbar}{4}$ three second	$\frac{6\hbar}{6}$ $1\hbar$ boson
+5	$\frac{5\hbar}{2}$ five second	$\frac{0\hbar}{2}$ zero spin	$\frac{5\hbar}{4}$ five fourth	$\frac{5\hbar}{6}$ five sixth
+4	$\frac{4\hbar}{2}$ $2\hbar$ boson	$\frac{0\hbar}{2}$ zero spin	$\frac{4\hbar}{4}$ $1\hbar$ boson	$\frac{4\hbar}{6}$ two third
+3	$\frac{3\hbar}{2}$ three second	$\frac{0\hbar}{2}$ zero spin	$\frac{3\hbar}{4}$ three fourth	$\frac{3\hbar}{6}$ $\frac{1\hbar}{2}$ fermion
+2	$\frac{2\hbar}{2}$ $1\hbar$ boson	$\frac{0\hbar}{2}$ zero spin	$\frac{2\hbar}{4}$ $\frac{1\hbar}{2}$ fermion	$\frac{2\hbar}{6}$ one third
+1	$\frac{1\hbar}{2}$ $\frac{1\hbar}{2}$ fermion	$\frac{0\hbar}{2}$ zero spin	$\frac{1\hbar}{4}$ one fourth	$\frac{1\hbar}{6}$ one sixth
0	$\frac{0\hbar}{4}$ $0\hbar$ boson	$\frac{0\hbar}{2}$ zero spin	$\frac{0\hbar}{4}$ $0\hbar$ boson	$\frac{0\hbar}{4}$ $0\hbar$ boson
–1	$\frac{-1\hbar}{2}$ N fermion	$\frac{0\hbar}{2}$ zero spin	$\frac{-1\hbar}{4}$ N one fourth	$\frac{-1\hbar}{6}$ N one sixth

Note: (16) and Table2 show**:** the new spin particle $\pi_{3;\ j/2,\ -1}$ or $\pi_{3;\ j/3,\ -1}$ maybe either a new boson or a new fermion, or neither a boson nor a fermion at all.

Example1 of the row labelled j= +4, indicates:

Combination of a $2\hbar$ boson and a zero spin particle (k=1), would form a $1\hbar$ boson

Combination of a $2\hbar$ boson and two zero spin particles (k=2), would form a three third spin $\frac{2\hbar}{3}$ particle

Example2: the row labelled j= +2, indicates:

Combination of a $1\hbar$ boson and a zero spin particle (k=1), would form a $\frac{1\hbar}{2}$ fermion

Combination of a $1\hbar$ boson and two zero spin particles (k=2), would form a one third spin $\frac{1\hbar}{3}$ particle

Example3: the row labelled j= +1, indicates:

Combination of a $\frac{1\hbar}{2}$ fermion and a zero spin particle (k=1), would form a one fourth spin $\frac{1\hbar}{4}$ particle

Combination of a $\frac{1\hbar}{2}$ fermion and two zero spin particles (k=2), would form a one sixth spin $\frac{1\hbar}{6}$ particle

Example4: the row labelled j= 0, indicates:

Combination of a $0\hbar$ boson and any number of $0\hbar$ boson (k=1, 2, ...), would still form a $0\hbar$ boson

Note: The mentioned above show, the original boson (k = 0) or the origianl fermion (k = 0) seems to be "dissolvable" (refer to (17) and (21) (22), the absolute values of Casimir oprtator of the boson, or the fermion is deminishing) when it combines with zero spin particle(s) (in state of k = 0) to form a new spin particle (k = 1, 2, ...), the amount of spin of the new spin particle is always less than the one of the original boson or the original fermion as below

$$\frac{j\hbar}{2(k+1)} < \frac{j\hbar}{2} \qquad k = 1, 2, 3, ... \qquad (17)$$

d) Casimir oprtator, the sum of square $\pi^2_{j/(k+1),\ -1}$ of $\vec{\pi}_{j/(k+1),\ -1}$

By means of (10) and (18), (19),

$$\pi_{3;\, j/(k+1),\ -1} = \frac{1}{2}\{\ \pi^{+}_{j/(k+1)}\pi^{-}_{-1}\ -\ \pi^{-}_{-1}\pi^{+}_{j/(k+1)}\ \} \qquad (18)$$

$$\pi^2_{1;\, j/(k+1),\ -1}\ +\ \pi^2_{2;\, j/(k+1),\ -1} = \frac{1}{2}\{\ \pi^{+}_{j/(k+1)}\pi^{-}_{-1}\ +\ \pi^{-}_{-1}\pi^{+}_{j/(k+1)}\ \} \qquad (19)$$

The total square $\pi^2_{j/(k+1),\ -1}$ is given

$$\pi^2_{j/(k+1),\ -1}\ =\ \pi^2_{1;\, j/(k+1),\ -1} + \pi^2_{2;\, j/(k+1),\ -1} + \pi^2_{3;\, j/(k+1),\ -1}\ =\ \frac{j\{j+2(k+1)\}\hbar^2}{4(k+1)^2} \qquad (20)$$

The concrete results of (20) with $k = 1, 2$ are given in Table3

Table3 Casimir Operators of Spin Particles with diffenent j and k

$\pi^2_{j/(k+1),\ -1}$	$\pi^2_{j,\ -1}$, k=0	$\pi^2_{0,\ -1}$, k=0	$\pi^2_{j/2,\ -1}$, k=1	$\pi^2_{j/3,\ -1}$, k=2
$\frac{j\{j+2(k+1)\}}{4(k+1)^2}$	$\frac{j\{j+2)\}}{4}$	$\frac{0}{4}$	$\frac{j\{j+4\}}{16}$	$\frac{j\{j+6\}}{36}$
j				
+6	$\frac{48\hbar^2}{4} = 3(3+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{15\hbar^2}{4} = \frac{3}{2}(\frac{3}{2}+1)$	$\frac{8\hbar^2}{4} = 1(1+1)$
+5	$\frac{35\hbar^2}{4} = \frac{5}{2}(\frac{5}{2}+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{45\hbar^2}{16} = \frac{5}{4}(\frac{5}{4}+1)$	$\frac{55\hbar^2}{36} = \frac{5}{6}(\frac{5}{6}+1)$
+4	$\frac{24\hbar^2}{4} = 2(2+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{8\hbar^2}{4} = 1(1+1)$	$\frac{10\hbar^2}{9} = \frac{2}{3}(\frac{2}{3}+1)$
+3	$\frac{15\hbar^2}{4} = \frac{3}{2}(\frac{3}{2}+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{21\hbar^2}{16} = \frac{3}{4}(\frac{3}{4}+1)$	$\frac{3\hbar^2}{4} = \frac{1}{2}(\frac{1}{2}+1)$
+2	$\frac{8\hbar^2}{4} = 1(1+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{3\hbar^2}{4} = \frac{1}{2}(\frac{1}{2}+1)$	$\frac{4\hbar^2}{9} = \frac{1}{3}(\frac{1}{3}+1)$
+1	$\frac{3\hbar^2}{4} = \frac{1}{2}(\frac{1}{2}+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{5\hbar^2}{16} = \frac{1}{4}(\frac{1}{4}+1)$	$\frac{7\hbar^2}{36} = \frac{1}{6}(\frac{1}{6}+1)$
0	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$
–1	$\frac{-1\hbar^2}{4} = \frac{-1}{2}(\frac{-1}{2}+1)$	$\frac{0\hbar^2}{4} = 0(\ 0+1)$	$\frac{-3\hbar^2}{16} = \frac{-1}{4}(\frac{-1}{4}+1)$	$\frac{-5\hbar^2}{36} = \frac{-1}{6}(\frac{-1}{6}+1)$

As k, the number of zero spin particles incresing, the new spin particle gradually "dissolve into" a zero spin particle

$$\lim_{k\to\infty}\ \pi^2_{j/(k+1),\ -1} = \lim_{k\to\infty}\ \frac{j\{j+2(k+1)\}}{4(k+1)^2}\ \to\ \lim_{k\to\infty}\ \frac{1}{k}\ \to\ 0(\ 0+1)\ =\ \pi^2_{0,\ -1} \qquad (21)$$

e) Losses of Casimir operator

Before the combinations (13), the contributions of a single spin particle $\vec{\pi}_{j,\ -1}$ and k zero spin particles $k\vec{\pi}_{0,\ -1}$ are $\pi^2_B = \pi^2_{j,\ -1} + k\pi^2_{0,\ -1} = \pi^2_{j,\ -1} = \frac{j(j+2)}{4}$, (11). And after the combinations (13), the contributions of spin particle $\pi_{3,\ j/(k+1),\ -1}$ are $\pi^2_A = \pi^2_{j/(k+1),\ -1} = \frac{j\{j+2(k+1)\}}{4(k+1)^2}$, (20).

$\Delta\pi(j, k)$ below, the difference between π^2_A and π^2_B, is called lose of Casimir oprtator

$$\Delta\pi(j, k)\ =\ \pi^2_{j/(k+1),\ -1}\ -\ \pi^2_{j,\ -1}\ =\ -\frac{kj}{4(k+1)^2}\{\ \frac{(j+2)(k+2)-2}{1}\ \} \qquad (22)$$

$$\Delta\pi(j, 1)\ =\ -\frac{j}{16}\{\ \frac{3(j+2)-2}{1}\ \} \qquad (22.1)$$

$$\Delta\pi(j, 2)\ =\ -\frac{2j}{36}\{\ \frac{4(j+2)-2}{1}\ \} \qquad (22.2)$$

Table4 $\Delta\pi(j, k)$ **Losses of Casimir oprtator with** k**=1, 2** (unit $\hbar^2$)

$\pi^2_{j/(k+1),\ -1}$	$\pi^2_{j/2,\,-1}$, k=1	$\pi^2_{j/3,\ -1}$, k=2	$\pi^2_{j,\,-1}$, k=0	$\Delta\pi(j, 1)$	$\Delta\pi(j, 2)$
$\frac{j\{j+2(k+1)\}}{4(k+1)^2}$	$\frac{j\{j+4\}}{16}$	$\frac{j\{j+6\}}{36}$	$\frac{j\{j+2)\}}{4}$	(22.1)	(22.2)
j					
+6	$\frac{15}{4}$	2	12	$-\frac{132}{16}$	-10
+5	$\frac{45}{16}$	$\frac{55}{36}$	$\frac{35}{4}$	$-\frac{95}{16}$	$-\frac{260}{36}$
+4	2	$\frac{10}{9}$	6	-4	$-\frac{176}{36}$
+3	$\frac{21}{16}$	$\frac{3}{4}$	$\frac{15}{4}$	$-\frac{39}{16}$	$-\frac{108}{36}$
+2	$\frac{3}{4}$	$\frac{4}{9}$	2	$-\frac{20}{16}$	$-\frac{56}{36}$
+1	$\frac{5}{16}$	$\frac{7}{36}$	$\frac{3}{4}$	$-\frac{7}{16}$	$-\frac{20}{36}$
0	0	0	0	0	0
-1	$\frac{-3}{16}$	$\frac{-5}{36}$	$\frac{-1}{4}$	$+\frac{1}{16}$	$+\frac{4}{36}$

Losses of Casimir oprtator (16) and (17) mean: **Something Plus Nothing, Equal To Something For Less**

f) The binding energy of spin particles

In STS, spin particles are symbolled by j and k. Formular (20) $\pi^2_{j/(k+1),\ -1} = \frac{j\{j+2(k+1)\}\hbar^2}{4(k+1)^2}$ is the attribute of the figure of spin particle lablled with different j and k. The attribute may be rewritten in the form of the rotational energy of spin particle as below.

$$E_r(j, k) = \frac{\pi^2_{j/(k+1),-1}}{2I_{j,k}} \qquad (23)$$

And

$$\Delta E_r(j, k) = \frac{1}{2I_{j,k}}\{\ \pi^2_{j/(k+1),-1} - \pi^2_{j,-1}\ \} = \frac{\Delta\pi(j,k)}{2I_{j,k}} \qquad (24)$$

(24) is called as the binding energy, the energy released when the constituent spin particles, $\vec{\pi}_{j,\ -1}$ and $k\,\vec{\pi}_{0,\ -1}$ come together to form spin particle $\vec{\pi}_{j/(k+1),\ -1}$.

5 Conclusions

So far zero spin particle is the only spin particle not possessing non-trivial spin angular momentum representation, because zero spin particle possesses no spin rotational construction, and plays the "nothing role" of spin interactions world.

This paper, researching the spin angular momentum coupling between zero spin particle and other spin particle, may be an approach to judge whether zero spin particle possesses spin rotational construction.

This paper shows: in Spin Topological Space, STS, zero spin particle was no long unable to do anything, in spin interactions. The idea of combination of " the nothing " of zero spin particle with " the something " of other spin particle provides heuristic math thought to understand many interesting physics phenomena [2], [3].

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