The Large Numbers in a Quantized Universe

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ABSTRACT

The article relates to a decades-old problem of the mysterious coincidence between various large numbers of the magnitude ranging from $10^{40}$ to $10^{120}$ which sometimes appears in cosmology and quantum physics. Using well known classical relations as well as the ideal Schwarzschild solution the exact relations of various large numbers, the fine structure constant \( \alpha \) and \( \pi \) were found. The new largest number law is claimed. The hypothetical approximations of the Hubble parameter—68.7457(82) km/s/Mpc, Hubble radius—14.2330(17) Gly, and some others were proposed. The exact formulae supporting P. Dirac’s large number hypothesis and H. Weyl’s proposition were found. It is shown that all major physical constants with the length dimension (from the Compton wave length of universe through the Planck and atomic scale up to the Hubble sphere radius) could be derived from each other, and the table of the specific conversion rules has been developed. The model shows that the Eddington-Weinberg relation can be transformed to precise identity. It is shown that both Bekenstein universal entropy bound and Bekenstein-Hawking Black Hole entropy bound are proportional to the largest number doubled.

Keywords: Large Numbers Hypothesis; Hubble Sphere; Eddington Number; Cosmological Constant

1. Introduction

The problem of Large Numbers dates back decades. The first problem statement and attempts at resolving it can be found in the studies by H. Weyl [1-3] and Sir A. Eddington [4,5], who drew attention to the incredibly large numbers of the dimensionless physical constants found in the cosmology, electrodynamics and quantum mechanics. The magnitude of these constants is so big \( \left( \sim 10^{20}, \sim 10^{40}, \sim 10^{80}, \sim 10^{120} \right) \) as compared with the conventional mathematical constants, like \( \pi = 3.14\ldots \) and Euler’s constant \( e = 2.71\ldots \), that boggles imagination as P. Davies [6] noted.

The first of the two most popular dimensionless large numbers is the classical ratio between the gravitational force and the electromagnetic force in any given distance, called by H. Weyl [2] “even more mysterious than the fine structure constant \( \alpha \)”: \(\frac{f_e}{f_g} = \left( \frac{r_e}{r_g} \right)^2 e \approx 4.16 \times 10^{12} \tag{1}\)

were:

\[ f_e = \frac{1}{4\pi \varepsilon_0} \frac{q^2}{r^2} \tag{2} \]
\[ f_g = -G \frac{m^2}{r^2} \tag{3} \]

The second is the ratio of the universe radius to the classical electron radius: \(N_{EB} = \left( \frac{R}{r_e} \right) \approx \frac{c}{H r_e} \approx 4.63 \times 10^{40} \tag{4}\)

where \( H \) is Hubble’s constant, \( r_e \)—classical electron radius, \( R \)—Hubble sphere radius or radius of event horizon, \( r_g \)—gravitational electron radius, \( f_e \)—electrostatic force between two electrons at a distance \( r \), \( \varepsilon_0 \)—permittivity of vacuum, \( q \)—electron charge, \( f_g \)—gravitational force between two electrons at a distance \( r \), \( G \)—Newton gravitational constant, \( m_e \)—electron mass.

The proximity of magnitude of \( \frac{r_e}{R} \) and \( \frac{r_g}{r_e} \) values led H. Weyl to the idea that the incredible weakness of gravitational interaction may be due to the ratio of the electron and the universe radiiues or to the total quantity of particles in the universe—the Eddington number [5].
We refer everybody interested in the history of studying the problem of large numbers to the reviews by S. Ray, U. Mukhopadhyay, P. P. Ghosh [7] and K. A. Tomilin [8].

The hypothesis by P.A.M. Dirac is one of the best known hypotheses put forward to explain the problem of large numbers [9-11]. He supposed that the reason for appearance of great magnitudes of dimensionless values is their reliance on the equally large value, so-called “cosmological time”. This led P. Dirac to the hypothesis of dependence of the gravitational constant and the universe mass on time:

$$ G \propto \frac{1}{T} $$

(5)

$$ M \propto T^2 $$

(6)

where $M$ is the mass of universe, $T$ is cosmological time.

The idea of time-varying constants was developed, in particular, by E. A. Milne [12]. To establish the ratios and laws between modern values of the fundamental constants, we suggest studying values of large numbers at the current point of time, without taking into account their time derivative. P. Dirac’s assumption that Newton’s gravitational constant and the universe mass are not true constants but change over time gave rise to an array of scientific discussions, experimental and theoretical studies devoted to verification of the fundamental constants in subsequent decades. No reliable proofs of variability of the physical constants were found yet.

In this article, we will use the Hubble time, the parameter inverse to the Hubble constant, as approximation of the cosmological time:

$$ T = \frac{1}{H} $$

(7)

We referred to the simplest mathematics in narrating the article; however, the results we obtained provide quite good approximation to the most precise and generally accepted values of physical parameters, such as $m_e$ and $r_e$, taking into account their uncertainties. In particular, CODATA 2010 [13] as well as the measurements of Mission Planck 2013 [14] were used.

2. Revealing the Large Numbers Ratios

To discover the correlation between large numbers in our epoch, let’s begin with the well known vacuum solution to the Einstein’s equations for spherically symmetric and static universe. According to Schwarzschild’s solution, the universe’s radius $R$ that coincides with the Black Hole radius with the mass of $M$ is determined by a well-known formula (Schwarzschild radius):

$$ R = \frac{2GM}{c^2} $$

(8)

On the other hand, we know the formula for the electron gravitational radius that includes more or less precisely measured physical values:

$$ r_g = \frac{Gm_e}{c^2} $$

(9)

Therefore, the formula for the classical electron radius that uses $G$ can be easily obtained from (8) and (9):

$$ r_e = \frac{N_{DR} Gm_e}{c^2} $$

(10)

We will not discuss now if the value of the classical electron radius has any real physical significance. It is enough that it is one of the energy status representations of an electron, a particle with the minimum self-energy among all charged particles.

For transition to energy values, we will use a large mass number introduced by H.Weyl:

$$ N_{we} = \frac{M}{m_e} = \frac{\lambda_e}{\lambda_U} = \frac{E_U}{E_e} $$

(11)

where $E_U = Mc^2$ —the universe self-energy, $E_e = m_ee^2$ —the electron self-energy, $\lambda_e = \frac{h}{Mc}$ —Compton wave length of the universe, $\lambda_U = \frac{h}{mc}$ —Compton wave length of the electron.

By dividing (8) by (10), one can get the following ratio:

$$ \frac{R}{r_e} = \frac{2M}{N_{DR} m_e} $$

(12)

Thus, we obtain the precise correlation among the three large numbers out of (4), (11) and (12):

$$ 2N_{we} = N_{DR} N_{DR} $$

(13)

Besides the above large numbers, we will use a large energy number $N_{w}$ representing the ratio of the universe’s self-energy $E_U$ and “the minimum vacuum energy” $E_w$ as proposed by J. Casado [15]:

$$ N_w = \frac{E_U}{E_w} $$

(14)

where $E_w = \frac{hc}{2\pi R} = hH$ —quantum of energy with wave length $2\pi R$, $h$ —Planck’s constant;

We will also need another large number equal to the cube of $N_{DR}$ value:

$$ N_U = \frac{R^3}{r_e^3} = N_{DR}^3 $$

(15)

Introduction of a new designation for a large number $N_U$ is quite justified, because, as we will see below, this
number has a particular and substantial value. One of the simplest classical interpretations of this number: “the large number $N_U$, represents the total number of, say, ‘elementary clusters’ in the universe—the ratio between volume $V_u$ of ball-like universe and the region $V_s$ folded by sphere of radius $r_s$”. Taking into account that this number has the greatest magnitude as compared with other large numbers ($\sim 10^{122}$), we suggest calling it “the Largest number”.

3. The Largest Number Law and Revealing of Dirac’s Proportionality

Now let us use well known physical parameters with their uncertainties (see Table 1). The parameters listed in Table 1 enable us to calculate large numbers values (see Table 2). To obtain $N_{DF}$, we use (10):

$$N_{DF} = \frac{1}{G m_e} r^2 c^2$$ (16)

We should pay an attention to the very close values of $N_U$ and $N_w$ of the magnitude $\sim 10^{21}$. The ratio between this two large numbers is

$$n_x = \frac{N_U}{N_w} \approx 3.185(36)$$ (17)

This is quite remarkable, taking into account the extremely large magnitudes of the numbers involved. Therefore, we can assume that this ratio is not just a coincidence but some specific physical law. In order to reveal the meaning of the ratio (17) one can rewrite it as follows:

$$n_x = \frac{R^3 E_w}{r_s^3 E_u} = \frac{R^3 m_e c^2 r_s 2 \pi}{r_s^3 M c^2 2 \pi \alpha} = \frac{1}{\alpha} \frac{R^2 m_e}{\alpha}$$ (18)

By simply grouping the cosmological parameters in the left-hand side, and the quantum ones in the right-hand one the ratio (17) can be transformed into the following relation:

$$n_x = \frac{M}{\alpha r_s^2}$$ (19)

The question is—what kind of a phisical law the last equation represents?

In order to find out the answer we would like to propose quite a simple classical model. We can apply the classical approach because we are dealing with constant macroscopic physical parameters and those ratios. Let us consider a really large number of non-interacting quanta. All quanta are moving in all directions with a speed of light, i.e. there are photons. Obviously, the set must be confined inside the Black Hole with a radius $R$. If the total energy of the whole quanta set is equal to $E_u$, we have to conclude that absolutely every photon must be reflected by the sphere’s bound in certain time. In other words the inner side of the Hubble sphere plays a role of an ideal diffusely reflecting surface. In a period of time $T = R/c$ we will see that ultimately all quanta had experienced a reflection from the bound. It means that the inner side of the Hubble sphere looks like a Lambertian light emitter for an internal observer. Thus the constant radiant emittance from the inner side of our Black Hole can be expressed by the formula:

$$W_u = \frac{E_u}{4 \pi R^2 T} = \frac{1}{4 \pi} M H^3$$ (20)

Now let us consider the observer—a spherical body in a vacuum with radius $r_o \ll R$ placed at the center of the Hubble sphere. The observer will find out a constant isotropic quanta flow that is falling to an every surface area $\delta S$ from a spatial hemisphere above the area.

According to the Lambert’s cosine law a radiant energy flux $\partial \Phi_{in}$ through the area $\delta S$ (i.e. irradiance) will be:

$$\frac{\partial \Phi_{in}}{\partial S} = \frac{a}{2} W_u \cos(\theta) \sin(\theta) \partial \phi \partial \theta$$ (21)

where $\theta$ is the angle between the beam and a line normal to the surface area $\delta S$.

The total radiant flux $\Phi_{in}$ related to the entire surface
of our spherical observer:

$$\Phi_m = \int \frac{\partial \Phi_m}{\partial S} \, dS$$  \hspace{1cm} (22)

$$\partial S = r_0^2 \sin(\beta) \partial \beta \partial \gamma$$  \hspace{1cm} (23)

where $\beta$ and $\gamma$ are spherical coordinates of the area $\partial S$ on observer’s surface. Thus:

$$\Phi_m = \int_0^{2\pi} \int_0^\pi \int_0^{r_0} W_U \cos(\theta) \sin(\theta) r_0^2 \sin(\beta) \partial \phi \partial \theta \partial \beta \partial \gamma$$  \hspace{1cm} (24)

The last integral (24) can be simplified as follows:

$$\Phi_m = 4\pi^2 r_0^3 W_U = \frac{E_U}{T} \frac{r_0^2}{R^2}$$  \hspace{1cm} (25)

The total amount of energy $E_{\text{int}}$ entered inward (or reflected by) the observer during the period of time $T$ is:

$$E_{\text{int}} = \Phi_m T = m_{\text{tot}} c^2$$  \hspace{1cm} (26)

where $m_{\text{tot}}$ is a total mass which our observer should have at present time if he absorbs (or reflects) the incoming energy flux $\Phi_m$ completely. Thus we can write down the following equation:

$$\frac{M c^2}{R^2} = \frac{m_{\text{tot}} c^2}{r_0^2}$$  \hspace{1cm} (27)

Now, making a comparison of the Equations (27) and (19) one would ultimately conclude that if the coefficient $n_e$ in the (19) equals exactly to $\pi$ then $m_{\text{tot}} / r_0^2$ must be equal to $m_e / \alpha r_e^2$. Thus:

$$M_R = \frac{1}{\alpha \pi r_e^2}$$  \hspace{1cm} (28)

Using (4), (11) and (28), we immediately obtain a noteworthy ratio between Weyl-Eddington-Dirac large numbers:

$$N_{\text{DR}}^2 = \alpha \pi N_M$$  \hspace{1cm} (29)

Hence, we can obtain the formula for the universe mass via the Hubble time:

$$M = \frac{m_e \alpha^2 c^2}{\pi} = \frac{m_e c^2}{\alpha \pi r_e^2} T^2$$  \hspace{1cm} (30)

The last one represents the proportional relation between $M$ and $T^2$ which was hypothesized by P. Dirac almost 80 years ago.

As we see from (29), both fundamental constants $\alpha$ and $\pi$ are deeply involved in large number relations and thus we can assume that it is an evidence that cosmological and quantum parameters of our model of the universe are closely connected through geometry.

The above Equations (13) and (29) readily yield the correlation of the best known large Weyl-Eddington-Dirac numbers, which include both the geometrical constant $\pi$ and the fine structure constant $\alpha$:

$$\alpha \pi N_{\text{DR}} = 2N_M$$  \hspace{1cm} (31)

This correlation is very notable. It enables one to calculate the approximations for Hubble sphere radius and the Hubble parameter via the correlation of gravitational and electrostatic forces:

$$R = \frac{1}{2} \alpha \pi N_M r_e$$  \hspace{1cm} (32)

$$H = \frac{2c}{N_M \alpha \pi r_e}$$  \hspace{1cm} (33)

Now let us introduce the “big” angular momentum of the Hubble sphere measured along any given direction $\mathbf{\ell}$:

$$L_{\mathbf{\ell}} = M R \mathbf{c}$$  \hspace{1cm} (34)

Thus, multiplying both sides of the Equation (28) by $\epsilon N_U$, we would propose the exact Largest number law in the following form:

$$N_U h = \pi L_{\mathbf{\ell}}$$  \hspace{1cm} (35)

The total sum of $N_U$ fundamental quanta of the angular momentum $h$ in universe equals exactly to the angular momentum of the Hubble sphere multiplied by $\pi$. This is a direct consequence of geometrical Lambert’s cosine law and rotational symmetry of space (conservation of angular momentum).

The following expression, as well as expressions (29) and (35), represent just another form of this law:

$$N_{\text{DR}} \pi \alpha N_M N_{\text{DR}}$$  \hspace{1cm} (36)

With the help of large numbers ratios and Largest number law, one can get various representations of the Newton constant of gravitation $G$ using initial expression:

$$G = \frac{1}{N_M} \frac{1}{4 \pi \epsilon_0} \frac{q_e^2}{m_e^2}$$  \hspace{1cm} (37)

The most elegant cases, in our opinion, are:

$$G = \frac{\alpha}{m_e^2 \pi} \frac{\epsilon_0}{8 \epsilon_0 N_{\text{DR}}}$$  \hspace{1cm} (38)

and:

$$G = \frac{\alpha \pi r_e^2 c^4}{2 m_e R} = \frac{h^2 \alpha^4}{8 \pi m_e^4 R^2} = \frac{\alpha \pi r_e^2 c}{2 m_e T}$$  \hspace{1cm} (39)

The last one represents the reverse proportional relation between $G$ and $R$, which was hypothesized by H. Weyl and between $G$ and $T$ which was hypothesized by P. Dirac.
By means of (39), one can get representation of the cosmological constant $\Lambda$:

$$\Lambda = \frac{8\pi G \omega_u}{c^4} = \frac{6GM}{c^2 R^2} = \frac{6\alpha \pi N_{DP}}{2N_{DP}R^2} = \frac{3}{R^2} \quad (40)$$

where $\omega_u = E_u / V_u$ — energy density of the universe.

The last one represents the de Sitter space—a vacuum solution of Einstein’s equation with cosmological constant—for the 4-dimensional case. Hence, we can obtain the formula for the cosmological constant $\Lambda$ via the fine structure constant $\alpha$ , Weyl-Eddington-Dirac large number $N_{DP}$, classical radius of electron $r_e$ and $\pi$:

$$\Lambda = \frac{12}{(N_{DP}\pi r_e)^2} \quad (41)$$

4. Calculations

The ratios between large numbers as described in Sections 2 to 3 enable us to calculate many cosmological parameters in the proposed model. The calculations has been carried out (by using the constants $\alpha, r_e, m_e, c, G, \pi$) as follows: $h, N_{DP}, E_u$, then $N_{DR}$, then $N_{L}, N_{M}, R, H, T$, then $M, N_{DR}, E_{DP}$ and then $E_u$.

The results of calculations of the constants based on referential data and information from the most recent measurements are shown in Table 3.

Table 3. Proposed values based on the the Largest number law.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Known value</th>
<th>Proposed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>67.80(77) km/s·Mpc</td>
<td>68.7457(82) km/s·Mpc</td>
</tr>
<tr>
<td>$H$</td>
<td>2.197(25)×10^{-8}s</td>
<td>2.22789(27)×10^{-8}s</td>
</tr>
<tr>
<td>$T$</td>
<td>4.551(52)×10^{-1}s</td>
<td>4.48853(54)×10^{-1}s</td>
</tr>
<tr>
<td>$T$</td>
<td>14.43(16)Gyr</td>
<td>14.2330(17)Gyr</td>
</tr>
<tr>
<td>$M$</td>
<td>9.188(10)×10^{11}kg</td>
<td>9.0606(22)×10^{11}kg</td>
</tr>
<tr>
<td>$R$</td>
<td>1.364(15)×10^{20}m</td>
<td>1.345629(16)×10^{20}m</td>
</tr>
<tr>
<td>$N_{DP}$</td>
<td>4.16589(50)×10^{60}</td>
<td>-</td>
</tr>
<tr>
<td>$N_{DR}$</td>
<td>4.842(55)×10^{40}</td>
<td>4.77522(57)×10^{40}</td>
</tr>
<tr>
<td>$N_L$</td>
<td>1.135(39)×10^{22}</td>
<td>1.08888(39)×10^{22}</td>
</tr>
<tr>
<td>$N_M$</td>
<td>1.009(11)×10^{41}</td>
<td>9.9465(24)×10^{40}</td>
</tr>
<tr>
<td>$N_e$</td>
<td>3.564(81)×10^{22}</td>
<td>3.4660(12)×10^{22}</td>
</tr>
<tr>
<td>$\omega_u = E_u / V_u$</td>
<td>7.65(26)×10^{-14} m^{-2}</td>
<td>7.97879(95)×10^{-15} m^{-2}</td>
</tr>
<tr>
<td>$E_u = Mc^2$</td>
<td>8.257(94)×10^{39}J</td>
<td>8.1433(19)×10^{39}J</td>
</tr>
<tr>
<td>$E_e$</td>
<td>2.317(26)×10^{-39} J</td>
<td>2.34947(28)×10^{-39} J</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1.612(35)×10^{-50} m^2</td>
<td>1.656801(39)×10^{-50} m^2</td>
</tr>
</tbody>
</table>

The ratios of large numbers described in the previous sections enable to link the Hubble volume radius and other constants with the length dimensions, including such parameters as the electron gravitational radius and the Planck’s length. All such constants can be calculated one from the other, via the fine structure constant $\alpha$, Weyl-Eddington-Dirac large numbers $N_{DP}$ or $N_{DR}$ and $\pi$. The Tables 4 and 5 show the mutual conversion ratios of the constants—from the Compton wave length of the universe to the Hubble sphere radius. To obtain the value of the top line parameter, one should multiply the initial parameter in the respective column by the formula in the cell at their crossing.

In Table 5, one can find one more elegant expression—the ratio between largest and smallest distances in universe-radius of the Hubble sphere and the Compton wave length of the entire universe. It is proportional to the Largest number:

$$\frac{R}{\lambda_U} = \frac{1}{2\pi^2} N_U \quad (42)$$

5. Examples of Applying the Large Number Ratios

The previous sections contain a rather simple derivation of the inter-dependence of all main large numbers. The Largest number law that links the Weyl-Eddington-Dirac numbers enable validation of whether the known hypothetic equations and inequalities conform to the large numbers combination we proposed or not. Below are several examples.

Example 1. J. Teller proposed [16] an interesting ratio between Planck’s values, the fine structure constant and Hubble’s cosmological parameter:

$$\kappa m_p c = \kappa \hbar = \frac{8\pi G}{\alpha} H = \exp \left( -\frac{1}{\alpha} \right) \quad (43)$$

where $\kappa = \frac{8\pi G}{c^4}$ — Einstein constant, $m_p, \hbar, \alpha$ — Planck units.

Having very large magnitude $\sim 10^{60}$, the left and right parts of the formula give us values which differ from each other by only 1.5%. It is really remarkable but it is about 70 times more uncertain than other values calculated by us earlier:

$$8\pi \frac{\hbar}{m_p c^2} \leq \frac{32\pi^2}{N_U} \leq 0.98588(12) \quad (44)$$

Teller’s formula cannot be recognized exact and expressing a fundamental physical law as it does not provide any strict derivation yet. A. Eddington’s contemplative assumptions on the quantity of particles in the universe, which is supposedly equal to precisely $2^{256}$,
Table 4. Calculating parameters with a length dimension \( N_{\text{em}} \).

<table>
<thead>
<tr>
<th>( \lambda_c )</th>
<th>( l_p )</th>
<th>( r_c )</th>
<th>( r_\lambda )</th>
<th>( r_\gamma )</th>
<th>( \lambda_0 )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \sqrt{\alpha' N_{\text{em}}} )</td>
<td>( \frac{\alpha' N_{\text{em}}^2}{2 \pi} )</td>
<td>( \frac{\alpha' N_{\text{em}}^3}{2 \alpha' \pi} )</td>
<td>( \frac{\alpha' N_{\text{em}}^4}{2 \alpha' \pi^2} )</td>
<td>( \frac{\alpha' N_{\text{em}}^5}{2 \alpha' \pi^3} )</td>
<td>( \frac{\alpha' N_{\text{em}}^6}{2 \alpha' \pi^4} )</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{1}{\alpha N_{\text{em}}} )</td>
<td>1</td>
<td>( \frac{N_{\text{em}}}{\alpha \pi} )</td>
<td>( \frac{N_{\text{em}}^2}{\alpha \pi^2} )</td>
<td>( \frac{N_{\text{em}}^3}{\alpha \pi^3} )</td>
<td>( \frac{N_{\text{em}}^4}{\alpha \pi^4} )</td>
</tr>
<tr>
<td>1</td>
<td>( \sqrt{\alpha N_{\text{em}}} )</td>
<td>1</td>
<td>( \frac{1}{\alpha N_{\text{em}}} )</td>
<td>( \frac{1}{\alpha N_{\text{em}}^2} )</td>
<td>( \frac{1}{\alpha N_{\text{em}}^3} )</td>
<td>( \frac{1}{\alpha N_{\text{em}}^4} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{\alpha' N_{\text{em}}} )</td>
<td>( \frac{1}{\alpha' N_{\text{em}}^2} )</td>
<td>1</td>
<td>( \frac{1}{\alpha' N_{\text{em}}^3} )</td>
<td>( \frac{1}{\alpha' N_{\text{em}}^4} )</td>
<td>( \frac{1}{\alpha' N_{\text{em}}^5} )</td>
</tr>
<tr>
<td>16</td>
<td>( \frac{1}{\alpha' N_{\text{em}}^2} )</td>
<td>( \frac{4}{\alpha' N_{\text{em}}^3} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^4} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^5} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^6} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^7} )</td>
</tr>
</tbody>
</table>

where \( l_p \)—Planck length, \( r_c \)—Bohr radius.

Table 5. Calculating parameters with a length dimension \( N_{\text{em}} \).

<table>
<thead>
<tr>
<th>( \lambda_c )</th>
<th>( l_p )</th>
<th>( r_c )</th>
<th>( r_\lambda )</th>
<th>( r_\gamma )</th>
<th>( \lambda_0 )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \sqrt{N_{\text{em}}^3} )</td>
<td>( \frac{\alpha N_{\text{em}}}{2 \pi} )</td>
<td>( \frac{N_{\text{em}}^2}{2 \alpha' \pi} )</td>
<td>( \frac{N_{\text{em}}^3}{2 \alpha' \pi^2} )</td>
<td>( \frac{N_{\text{em}}^4}{2 \alpha' \pi^3} )</td>
<td>( \frac{N_{\text{em}}^5}{2 \alpha' \pi^4} )</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{1}{\alpha N_{\text{em}}} )</td>
<td>1</td>
<td>( \frac{2}{\alpha N_{\text{em}}} )</td>
<td>( \frac{2}{\alpha N_{\text{em}}^2} )</td>
<td>( \frac{2}{\alpha N_{\text{em}}^3} )</td>
<td>( \frac{2}{\alpha N_{\text{em}}^4} )</td>
</tr>
<tr>
<td>16</td>
<td>( \frac{1}{\alpha' N_{\text{em}}} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^2} )</td>
<td>1</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^3} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^4} )</td>
<td>( \frac{2}{\alpha' N_{\text{em}}^5} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{\pi}{N_{\text{em}}} )</td>
<td>( \frac{4}{N_{\text{em}}} )</td>
<td>( \frac{8}{N_{\text{em}}} )</td>
<td>( \frac{16}{N_{\text{em}}} )</td>
<td>( \frac{32}{N_{\text{em}}} )</td>
<td>( \frac{64}{N_{\text{em}}} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{\pi}{N_{\text{em}}} )</td>
<td>( \frac{4}{N_{\text{em}}} )</td>
<td>( \frac{8}{N_{\text{em}}} )</td>
<td>( \frac{16}{N_{\text{em}}} )</td>
<td>( \frac{32}{N_{\text{em}}} )</td>
<td>( \frac{64}{N_{\text{em}}} )</td>
</tr>
<tr>
<td>16</td>
<td>( \frac{1}{N_{\text{em}}} )</td>
<td>( \frac{1}{N_{\text{em}}} )</td>
<td>( \frac{1}{N_{\text{em}}} )</td>
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</tr>
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</table>

are even further from the reality.

**Example 2.** There is known Eddington-Weinberg approximate relation [17]:

\[
\hbar^2 H \approx G c m_p^3
\]

where \( m_p = 1.672621777(17) \times 10^{-27} \text{kg} \)—proton mass (CODATA 2010).

Using the approximation of \( H \) calculated above (Table 3) one can get the ratio, which is quite far from expected 1.0:

\[
\frac{\hbar^2 H}{G c m_p^3} = 0.000264629(45) \neq 1
\]

Thus we can conclude that Eddington-Weinberg approximate identity is not confirmed in our model. But, we should say that Eddington-Weinberg formula does have physical meaning in a nutshell. One can get the exact identity by replacing \( m_p \) by \( m_e \) and applying
the large number ratios to this hypothetical formula:

$$\frac{h^2H}{Gcm^2} = \frac{2}{\alpha^2\pi}$$  \hspace{1cm} (47)

**Example 3.**

J. Bekenstein proposed [18] universal entropy bound for a complete physical system whose total mass-energy (in our case) is \(E_U\), and which fits inside a sphere of radius \(R\). Applying the large numbers ratios, we can get the universal entropy bound value:

$$S_B \leq \frac{2\pi k E_U R}{\hbar c} = \frac{2\pi k E_U}{E_w} = \frac{2\pi k N_U}{2k N_U} = 2k N_U$$  \hspace{1cm} (48)

Where \(k\) is Boltzmann constant.

On the other hand, the Bekenstein-Hawking entropy bound for the Black Hole with radius \(R\) is:

$$S_{BH} \leq \frac{\pi k c^2 R^2}{\hbar G}$$  \hspace{1cm} (49)

By dividing (48) by (49) and using (8) one can get:

$$\frac{S_B}{S_{BH}} = \frac{2E_G G}{c^2 R} = \frac{2M G}{c^2 R} = 1$$  \hspace{1cm} (50)

The last one says that both Bekenstein universal entropy bound and Bekenstein-Hawking Black Hole entropy bound have the same value in our model and equal exactly to \(2k N_U\):

$$S_B \approx k \cdot 2.17776 \cdot 10^{122} \approx 3.0067 \cdot 10^{99} \text{ J/K}$$  \hspace{1cm} (51)

Using the large number ratios it is also easy to express the Hawking radiation \(E_H\) of our Black Hole with energy \(E_U\) via Largest number \(N_U\) or Hubble parameter:

$$E_H = T_H k = \frac{h c^3}{8 \pi G M} = \frac{E_U}{4N_U} = \frac{E_w}{4\pi} = \frac{h H}{4\pi}$$  \hspace{1cm} (52)

where \(T_H\) is Hawking radiation temperature.

**6. Discussion and Conclusions**

The two currently prevailing physical theories, i.e. quantum mechanics and the general relativity, describe the reality very precisely, each in its range of energy and spatial scale. It is presumed that sometimes in the future, the value of \(N_{DF}\) will be obtained in theory directly as a direct result of consolidation of gravitation with other known interactions, strong and electroweak.

The fruitless attempts at explaining the proximity of \(N_{DF}\) and \(N_{DR}\) resulted in a broad application of the term of “coincidence” that somehow highlights the randomness of the event. As shown in previous sections it is not random.

As we see, the large number ratios proposed in the article provide the powerful means for finding relations among various information and physical parameters of our model universe. However, it does not help answer the main question: where do these enormous numbers come from in physics? Hopefully, the law and hypotheses proposed in this article will let find the correct answer in the foreseeable future.

The ratios we suggested impose rather stringent limitations on the way physical constants may change over time. We must note that the sharply tuned combination of large numbers, including the mentioned approximations for Hubble time \(T\), mass of the universe \(M\) and Hubble limit \(R\) correspond to the values of the very precisely measured physical constants of quantum scale. Looking at (39) and (40), it is obvious that the gravity is closely connected with the Hubble radius, Hubble time and the properties of electron. However, there are rather reliable measurements that establish a very low limit for the Newton constant change rate in the long-term (<1%). Thus, if Hubble parameters \(T, R\) varies with time then the corresponding variation of \(\alpha\) or/and electron energy \(m_e c^2\) should preserve the constancy of \(G\).

If the entire universe energy \(E_U\) comprises (or once comprised) quants of the minimum energy \(E_w\), we should make the conclusion that number (14) is a natural number. The same is true about (15). Hence, we can establish the hypothesis that all large numbers in the real (not infinite) universe should be naturals or rationals. However, the denominator and the numerator in these fractional numbers are so great that one can confidently presume the real large numbers approximate some perfect limit of fundamental importance infinitely closely. Taking into account the transcendentalism of (29), (31) and (35), we dare to express one more hypothesis: all Large numbers are infinitely approaching their limits and these very limits are transcendent mathematical constants.

The Schwarzschild solution is one of the well known models of our universe, representing a Black Hole. Avoiding creation of new essences, we just consider the interaction between internal radiation and a small spherical observer. It appears that such a simple model reveals a lot of interesting identities and conservation laws. For example, the value of \(m_{er}/\alpha^2\) should be conserved for a sphere with a radius \(r_e\). Furthermore we found that identity \(m_{er} = m_e/\alpha\) is valid for a sphere with classical electron radius \(r_e\).

The last derivation allowed us to propose a list of exact ratios between well known Eddington-Weyl-Dirac large numbers which are listed in Table 2. The ratios (30) and (39) reveals the exact formulae supporting P. Dirac’s hypothesis and propositions which were announced many years ago and never were written in a clear mathematical...
form.

Using these large numbers ratios we have claimed the new Largest number law (35) which is based on Lambert’s cosine law and the rotational symmetry of space. It is quite important that this law precisely unites the major cosmological and quantum parameters of our universe. It could be interpreted as follows: our universe comprises of the mathematically determined number of elementary spatial clusters which can be matched up to energy quanta and angular momentum quanta. We should note that basic parameters of our model universe have been obtained from the properties of electron, speed of light and fine structure constant.

If Equations (13), (28) and (31) are valid inside the Hubble sphere, which is very probable, we should make a conclusion that all cosmological parameters are fully and unambiguously determined by quantum and mathematical constants. Simply put, all of us are very likely to live in an extremely precisely self-tuned quantized universe.

REFERENCES