Well Behaved Class of Charge Analogue of Adler’s
Relativistic Exact Solution

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Abstract

We present a well behaved class of charge analogue of Alder’s (1974) [1]. This solution describes charge fluid balls with positively finite central pressure and positively finite central density; their ratio is less than one and causality condition is obeyed at the centre. The outmarch of pressure, density, pressure-density ratio and the adiabatic speed of sound is monotonically decreasing, however, the electric intensity is monotonically increasing in nature. The solution gives us wide range of parameter $K$ (0.96 $\leq$ $K$ $\leq$ 5.2) for which the solution is well behaved and appropriate for relativistic theory; therefore, suitable for modeling of super dense star. For this solution the mass of a star is maximized with all degrees of suitability and by assuming the surface density $\rho_b = 2 \times 10^{14}$ g/cm$^3$. Corresponding to $K = 0.96$ and $X = 0.35$, the maximum mass of the star comes out to be 3.43 $M_\odot$ with linear dimension 32.66 Km and central redshift and surface redshift 1.09374 and 0.5509 respectively.

Keywords: Charge Fluid, Reissner-Nordstrom, General Relativity, Exact Solution

1. Introduction

It is well known that the Reissner-Nordstrom solution for the external field of a ball of charged mass has two distinct singularities at finite radial positions other than at the centre. Thus the solution describes a bridge (worm hole) between two asymptotically flat spaces and an electric flux flowing across the bridge. Graves and Bill [2] pointed out that the region of minimum radius or the throat of worm hole pulsates periodically between these two surfaces due to Maxwell pressure of the electric field. Consequently, unlike Schwarzschild’s exterior solution of chargeless matter in Reissner-Nordstrom solution has no surface which can catastrophically hit the geometric singularity at $r = 0$. All these aspects show that the presence of some charge in a spherical material distribution provides an additional resistance against the gravitational contraction by means of electric repulsion, thereby the catastrophic collapse of the entire mass to a point singularity can be avoided.

The above result has been supported by a physically reasonable charge spherical model of Bonnor [3], that a dust distribution of arbitrarily large mass and small radius can remain in equilibrium against the pull of gravity by a repulsive force produced by a small amount of charge. Thus it is desirable to study the implications of Einstein-Maxwell field equations with reference to the general relativistic prediction of gravitational collapse. For this purpose charged fluid ball models are required. Eventually, the external field of such ball is to be matched with Reissner-Nordstrom solution.

For obtaining significant charged fluid ball models of Einstein-Maxwell field equations, the Astrophysicists have been using exact solutions with finite central parameters of Einstein field equations, as seed solutions. There are two type of exact solutions of this category.

Type 1. If the solutions are well behaved (Delgaty-Lake [4], Pant [5]). These solutions their self completely describe interior of the Neutron star or analogous super dense astrophysical objects with chargeless matter. Delgaty-Lake [4] studied most of the exact solutions so far obtained and pointed out that only nine solutions are regular and well behaved; out of which only six of are well behaved in curvature coordinates. Pant [5] obtained two new well behaved solutions in curvature coordinates.

Type 2. If the solutions are not well behaved but with finite central parameters; such solutions are taken as seed solutions of super dense star with charge matter since at centre charge distribution is zero.

1) Schwarzschild’s interior solution-The solution is
insignificant as it gives us infinite speed of sound throughout with in the ball. However, charge Analogues of the solution is well behaved for wide range of constant (Gupta-kumar [6], Gupta-Gupta [7], Florides [8] etc).

2) Many of the authors electrified the well known exact solutions which are not well behaved, as seed solutions e.g. Kuchowicz solutions by Nduka [9], Tolman solution by Cataldo-Mitskievic [10] and Durga-pal-Fuloria solution [11] by Gupta-Maurya [12] Durgapal solution [13] by Pant [14] etc. These coupled solutions are well behaved and completely describe interior of the Neutron star or pulsar with charge matter.

In the present paper we have charged the Adler [1] solution, which is not well behaved with chargeless matter as the speed of sound is monotonically increasing from centre to boundary Durgapal [13]. The present paper charge analogue of Adler solution is well behaved in all respects and simple in terms of expression. Though the charge Analogues of the Adler solution has also been studied by Singh-Yadav [15], however the solution is silent about its well behaved nature and not simple in terms of expression.

For well behaved nature of the solution in curvature coordinates, the following conditions should be satisfied (Pant N [5]).

1) The solution should be free from physical and geometrical singularities i.e. finite and positive values of central pressure, central density and non zero positive values of $e'$ and $e''$. i.e. $p_0 > 0 \text{ and } \rho_0 > 0$. For such solutions the tangent $-3$ space at the centre is flat and it is an essential condition. For curvature coordinates, mathematically it is expressed as $(e^\gamma)_{r=0} = 1$ and $(e')_{r=0} = \text{positive constant (Leibovitz [16] Pant [5])}.$

2) The solution should have positive and monotonically decreasing expressions for fluid parameters ($p$ and $\rho$) with the increase of $r$ i.e.

\begin{align*}
\text{a) } & \left( \frac{dp}{dx} \right)_{r=0} < 0 (-ve), \text{ b) } \left( \frac{d\rho}{dx} \right)_{r=0} < 0 \\
\text{3) The solution should have positive and monotonically decreasing expression for fluid parameter } & \frac{p}{\rho c^2} \text{ with the increase of } r \text{ i.e.} \\
& \frac{1}{c^2} \left[ \frac{d}{dx} \left( \frac{p}{\rho} \right) \right]_{r=0} < 0 . \\
\text{4) The solution should have positive and monotonically decreasing expressions for fluid parameter } & \frac{dp}{d\rho} \text{ with the increase of } r \\
\end{align*}

\begin{align*}
\left( \frac{d}{dx} \left( \frac{dp}{d\rho} \right) \right)_{r=0} < 0 .
\end{align*}

5) The solution should have causality condition at centre of the ball i.e.

\begin{align*}
0 < \left( \frac{dp}{\rho^2} \right)_{r=0} \leq 1.
\end{align*}

6) The solution should have positive value of ratio of pressure-density and less than 1 at the centre of the ball i.e.

\begin{align*}
0 < \frac{p_0}{\rho_0 c^2} \leq 1.
\end{align*}

7) The central red shift $Z_0$ and surface red shift $Z_0$ should be positive and finite i.e. $Z_0 = \left( e^{\delta/2} - 1 \right)_{r=0} > 0$ and $Z_0 = \left[ e^{\delta/2} - 1 \right]$; both should be bounded and red shift should be monotonically decreasing with the increase of $r$ i.e.

\begin{align*}
\left[ \frac{dz}{dr} \right]_{r=0} < 0 .
\end{align*}

8) Electric intensity is positive and monotonically increasing from centre to boundary and at the centre the Electric intensity is zero i.e.

\begin{align*}
\left[ \frac{d}{dx} \left( E^2 \right)_{i} \right]_{r=0} > 0 .
\end{align*}

2. Einstein-Maxwell Equation for Charged Fluid Distribution

Let us consider a spherical symmetric metric in curvature coordinates

\begin{align*}
\frac{dx_i}{e^{r}} = -e^{2}dr^{2} - r^{2} \left( dt^{2} + \sin^{2} \theta d\phi^{2} \right) + e^{r} dr^{2}
\end{align*}

(1)

where the functions $\lambda(r)$ and $v(r)$ satisfy the Einstein-Maxwell equations

\begin{align*}
-\frac{8\pi G}{c^4} T^t_{t} = R_{t} - \frac{1}{2} R S_{t} = -\frac{8\pi G}{c^4} \left[ \left( c^2 \rho + p \right) v' v_j - p \delta'_j \right. \\
+ \frac{1}{4\pi} \left( -F_{\mu j}^{\nu} F_{\mu j}^{\nu} + \frac{1}{4} \delta^{\nu}_{j} F_{\mu \nu} F_{\mu \nu} \right) \left. \right]
\end{align*}

(2)

where $\rho, p, v', F_{\mu j}$ denote energy density, fluid pressure, velocity vector and skew-symmetric electromagnetic field tensor respectively.

In view of the metric (1), the field equation (2) gives (Dionysiou, [17])

\begin{align*}
\frac{v'}{r} e^{-\frac{1}{2}} - \frac{\left( 1 - e^{-\frac{1}{2}} \right)}{r^{2}} = \frac{8\pi G}{c^4} p - \frac{g^{2}(r)}{r^4}
\end{align*}

(3)

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$$\left( \frac{v^2}{2} - \frac{A'v'}{4} + \frac{v'^2}{4} - \frac{v - 2'}{2r} \right) e^z = \frac{8\pi G}{c^4} \rho + \frac{q^2(r)}{r^4}$$  \hspace{1cm} (4)$$

$$\frac{2'}{r} e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \frac{8\pi G}{c^4} \rho + \frac{q^2(r)}{r^4}$$  \hspace{1cm} (5)

where, prime (') denotes the differentiation with respect to r and q(r) represents the total charge contained within the sphere of radius r.

Now let us set

$$e' = B\left(1 + c_r r^2\right)^2$$  \hspace{1cm} (6)

which is the same as that of the metric obtained by Adler [1].

Putting (6) into (3)-(5), we have

$$4Z = 1 + \frac{\left(1 - Z\right)}{x} + \frac{c_i q^2}{x^2} = \frac{1}{c_i} \frac{8\pi G}{c^4} \rho$$  \hspace{1cm} (7)

$$-2 \frac{dZ}{dx} = \frac{c_i q^2}{x^2} = \frac{1}{c_i} \frac{8\pi G}{c^4} \rho$$  \hspace{1cm} (8)

and Z satisfying the equation

$$\frac{dZ}{dx} - \frac{x+1}{x(x+3)}Z = \frac{(1+x)}{x(x+3)} \left(\frac{2c_i q^2}{x} - 1\right)$$  \hspace{1cm} (9)

where \( x = c_r r^2 \), \( e^{-\lambda} = Z \).

Our task is to explore the solutions of equation (9) and obtain the fluid parameters \( \rho \) and \( \rho \) from Equation (7) and Equation (8).

### 3. New Class of Solutions

In order to solve the differential Equation (9), we consider the electric intensity \( E \) of the following form

$$E^2 = \frac{c_i q^2}{x^2} = \frac{K}{2} x(1+3x)^3$$  \hspace{1cm} (10)

where \( K \) is a positive constant. The electric intensity is so assumed that the model is physically significant and well behaved i.e. \( E \) remains regular and positive throughout the sphere. In addition, \( E \) vanishes at the centre of the star.

In view of Equation (10) differential Equation (9) yields the following solution

$$Z = e^{-\lambda} = 1 + \frac{K}{2} \frac{x(1+x)^2}{(1+3x)^3} + \frac{Ax}{(1+3x)^3}$$  \hspace{1cm} (11a)

where \( A \) is an arbitrary constant of integration.

$$e' = B\left(1 + x\right)^2$$  \hspace{1cm} (11b)

Using (11a), (11b) into Equations (7) and (8), we get the following expressions for pressure and energy density

$$\frac{1}{c_i} \frac{8\pi G}{c^4} \rho = \frac{K}{2} \frac{\left(8x^2 + 7x + 1\right)}{(1+3x)^3} + \frac{A(1+5x)}{(1+x)(1+3x)^3} + \frac{4}{(1+x)}$$  \hspace{1cm} (12)

$$\frac{1}{c_i} \frac{8\pi G}{c^4} \rho = -\frac{A(3+5x)}{(1+3x)^3} \hspace{1cm} (13)$$

### 4. Properties of the New Class of Solutions

The central values of pressure and density are given by

$$\frac{1}{c_i} \frac{8\pi G}{c^4} p_{0} = \frac{K}{2} + A + 4$$  \hspace{1cm} (14)

$$\frac{1}{c_i} \frac{8\pi G}{c^4} \rho_{0} = -\frac{3}{2} K - 3A$$  \hspace{1cm} (15)

For \( p_{0} \) and \( \rho_{0} \) must be positive and \( \frac{p_{0}}{\rho_{0}} \leq 1 \), we have

$$\frac{K}{2} < A \leq \frac{K}{2} \hspace{0.5cm} \text{and} \hspace{0.5cm} K \geq 0, A < 0$$  \hspace{1cm} (16)

Differentiating (12) and (13) w.r.t. \( x \), we get;

$$\frac{1}{c_i} \frac{8\pi G}{c^4} \frac{dp}{dx} = \frac{K}{2} \frac{(32x^3 + 55x^2 + 28x + 5)}{(1+3x)^3(1+x)} + \frac{2A}{(1+x)^2} \frac{(1-5x^2)}{(1+3x)^3} + \frac{4}{(1+x)^2}$$  \hspace{1cm} (17)

$$\frac{1}{c_i} \frac{8\pi G}{c^4} \frac{d\rho}{dx} = -\frac{K}{2} \frac{(104x^4 + 83x^3 + 38x + 1)}{(1+3x)^3(1+3x)^3} + 10A \frac{(x+1)}{(1+3x)^3}$$  \hspace{1cm} (18)

$$\left(\frac{1}{c_i} \frac{8\pi G}{c^4} \frac{dp}{dx}\right)_{x=0} = \frac{5K}{2} + 2A - 4$$  \hspace{1cm} (19)

The expression of right hand side of (19) is negative, thus by virtue of theorem, the pressure \( p \) is maximum at the centre and monotonically decreasing.
\[
\left(1 + \frac{8\pi G}{c^2} \frac{d\rho}{dx}\right)_{x=0} = -\frac{K}{2} + 10A \tag{20}
\]

\[
\left(1 + \frac{8\pi G}{c^2} \frac{d\rho}{dx}\right)_{x=0} < 0 \tag{21}
\]

The expression of right hand side of (20) is negative, thus the density \(\rho\) is maximum at the centre and monotonically decreasing.

and hence the velocity of sound \(v\) is given by the following expression

\[
v^2 = \frac{d\rho}{dp}
\]

\[
\frac{1}{c^2} \frac{dp}{d\rho} = \left(1 + 3x\right) \left[ K \left(32x^3 + 55x^2 + 28x + 5\right) \left(1 + x\right) + 4A \left(1 - 5x^2\right) \right] - 8\left(1 + 3x\right)^7 \\
\left[-K \left(104x^3 + 83x^2 + 38x + 1\right) + 20A \left(1 + x\right)^2 \right] \left(1 + x\right)^2
\tag{22}
\]

\[
\frac{1}{c^2} \left( \frac{d\rho}{dx}\right)_{x=0} = \frac{5K + 4A - 8}{-K + 20A} \leq 1 , \text{ for all values of } K \text{ and } A \text{ satisfied by (16)}
\]

Using Equations (12) and (13)

\[
\frac{1}{c^2} \frac{dp}{d\rho} = \frac{\alpha}{\beta}
\tag{23}
\]

where

\[
\alpha = \frac{K}{2} \left(8x^2 + 7x + 1\right) + \frac{A}{\left(1 + 3x\right)^2} \left(1 + x\right) + \frac{4}{\left(1 + 3x\right)^2}
\]

\[
\beta = \frac{K}{2} \left(26x^3 + 35x^2 + 16x + 3\right) - \frac{A}{\left(1 + 3x\right)^2}
\]

Differentiating (22) w.r.t. \(x\)

\[
\psi = \left(1 + 3x\right) \left[ K \left(32x^3 + 55x^2 + 28x + 5\right) \left(1 + x\right) + 4A \left(1 - 5x^2\right) \right] - 8\left(1 + 3x\right)^7 \\
\xi = -K \left(104x^3 + 83x^2 + 38x + 1\right) + 20A \left(1 + x\right)^2 \left(1 + x\right)^2
\tag{26a}
\]

\[
\xi = \left(-14K^2 - 60AK + 32K - 112A - 64A^2 \right) \left(3K + 6A\right)^2
\tag{26b}
\]

The expression of right hand side of (27) is negative, thus the square of adiabatic speed of sound \(d\rho/d\rho\) is maximum at the centre and monotonically decreasing.

The expression for gravitational red-shift \((z)\) is given by

\[
z = \left(1 + x\right)^3 - \frac{1}{\sqrt{B}} - 1
\tag{28}
\]

The central value of gravitational red shift to be non zero positive finite, we have

\[
1 > \sqrt{B} > 0
\tag{28a}
\]

Differentiating Equation (28) w.r.t. \(x\), we get,

\[
\frac{dz}{dx} \bigg|_{x=0} = -\frac{1}{\sqrt{B}} < 0
\tag{28b}
\]

The expression of right hand side of (28b) is negative, thus the gravitational red-shift is maximum at the centre and monotonically decreasing.

Differentiating equation (10) w.r.t. \(x\), we get,

\[
\frac{d\left(E^2\right)}{dx} \bigg|_{x=0} = K \left(1 + 4x\right) \left(1 + 3x\right)^2
\tag{29}
\]

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The expression of right hand side of (29a) is positive, thus the electric intensity is minimum at the centre and monotonically increasing for all values of $K > 0$. Also at the centre it is zero.

5. Boundary Conditions

The solutions so obtained are to be matched over the boundary with Reissner-Nordstrom metric;

$$ds^2 = - \left(1 - \frac{2GM}{r} + \frac{e^2}{r^2}\right)dt^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right) + \left(1 - \frac{2GM}{r} + \frac{e^2}{r^2}\right)dr^2$$  \hspace{1cm} (30)

which requires the continuity of $e^x$, $e^y$ and $q$ across the boundary $r = r_b$

$$e^{-x(r_b)} = 1 - \frac{2GM}{c^2 r_b} + \frac{e^2}{r_b^2}$$  \hspace{1cm} (31)

$$e^{x(r_b)} = 1 - \frac{2GM}{c^2 r_b} + \frac{e^2}{r_b^2}$$  \hspace{1cm} (32)

$$q(r_b) = e$$  \hspace{1cm} (33)

$$p(r_b) = 0$$  \hspace{1cm} (34)

The condition (34) can be utilized to compute the values of arbitrary constants $A$ as follows:

On setting $x_{r = b} = X = c r_b^2$, ($r_b$ being the radius of the charged sphere)

$$\frac{8\pi G}{c^2} \rho_b r_b^2 = X \left[-\frac{K}{2} \left(\frac{26X^3 + 35X^2 + 16X + 3}{(1 + 3X)^5} - A(3 + 5X)^{\frac{4}{5}}\right) \right]$$  \hspace{1cm} (38)

Table 1. The variation of various physical parameters at the centre, surface density, electric field intensity on the boundary, mass and linear dimension of stars with different values of $K$ and $X$.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$X/c r_b^2$</th>
<th>$8\pi G/c^2 \rho_b$</th>
<th>$1/2\rho_b/c r_b^2$</th>
<th>$1/p_b/c r_b^2$</th>
<th>$1/c^2$</th>
<th>$(\partial \rho_b/\partial r_b)_r$</th>
<th>$z_0$</th>
<th>$\left(E^0/c r_b\right)_r$</th>
<th>$8\pi G/c^2 \rho_b r_b^2$</th>
<th>$M/M_\odot$</th>
<th>$2r_b$ in Km</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96</td>
<td>0.1</td>
<td>0.6607</td>
<td>9.9638</td>
<td>0.0679</td>
<td>0.2390</td>
<td>0.2889</td>
<td>0.0523</td>
<td>0.705</td>
<td>1.28</td>
<td>27.52</td>
<td></td>
</tr>
<tr>
<td>0.96</td>
<td>0.23</td>
<td>1.00720</td>
<td>8.9777</td>
<td>0.1122</td>
<td>0.2427</td>
<td>0.6694</td>
<td>0.1315</td>
<td>0.974</td>
<td>2.60</td>
<td>32.35</td>
<td></td>
</tr>
<tr>
<td>0.96</td>
<td>0.35</td>
<td>1.08886</td>
<td>8.73342</td>
<td>0.12467</td>
<td>0.24373</td>
<td>1.09374</td>
<td>0.213</td>
<td>0.993</td>
<td>3.43</td>
<td>32.66</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.5183</td>
<td>10.44</td>
<td>0.0496</td>
<td>0.1738</td>
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<td>0.1091</td>
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<td>10.16</td>
<td>0.0603</td>
<td>0.1732</td>
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</tr>
<tr>
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<td>0.3</td>
<td>0.559141</td>
<td>10.32258</td>
<td>0.05416</td>
<td>0.17357</td>
<td>1.03058</td>
<td>0.985</td>
<td>0.857</td>
<td>3.34</td>
<td>30.34</td>
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<tr>
<td>5.2</td>
<td>0.05</td>
<td>0.0954</td>
<td>11.71</td>
<td>0.0081</td>
<td>0.05926</td>
<td>0.1501</td>
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<tr>
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<td>7</td>
<td>0.01</td>
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<td>0.0080</td>
<td>0.01888</td>
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<td>0.112</td>
<td>0.07</td>
<td>9.04</td>
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Table 2. The Derivatives of various physical parameters at the centre with different values of $K$ and $X$.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$X$</th>
<th>$c_G r$</th>
<th>$\left(1 - \frac{8\pi G \rho}{c^2} \frac{d}{dr} \right)_{v=0}$</th>
<th>$\left(1 - \frac{8\pi G \rho}{c^2} \frac{d}{dr} \right)_{v=1}$</th>
<th>$\left(1 - \frac{d}{dr} \left( \frac{p}{\rho} \right) \right)_{v=0}$</th>
<th>$\left(1 - \frac{d}{dr} \left( \frac{p}{\rho} \right) \right)_{v=1}$</th>
<th>$\frac{d}{dr} \left( \frac{\rho}{\rho c^2} \right)$</th>
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<td>0.1</td>
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<td>5.2</td>
<td>0.1</td>
<td>4.14</td>
<td>-68.30</td>
<td>-0.3333</td>
<td>-1.048</td>
<td>-1.30</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. The march of pressure, density, pressure-density ratio and square of adiabatic sound speed within the ball corresponding to $K = 0.96$ with $X = 0.35$.

<table>
<thead>
<tr>
<th>$r/r_b$</th>
<th>$\frac{8\pi G}{c^2} \rho^2$</th>
<th>$\frac{8\pi G}{c^2} \rho c^2$</th>
<th>$\frac{p}{\rho c^2}$</th>
<th>$\frac{1}{c^2} \frac{d}{dr} \left( \frac{\rho}{\rho c^2} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.08886</td>
<td>3.056697</td>
<td>0.356221</td>
<td>0.243732</td>
</tr>
<tr>
<td>0.1</td>
<td>1.059853</td>
<td>3.01504</td>
<td>0.351522</td>
<td>0.243685</td>
</tr>
<tr>
<td>0.2</td>
<td>0.976679</td>
<td>2.895476</td>
<td>0.337312</td>
<td>0.243187</td>
</tr>
<tr>
<td>0.3</td>
<td>0.849993</td>
<td>2.712458</td>
<td>0.313366</td>
<td>0.24119</td>
</tr>
<tr>
<td>0.4</td>
<td>0.694975</td>
<td>2.485169</td>
<td>0.279649</td>
<td>0.236025</td>
</tr>
<tr>
<td>0.5</td>
<td>0.528536</td>
<td>2.232796</td>
<td>0.236715</td>
<td>0.225541</td>
</tr>
<tr>
<td>0.6</td>
<td>0.366874</td>
<td>1.971201</td>
<td>0.186117</td>
<td>0.207331</td>
</tr>
<tr>
<td>0.7</td>
<td>0.223863</td>
<td>1.711536</td>
<td>0.130796</td>
<td>0.179042</td>
</tr>
<tr>
<td>0.8</td>
<td>0.110333</td>
<td>1.460396</td>
<td>0.07555</td>
<td>0.138766</td>
</tr>
<tr>
<td>0.9</td>
<td>0.034033</td>
<td>1.220761</td>
<td>0.027878</td>
<td>0.085434</td>
</tr>
<tr>
<td>1.0</td>
<td>0.000000</td>
<td>0.993111</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Centre red shift is given by

$$z_0 = B^{-1/2} - 1$$

In view of and Table 3 We observe that pressure, density, pressure-density ratio and square of adiabatic sound speed decrease monotonically with the increase of radial coordinate.

We now present here a model of Neutron star based on the particular solution discussed above. The Neutron star is supposed to have a surface density, $\rho_b = 2 \times 10^{14}$ g/cm$^3$. The resulting well behaved model has the mass $M = 3.43$ $M_\odot$ and the linear dimension, $2 r_b \approx 32.66$ km. The surface red shift $Z_b \approx 0.5509$.

6. Discussion

In view of and Table 1, it has been observed that all the physical parameters ($p$, $\rho$, $\frac{p}{\rho c^2}$, $\frac{dp}{d\rho}$, and $z$) are positive at the centre and within the limit of realistic equation of state. From Table 2, the first derivative of all the parameters are negative at the centre. Thus by virtue of the
In view of (29a), the electric intensity is minimum at the centre and monotonically increasing. for all values of $K > 0$. Therefore, the solution is well behaved for $(0.96 \leq K \leq 5.2)$, however, corresponding to any value of $K$ satisfying the inequalities $0 \leq K < 0.96$ the nature of adiabatic sound speed is non decreasing in nature.

Corresponding to any value of $K > 7$ there exist no value of $X$ for which centre pressure is positive. However, for $(7 > K > 5.2)$, the length of interval for $X$ converges to zero and the value of $X$ is also approaching to zero as $K$ approaches to $7$.

It has been observed that for higher values of $K$ approaching to $7$, though the solution is well behaved but the mass of the star is very less than the Chandrasekhar limit.