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Bank Portfolio Management under Credit Market Imperfections

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Abstract

This paper examines bank portfolio management under banking regulation and asymmetric information about borrower types and screening by banks and imperfect competition in the credit market. A bank tries to maximize expected profit subject to a portfolio variance constraint. The analysis yields the following results: For a monopoly bank, the incentive constraint of the efficient type of borrowers will be binding and the participation constraint of the inefficient type of borrowers will be binding. Further, given the variance constraint being binding, the optimal portfolio will be on the efficiency frontier. The paper also examines duopoly competition between aggressive (predator) and defensive (prey) banks and the scope for potential cooperation and reveals that among the alternatives of natural monopoly, entry deterrence, takeovers and efficient portfolio diversification through mergers or interest swaps, the cooperative efficient portfolio diversification strategy will dominate any non-cooperative strategy whenever portfolio returns are negatively correlated between any pair of interacting banks as it reduces portfolio variance for a given package of interest and loans.

Keywords

Bank, Portfolio Management, Credit Market Imperfections

1. Introduction

This paper primarily addresses the key issues in designing and managing bank portfolio management under credit market imperfections given all the potential problems and given the constraints imposed by prudential regulation. Two sources of credit market imperfections are addressed. First, the risk-return structure of the borrowers are known only to the borrowers and the bank has to use screening technology in order to reveal the borrower types and allocate selective loan-interest packages with respect to each type of borrowers. I assume that banks are risk averse financial intermediaries and therefore their lending to high return high risk borrower types is constrained by the self imposed preference on maximum variance that bank portfolios can tolerate. Second, banks face limited competition in the domain of an incumbent bank constraining the scope for competitive undercutting of interest rates and financial innovation. Third, two banks engaging in potential competition may have negatively correlated returns on their respective portfolios which may allow cooperation in the form of risk sharing using credit swaps or a merger. This relaxes the constraints on portfolio variances of the two banks that are deliberating over the choice of competition versus cooperation. The paper shows that under certain conditions, cooperation dominates competition in banking arrangements through risk sharing arrangements like mergers and interest swaps that ensure variance preserving higher expected portfolio returns.

This paper is unique in integrating bank screening mechanisms under adverse selection problems with bank portfolio management in a mean-variance framework. Thus it contributes to bank portfolio management theory. The paper also develops the industrial organization theory of banking by highlighting the tradeoffs between traditional interest rate competition and cooperative financial innovations like swaps and mergers. The paper does not deal with moral hazard, liquidity risks and banking crises. However, it makes possible the extensions along those lines by creating a basic platform which can be improved upon by contract theory innovations and by using inter-temporal tradeoffs in macroeconomics.

The paper is organized as follows. The next section does a literature review. The following section describes and analyzes the basic model. The basic framework and assumptions are first described. Next the optimal program of the monopoly bank is described and analyzed. After that, duopoly banking competition and possible cooperative solution through portfolio diversification and financial innovation are analyzed. This section is followed by a section which discusses possible extensions. The last section concludes.

2. Literature Review

Banks play a fundamental role in the allocation of resources in the economy. They provide payments services ([1], [2] and [3]), liquidity insurance [4], risk management ([3] and [5]), term transformation [6] and efficient allocation of risk bearing ([3] and [4]) which other financial institutions and financial markets cannot replicate effectively. However, the essential problem in banking operations and strategy throughout the world is that banks face acute portfolio management problems in the face of known and unknown asset and liability side risks ([1], [2], [3] and [4]). These problems have elicited different regulatory responses. Today, prudential regulation has thus taken the form of capital adequacy norms ([7] and [8]), risk regulation ([1], [2] and [3]), liquidity requirements [9], efficient bank corporate governance [10] and the maintenance and innovations in the existing structure of deposit insurance ([1] and [2]).

Both formal and informal credit market are characterized by rationing and market segmentation though credit constraints are relaxed during the booms and to some extent, market integration occurs over time through takeovers and mergers ([11], [12] and [13]) and financial innovations like swaps ([14] and [15]). This paper provides a rationale for these phenomena. The efficiency oriented rationale for credit rationing is highlighted in bank portfolio design and management. The nature of credit market imperfections lead to the following: inefficient borrowers exert negative externality on banks and thus are constrained by participation constraints while efficient borrowers get a rent through incentive constraints being binding for them.

Next, I discuss banking policy and prudential regulation. Without adequate regulation, there is too much risky lending and too little liquid reserves and thus risk weighted capital constraints and liquidity requirements are imposed on bank lending. The nature of risk stems essentially from the fact that high risk and high return borrowers may default with a certain probability and the corresponding nature of the problem is shifted to bank regulators and taxpayers who may have to bailout insolvent banks. To avoid these problems, a regulated bank imposes credit constraints and interest rate restrictions on different types of borrowers and deal with the problem by applying incentive constraints and participation constraints Prudential regulation however, does not solve the problem of credit market imperfections in its entirety and complementary market innovations like diversification through financial innovations like credit default swaps and financial restructuring through mergers have to be brought into play.

I characterize prudential regulation in its salient aspects with due references as to institutions and timing. While Basel I [7] and Basel II [8] focused on setting standards for risk weighted capital adequacy ratios and standards of disclosure by the banks coming under their purview, Basel III [9] focused on measuring and minimizing liquidity risks across banks globally ranging from illiquidity to high leverage to runs on banks during a financial crisis. It is instructive to compare the days of self-regulation and old fashioned crises with the modern complex days of prudential regulation under competitive pressures and adverse selection, moral hazard and transaction costs. This paper indicates some results from these reflections and leaves more room for future investigations.

Portfolio analysis has been quite comprehensively analyzed by many authors ([16], [17], [18] and [19]) for the decision problem of individuals and institutions of allocating wealth in different assets under conditions of risk and return and given the structure of the covariance of different available capital assets. Portfolio analysis of depository financial intermediaries has been conducted by some authors in the same spirit where liabilities have been denoted as negative assets and the standard restrictions on assets and liabilities have been incorporated to obtain a separation theorem and intuitive comparative statics results [3].

However, portfolio theory has not analyzed in the context of adverse selection problem and imperfect competition in credit markets. On the other hand there exists a well developed literature which deals with the adverse selection problem in credit markets and imperfect competition in credit markets ([3], [20]) but do not deal with mean-variance analysis in the design of bank portfolios under adverse selection. The present paper integrates the mean-variance analysis of the bank portfolio with the contractual screening mechanisms under the adverse selection in credit markets and imperfect competition in credit markets. Different complexities and tradeoffs emerge in such an analysis.

3. The Model

3.1. The Framework

Assumption I. Firms

There are N_f firms which want to borrow from a bank. Only α fraction of borrower firms are efficient but risky with expected return $pR_{ef}(L_1)$ from a loan size L_1 while (1- α) fraction of borrower firms are inefficient but safe with expected return $R_{if}(L_2)$ from a loan size L_2. In other words ($N_f\alpha$) number of borrower firms are efficient but risky while (1- α)N_f number of borrower firms are inefficient but safe. Each borrower knows its type but the bank does not know the borrower type without screening.

It is assumed that α is small enough since highly efficient firms are hard to obtain.

The following standard assumptions are made:

$$p\left(\mathrm{d}R_{ef}\left(L_1\right)/\mathrm{d}L_1\right) = p\left(R_{ef}\right) > 0 \quad (1)$$

(Constant expected returns to scale for efficient borrowers)

$$p\left(\mathrm{d}^2 R_{ef}\left(L_1\right)/\mathrm{d}L_1^2\right) = 0 \quad (2)$$

(Linear returns and no diminishing returns for efficient borrowers)

$$\left(R_{if}\left(L_2\right)\right) > \left(R_{sf}\right) > 0 \quad (3)$$

(There is a minimum Return for the inefficient borrowers)

$$\left(\mathrm{d}R_{if}\left(L_2\right)/\mathrm{d}L_2\right) > 0 \text{ and } \left(\mathrm{d}^2 R_{if}\left(L_2\right)/\mathrm{d}L_2^2\right) < 0 \quad (4)$$

(Return for any inefficient borrower is an increasing and concave function of the loan input or there is diminishing returns for inefficient borrowers)

These assumptions are realistic and entail very little loss of generality in this type of analysis as will be seen as we proceed.

Risky type: demand within an industry is distributed such that with probability p each of the efficient but risky firms earn revenue (R_{ef}) (L_i) for each loan amount L_i but with probability 1-p each risky firm earns zero revenue for each loan amount.

Safe type: return lower than the expected return from the efficient type for each loan amount. However, the return on the safe type is certain (as opposed to the efficient but risky type). One reason is that the safe type has enough collateral whereas the risky type does not have sufficient collateral).

Assumption. Relative Efficiency or the Single Crossing Condition

$$p\left[\mathrm{d}R_{ef}(L)/\mathrm{d}L\right] = p\left(R_{ef}\right) > \left(\mathrm{d}R_{if}(L)/\mathrm{d}L\right) \tag{5}$$

So if $L_1 > L_2$, then

$$p\left[R_{ef}(L_1) - R_{ef}(L_2)\right] > R_{if}(L_1) - R_{if}(L_2) \tag{6}$$

Assumption. Absolute Efficiency

For all L,

$$p\left[R_{ef}(L)\right] > R_{if}(L) \tag{7}$$

Collateral: For the inefficient but safe type, for each loan amount L_2, there is adequate collateral C_{if} to cover the gross interest payment R_2L_2,

$$C_{if} > R_2L_2 \tag{8}$$

but for the efficient but risky type, for each loan amount L_1, there is no collateral so that

$$C_{ef} = 0 \tag{9}$$

This assumption reflects the fact that safe types usually invest in standardized capital goods and inventories which are valuable outside the business but the risky types invest in specific capital goods and inventories which have no value outside the business. This assumption approximates the reality quite well ([21], [22]).

Assumption II. Bank Finance

This assumption discusses the contracts with respect to loans and gross interest rates.

The bank raises finance from the depositor at a fixed rate of interest. A bank allocates a loan and gross interest package (L_1, R_1) for each efficient (but risky) borrower it finances. A bank allocates a loan and gross interest package (L_2, R_2) for each inefficient (but safe) borrower it finances. Banking contracts to borrowers are designed so that for each type of borrower, the participation constraint of the borrower is satisfied so that it obtains a non-negative expected profit (assuming no outside opportunity for the borrower) from the contract (individual rationality) and incentive constraint of the borrower of a given type is satisfied such that it selects the contract meant for the borrower type (self selection).

Assumption III. Bank Regulation

If a bank allocates L_1 for the efficient (but risky) borrower then it has to pay the gross interest cost R_dL_1 and has to raise risk weighted capital C_kL_1 and has to maintain sufficient liquidity at the cost $C_{LA}L_1$. A bank allocating L_2 for the inefficient (but safe) borrower at the gross interest cost R_dL_2 does not have to raise risk weighted capital but has to maintain sufficient liquidity at the cost $C_{LA}L_2$.

3.2. The Problem of the Monopoly Bank

A bank maximizes expected profit subject to the participation constraints and incentive constraints of the borrower firms and subject to a self imposed maximum variance of its portfolio (the banker is risk averse).

P is the bank portfolio and $E[\pi_b(P)]$ is the expected profit from the portfolio.

The self imposed maximum variance is $Var(P^*)$ set by the bank.

The optimization program of the bank is:

Max

$$E(\pi_b(P)) = N_f\left[\left\{\alpha\left(R_1L_1 - R_dL_1 - C_kL_1 - C_{LA}L_1\right)\right\} + \left\{(1-\alpha)\left(R_2L_2 - R_dL_2 - C_{LA}L_2\right)\right\}\right] \tag{10}$$

s.t

$$Var\left(P^*\right) \geq Var(P) = Var\left(P(L_1)\right) \tag{11}$$

which states that there is a maximum value that portfolio variance can take. w.r.t. $\{R_1, L_1, R_2, L_2, \lambda\}$ where λ is the shadow price of the variance of the portfolio such that the following conditions have to be met (given asymmetric information, the conditions are standard) which are the participation and incentive constraints which need no special explanation:

$$pR_{ef}(L_1) - R_1L_1 \geq 0 \tag{12}$$

$$pR_{ef}(L_1) - R_1L_1 \geq pR_{ef}(L_2) - R_2L_2 \tag{13}$$

$$R_{if}(L_2) - R_2L_2 \geq 0 \tag{14}$$

$$R_{if}(L_2) - R_2L_2 \geq R_{if}(L_1) - R_1L_1 \tag{15}$$

Proposition 1. The incentive constraint of the efficient type of borrower will be binding and the participation constraint of the inefficient Type of Borrower will be binding.

Now, rearrange the two incentive or self-selection constraints. We get $p\left[R_{ef}(L_1) - R_{ef}(L_2)\right] \geq R_1L_1 - R_2L_2 \geq R_{if}(L_1) - R_{if}(L_2)$. Given the relative efficiency or the single crossing condition we get $L_1 \geq L_2$.

Case 1. $L_1 > L_2$

Compare the participation and incentive constraints of the efficient type:

$$pR_{ef}(L_1) \geq R_1L_1$$

and

$$pR_{Ef}(L_1) \geq R_1L_1 + \left(pR_{ef}(L_2) - R_2L_2\right) \geq R_1L_1 + \left(pR_{ef}(L_2) - R_{if}(L_2)\right)$$

where the last term on RHS in the bracket is non-negative given the assumption on absolute efficiency.

Lemma. The incentive constraint of the efficient type will be binding if $L_1 > L_2$.

Proof. Now, a bank will try to increase revenue from each sub-segment. Thus it will try to maximize R_1L_1 subject to the above constraints. It follows that at least one of the constraints must be binding. Given the discussion above we see that the incentive constraint of the efficient type binds and

$pR_{ef}(L_1) = R_1L_1 + \left(pR_{ef}(L_2) - R_2L_2\right)$. **Q.E.D.**

Now similarly compare the participation and incentive constraints of the inefficient type: $R_{if}(L_2) \geq R_2 L_2$ and $R_{if}(L_2) \geq R_2 L_2 + (R_{if}(L_1) - R_1 L_1)$ Now, a bank will try to increase revenue from each sub-segment. Thus it will try to maximize $R_2 L_2$ subject to the above constraints. It follows that at least one of the constraints must be binding.

Lemma. The participation constraint of the inefficient type will be binding if $L_1 > L_2$

Proof: Suppose not. Then $R_{if}(L_2) > R_2 L_2$. Then the incentive constraint must bind. So, $R_{if}(L_2) - R_2 L_2 = (R_{if}(L_1) - R_1 L_1)$ or $R_{if}(L_2) - (R_{if}(L_1) - R_1 L_1) = R_2 L_2$ Now from the incentive constraint of the efficient type, we get $pR_{ef}(L_1) = R_1 L_1 + (pR_{ef}(L_2) - R_2 L_2)$ or $pR_{ef}(L_1) - pR_{ef}(L_2) = R_1 L_1 - R_2 L_2 = R_{if}(L_1) - R_{if}(L_2) > 0$ since $L_1 > L_2$ So $R_1 L_1 = pR_{ef}(L_1) - pR_{ef}(L_2) + R_2 L_2$ which indicates that the relation $R_{if}(L_2) - (R_{if}(L_1) - R_1 L_1) = R_2 L_2$ or $(R_{if}(L_2) - R_{if}(L_1)) + R_1 L_1 = R_2 L_2$ must satisfy $(R_{if}(L_2) - R_{if}(L_1)) + (pR_{ef}(L_1) - pR_{ef}(L_2) + R_2 L_2) = R_2 L_2$ which implies $(R_{If}(L_2) - R_{If}(L_1)) = pR_{Ef}(L_2) - pR_{Ef}(L_1)$ which is a contradiction given the single crossing property or relative efficiency. So the participation constraint of the inefficient type of the borrower will bind. **Q.E.D.**

Case 2. $L_2 = L_1$

Lemma. The incentive constraint of the efficient type of borrowers and the participation constraint of inefficient type of borrowers will be binding if $L_2 = L_1$.

Proof: The incentive constraint of the efficient type of the borrower is satisfied if $R_1 = R_2$. Assume that the participation constraint of the inefficient type of borrower is slack, then if the bank charges $(R_2 + \varepsilon)$ then the participation constraint: $R_{if}(L_2) - (R_2 + \varepsilon)L_2 \geq 0$ can also be satisfied and the bank can increase the expected profit. Then the incentive constraint of the inefficient type of the borrower must bind which means $R_{if}(L_2) - (R_2)L_2 = R_{if}(L_1) - (R_1)L_1$. Thus $R_1 = R_2$. Therefore we must have the participation constraint binding. **Q.E.D.**

Given the above conditions on the set ($\{R_1, L_1, R_2, L_2\}$) the expected profit from the bank portfolio can be written as

$$\begin{aligned} &E(\pi_b(P)) \\ &= N_f\left[\alpha\left\{pR_{ef}(L_1) + R_{If}(L_2) - pR_{ef}(L_2) - R_d L_1 - C_k L_1 - C_{LA} L_1\right\}\right. \\ &\quad \left. + (1-\alpha)\left\{R_{if}(L_2) - R_d L_2 - C_{LA} L_2\right\}\right] \end{aligned} \tag{16}$$

where variance of the portfolio is:

$$var(P(L_1)) = p(1-p)\left[(N_f \alpha) R_{ef}(L_1)\right]^2 \tag{17}$$

and

$$\left(\mathrm{d}var(P(L_1))/\mathrm{d}L_1\right) = \left\{N_f p(1-p)\alpha 2 R_{ef}(L_1)\right\}\left(R_{ef}\right) > 0 \tag{18}$$

$$\mathrm{d}^2 var(P(L_1))/\mathrm{d}L_1^2 > 0 \tag{19}$$

The Lagrangian to be maximized is:

$$L = E\left(\pi_b\left(P\right)\right) + \lambda\left(varP^* - varP\left(L_1\right)\right) \tag{20}$$

Assumption. The shadow price of the variance constraint is positive or $\lambda > 0$.

This assumption implies that there is a potential diversification motive for each monopoly bank. This allows a room for financial innovation in the interaction between banks as we shall see in the next section. Financial innovation to achieve better diversification is the cornerstone of finance and this assumption allows for it in the context of interbank interaction and enriches the banking literature.

The following first order conditions are obtained

The first order condition with respect to λ:

$$var\left(P^*\right) - \mathrm{var}\left(P\left(L_1\right)\right) = 0 \tag{21}$$

From this first order condition, the optimal value of $L_1^* > 0$ is obtained. Thus, $p\left(R_{ef}\right) - R_d - C_k - C_{LA} > 0$. In other words, the marginal value of loan to the efficient type is positive for each value of loan to the efficient type or equivalently, the expected interest return from a unit loan to the efficient type of the borrower for the bank is greater than the sum of the interest cost on deposits plus the unit cost of risk-weighted capital gross interest plus cost the unit cost of having access to liquidity.

The first order condition with respect to L_2 is

$$R_{if}\left(L_2\right) - R_d - C_{LA} = 0 \tag{22}$$

This requires no explanation.

Proposition 2. The Optimum Portfolio is on the Efficiency Frontier.

Proof: Suppose not. Then, given the same variance, there is a feasible mean or expected bank profit which is higher. But this is a contradiction by construction. **Q.E.D.**

3.3. Equilibrium Portfolio Allocation under Duopoly

The Structure of the Banking Network. There is a loose network or a cluster of banks with each bank being represented as a node in the cluster. Each sub-network consists of 2^n banks where n is a positive integer. Half of the banks are of type A while the rest are of type D. A bank of type A is always matched with a bank of type D. The order of the linear distances between the matched pairs of A and D types in any sub-network is such that the distances between adjoining pairs keep increasing so that there is no possibility of triangular or any other asymmetric matching. This means that each matched pair can be treated in isolation. The deposit market, equity market and market for short term securities (liquid assets) are integrated throughout the network. However, the credit market is segmented with each type t of bank $B(t)$. Some banks in the cluster or the network are predators in the credit market segments who try to snatch borrowers from other banks while holding onto their own segment of borrowers. They are denoted as aggressive banks such that $t = A$. The rest of the banks try to hold onto the borrowers in their respective credit segments. They are denoted as defensive banks such that $t = D$. For bank $B(D)$, at most one competitor $B(A)$ can enter and compete in terms of contracts defined over interest rates and loan sizes provided that for each aggressive bank, the profit from entry is strictly positive. The competitor $B(A)$ which can enter the segment of $B(D)$ has the least distance from $B(D)$ which is denoted by $\min\left\{d\left[B(D),B(A)\right]\right\}>0$. The fixed cost of entry is F. I assume that the fixed cost of entry has to be financed from internal funds I. To keep the analysis simple, I assume that $F > 0$ and $2F > I > F$. This implies that if entry at all occurs by the aggressive bank, it will choose only one defensive bank, that which is closest to it. I also assume symmetry in bank optimization problems except for the fact that in order to enter the market of another bank, the aggressive bank must pay a fixed cost F while the incumbent defensive bank does not have to pay any fixed cost at all and only bear standard costs.

Suppose that fraction a is owned in market of bank I with portfolio return X and fraction b is owned in market of bank E with portfolio return Y. We apply the formula

$$Var\left(P\left(aX + bY\right)\right) = a^2 Var\left(X\right) + b^2 Var\left(Y\right) + 2abcov\left(X,Y\right) \tag{23}$$

to find the variance of the portfolio of a bank which is active in both markets. Using symmetry

$$Var\left(P\left(aX+bY\right)\right) = 2a^2 Var\left(X\right) + 2abcov\left(X,Y\right) \tag{24}$$

Under complete entry $a = 1$ and $b = 1$. Under risk sharing without entry $a = 1/2$ and $b = 1/2$.

We thus restrict the analysis to two competing banks $B(D)$ and $B(A)$ without any loss of generality. It is assumed that each bank has identical situation in terms of assets and liabilities and risks and returns. This assumption allows one to examine the essential nature of duopoly competition and cooperation and this assumption can be relaxed to allow for a richer set of pricing strategies, institutional arrangements and risk sharing arrangements. Given the above assumptions the analysis can be extended to the entire cluster or the network of banks with different credit market segments. Note that if the shadow price of the variance is zero then the aggressive bank will not try to get out of its boundaries and similarly the defensive bank will not try to enter into any risk reduction strategy. The equilibrium concept is as follows: given any pair of separating contracts (pooling contracts are ruled out by standard analysis) and financial innovation by the aggressive (defensive) bank, the pair of separating contracts and financial innovation of the defensive (aggressive) bank is optimal.

Recall that the Optimal Separating Contract of the Monopoly Bank is the contract set (R_1, L_1, R_2, L_2) governed by the optimization relations as given before.

Case 1. The Incumbent Bank is a Natural Monopoly (IBNM)

Consider the OSC or the Optimal Separating Contract offered by $B(D)$ characterized by the following conditions: The incumbent bank offers the following interest and loan packages: The gross interest charged to the efficient type of borrowers by the incumbent natural monopolist bank is $R(\text{ef, IBNM}) = (R_1)$. The loan amount offered to the efficient borrower type by the incumbent natural monopolist bank is $L(\text{ef, IBNM}) = (L_1)$. The gross interest charged to the inefficient type of borrowers by the incumbent natural monopolist bank is $R(\text{if, IBNM}) = (R_2)$. The loan amount offered to the inefficient type of borrowers by the incumbent natural monopolist bank is $L(\text{if, IBNM}) = (L_2)$.

$R_1L_1 < pR_{ef}(L_1)$ or the efficient type makes positive profit $R_2L_2 = R_{if}(L_2)$ or the inefficient type makes zero profit. Now consider a contract $(R_1 - \varepsilon, L_1)$ offered by $B(A)$ where $\varepsilon > 0$ is a small positive number. The efficient type will accept this contract. Now consider a contract $(R_2 - \varepsilon, L_2)$ offered by $B(A)$ where $\varepsilon > 0$ is a small positive number. The inefficient type will accept this contract. It is optimal to undercut for both types if it is at all optimal to enter the market segment of $B(D)$.

Thus we have the following proposition.

Proposition 3. IBNM is a possible equilibrium if all of the following conditions are satisfied for any $\varepsilon > 0$

$$N_f\Big[\big\{\alpha\left(R_1L_1 - R_dL_1 - C_kL_1 - C_{LA}L_1\right)\big\} + \big\{(1-\alpha)\left(R_2L_2 - R_dL_2 - C_{LA}L_2\right)\big\}\Big] > 0 \tag{25}$$

$$N_f\Big[\alpha\big\{(R_1-\varepsilon)L_1 - R_dL_1 - C_k(L_1) - C_{LA}(L_1)\big\} + (1-\alpha)\big\{(R_2-\varepsilon)(L_2) - R_dL_2 - C_{LA}(L_2)\big\}\Big] \le F \tag{26}$$

The possibility of equilibrium is contingent on the incumbent (defensive) bank earning a positive expected profit from not undercutting the monopoly interest rates and the potential entrant (aggressive) bank finding it impossible to earn a positive expected profit from undercutting interest rates (no matter how small the undercutting). **Q.E.D.**

The above inequalities ensure the possibility that there is natural monopoly with the optimal separating contract offered by the monopoly bank. However, if the cost of operating in both markets was less than the expected revenue obtained by the aggressive bank, then we would have to consider the possibility of entry deterrence as a possible equilibrium.

Case 2. Entry Deterrence (ED)

The incumbent offers the following interest and loan packages:

$$R(\text{ED1}) = (R_1 - \varepsilon),\ L(\text{ED1}) = (L_1) \text{ and } R(\text{ED2}) = (R_2 - \varepsilon),\ L(\text{ED2}) = (L_2)$$

Proposition 4. Entry Deterrence is a possible equilibrium if all of the following conditions are satisfied for any $\varepsilon > 0$

$$N_f\Big[\alpha\big\{(R_1-\varepsilon)(L_1) - R_dL_1 - C_k(L_1) - C_{LA}(L_1)\big\} + (1-\alpha)\big\{(R_2-\varepsilon)(L_2) - R_dL_2 - C_{LA}L_2\big\}\Big] > 0 \tag{27}$$

$$N_f\Big[\alpha\big\{(R_1-\varepsilon)(L_1) - R_dL_1 - C_k(L_1) - C_{LA}(L_1)\big\} + (1-\alpha)\big\{(R_2-\varepsilon)(L_2) - R_dL_2 - C_{LA}L_2\big\}\Big] - F \le 0 \tag{28}$$

The above inequalities ensure that the incumbent (defensive) bank can limit price the potential entrant (aggressive) bank by cutting interest rates sufficiently while making positive expected profits. Since the entrant has to pay the fixed cost of entry, it cannot earn positive expected profit if the interest rates set by the incumbent are low enough.

Case 3. Takeover (T)

The entrant offers the following interest and loan packages: $R(\mathrm{T1}) = (R_1 - \varepsilon)$, $L(\mathrm{T1}) = (L_1)$ and $R(\mathrm{T2}) = (R_2 - \varepsilon)$, $L(\mathrm{T2}) = (L_2)$

Proposition 5. Takeover is a possible equilibrium if all of the following conditions are satisfied for the potential entrant for any $\varepsilon > 0$ and $F < 0$.

$$N_f\left[\alpha\left\{(R_1-\varepsilon)(L_1) - R_d L_1 - C_k(L_1) - C_{LA}(L_1)\right\} + (1-\alpha)\left\{(R_2-\varepsilon)L_2 - R_d L_2 - C_{LA}(L_2)\right\}\right] > F \tag{29}$$

$$N_f\left[\alpha\left\{(R_1-\varepsilon)(L_1) - R_d L_1 - C_k(L_1) - C_{LA}(L_1)\right\} + (1-\alpha)\left\{(R_2-\varepsilon)L_2 - R_d L_2 - C_{LA}(L_2)\right\}\right] \le 0 \tag{30}$$

$$2Var(B(A)) + 2cov(B(D), B(A)) \le Var(P^*) \tag{31}$$

The first two inequalities state that if the entrant sufficiently cuts interest rates such that the incumbent is unable to earn positive expected profit while the entrant can make positive expected profit with F being sufficiently negative, then takeover can occur. The third inequality ensures that the maximum variance constraint is not exceeded. **Q.E.D.**

Thus, a necessary and sufficient condition for takeover to be an equilibrium is that $F < 0$ and F be a sufficiently high negative number and the maximum variance condition must be exceeded. The interpretation is that for takeover to be equilibrium condition, there must be economies of scope for $B(A)$ in the domain of $B(D)$. However, given the assumption that $F > 0$ in the present paper, takeover is ruled out as an equilibrium condition.

Case 4. FIIBD (Financial Innovation and Inter-Bank Diversification)

However, if the covariance of portfolio returns is negative, then there exists a better allocation than natural monopoly or entry deterrence in the sense that there exists a better strategy combination that is incentive compatible for the banks. This can be implemented by the contract of interest swaps or merger which is discussed below. In this case there is no entry in the market of the incumbent. However, the two banks maximize in their respective regions and agree to swap half of the portfolio returns. Since the covariance between two interest streams is negative, a welfare improvement is obtained with respect to mean or expected returns of banks and the allocation avoids fixed cost investments. Note that, interest swaps can take the form of default swaps or asset portfolio diversification through mergers.

Proposition 6. FIIBD is a possible equilibrium if the following conditions hold:

$$\begin{aligned}
&(1/2) N_f \left[\alpha \left\{ (R_1) L_1 - R_d L_1 - C_k (L_1) - C_{LA} (L_1) \right\} \right. \\
&\left. +(1-\alpha) \left\{ L_2 R_2 - R_d L_2 - C_{LA} (L_2) \right\} \right] \\
&+(1/2) N_f \left[\alpha \left\{ (R_1) L_1 - R_d L_1 - C_k (L_1) - C_{LA} (L_1) \right\} \right. \\
&\left. +(1-\alpha) \left\{ L_2 R_2 - R_d L_2 - C_{LA} (L_2) \right\} \right] \\
&= N_f \left[\alpha \left\{ (R_1) L_1 - R_d L_1 - C_k (L_1) - C_{LA} (L_1) \right\} \right. \\
&\left. +(1-\alpha) \left\{ L_2 R_2 - R_d L_2 - C_{LA} (L_2) \right\} \right] > 0
\end{aligned} \tag{32}$$

$$(1/2) Var\left(P^*\right) + (1/2) cov\left(B(A), B(D)\right) \le Var\left(P^*\right) \tag{33}$$

Proof: Inequality (33) is always satisfied if $cov\left(B(A), B(D)\right) < 0$ since $Var\left(P^*\right) > 0 > cov\left(B(A), B(D)\right)$. The above inequalities ensure positive expected profit for each bank participating in the diversification strategy discussed above without violating the variance constraint. **Q.E.D.**

Note that the strategy combination is cooperative in nature which does not dissipate expected profit through limit pricing or entry through incurring fixed cost. Further, when covariance of returns between the banks is negative, the cooperation strategy combination is not only feasible but also dominates any other feasible strategy non-cooperative combinations.

Proposition 7. Whenever $Cov\left(B(A), B(D)\right) < 0$, FIIBD dominates the IBNM and ED

Proof: First we compare IBNM with FIIBD. With the separating contract of the monopoly bank, the variance constraint is binding. With diversification, the variance constraint with the above separating contract is slack for any of the two banks. Thus each bank will increase the investment in risky loans until it finds its variance constraint binding. Thus,

$$E\left(\pi_b\left[P(\text{IBNM})\right]\right) < E\left(\pi_b\left[P(\text{FIIBD})\right]\right) \tag{34}$$

$$Var\left(P^*\right) = V(\text{IBNM}) = Var(\text{FIIBD}) \tag{35}$$

Next we compare ED with FIIBD. Entry deterrence (ED) involves some interest concessions which FIIBD does not.

$$E\left(\pi_b\left[P(\text{ED})\right]\right) < E\left(\pi_b\left[P(\text{FIIBD})\right]\right) \tag{36}$$

Q.E.D

4. Possible Extensions

While this paper has characterized optimal risk sharing through diversification arrangements under interest swaps, the credit market structure has been assumed to be extremely fragmented so as to rule out other mechanisms for risk sharing and value addition. A more connected credit network could accommodate the role of bank mergers in light of economies of scale and scope due to technological advances, liquidity reasons and regulatory flexibility and such mergers or financial intermediary coalition formations would also make the best of both worlds of specialization and diversification.

However, the social optimality of bank mergers is ambiguous due to monopoly effect running counter to the economies of scale and scope effect and reduced competition effect induced by mergers favoring financial stability while financial fragility is increased due to the possible failure of large banks which can have a large effect on the financial system. This structure of tradeoffs remains a direction for future research.

Another possible extension is to allow liquidity insurance by banks and the possibility of liquidity runs. This really renders meaningful the liquidity cost imposed by the Basel Accords. It should be borne in mind that under liquidity insurance services provided by banks, deposit insurance and lender of last resort have to come to the fore and together with liquidity requirements, provide the essential safety net. However, the analysis gets a lot complicated and tight results would be hard to find. Nonetheless, it is extremely important to extend this line of analysis and integrate with the asymmetric information structure and moral hazard issues.

A last suggestion is to evaluate dynamic portfolio management with learning about borrower types and changing scope and scale of moral hazard under changing macroeconomic and financial market conditions. Dynamic portfolio management involves reshuffling credit between borrowers, between industry sectors and inter-temporally with reserves and high degree of credit rationing under recessions and credit easing under booms.

It goes without saying that a lot depends on how one redefines the structure of the banking network and the regulation costs and incentives. Prudential handling of credit booms and predator-prey cycles play a very important role. The financial innovations and technological innovations of the banking industry together with the macro-economic and regulatory concerns can lead to complex evolutionary forces.

Having suggested that these extensions can enrich our baseline model of bank portfolio management under credit market imperfections we do think that the baseline model is expected to be robust to the changes and innovations.

5. Conclusions

This paper examines bank portfolio management under banking regulation and asymmetric information about borrower types and screening by banks and imperfect competition in the credit market. A bank tries to maximize expected profit subject to a portfolio variance constraint. The analysis yields the following results: for a monopoly bank, the incentive constraint of the efficient type of borrowers will be binding and the participation constraint of the inefficient type of borrowers will be binding. Further, given the variance constraint being binding, the optimal portfolio will be on the efficiency frontier. The paper also examines duopoly competition between aggressive (predator) and defensive (prey) banks and the possibility of potential cooperation through diversification motivated by higher expected profitability. The analysis reveals that among the alternatives of natural monopoly, entry deterrence, takeovers and efficient portfolio diversification through mergers or interest swaps, the cooperative efficient portfolio diversification strategy will dominate whenever portfolio returns are negatively correlated between any pair of interacting banks as it reduces portfolio variance for a given package of interest and loans. It should be again emphasized that improving portfolio diversification and higher expected portfolio returns can take place either on the basis of credit default swaps or through risk sharing arrangements under mergers.

The extensions suggested allowing robustness of the baseline model in terms of the basic intuitions and qualitative results but do allow a richer platform to be constructed.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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