The Photometric Maximum in Cosmicflows-2

Lorenzo Zaninetti

Physics Department, via P. Giuria 1, Turin, Italy
Email: zaninetti@ph.unito.it

Abstract

Based on well known photometric rules for the number of galaxies as function of the distance in Mpc we model the so called “Great Wall” which is visible on the Cosmicflows-2 catalog. The gravitational field is evaluated at the light of the shell theorem and a finite value for the gravitational field is numerically derived.

Keywords

Cosmology, Observational Cosmology, Distances, Redshifts, Radial Velocities, Spatial Distribution of Galaxies, Magnitudes and Colors, Luminosities

1. Introduction

Before to start we briefly review the Hubble law, after [1], which is a linear relationship between expansion velocity and distance in

\[ V = H_0 D = cz, \]  (1)

where \( H_0 \) is the Hubble constant \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\), with \( h = 1 \) when \( h \) is not specified, \( D \) is the distance in Mpc, \( c \) is the light velocity and \( z \) is the redshift. As an example a recent evaluation, see [2], quotes

\[ H_0 = (74.4 \pm 3) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}. \]  (2)

The original Hubble’s law was based on \( \approx 20 \) galaxies of which was known the distance and the velocity; conversely in the last years the number of catalogs for galaxies with redshift available for public downloading has progressively grown. This huge amount of data allows setting up critical tests between different theoretical models over the various aspects of the Large Scale Structures. The layout of the paper is as follows. In Section 2, we describe some representative catalogs of galaxies. In Section 3, we introduce the adopted luminosity function for galaxies. In Section 4, we model the maximum in the number of galaxies as func-
tion of the distance. In Section 5, we evaluate the gravitational field as produced by the visible galaxies of Cosmicflows-2 catalog. We conclude in Section 6.

2. The Catalogs of Galaxies

A first kind of catalog for galaxies is represented by those focused as slice as the two-degree Field Galaxy Redshift Survey, in the following 2dFGRS, see [3], or the Sloan Digital Sky Survey (SDSS), see [4]. A second classification is about the all-sky catalogs such as the 2MASS Redshift Survey (2MRS), see [5], or the Cosmicflows-2, see [2]. As a first example a strip of the 2dFGRS is shown in Figure 1. The below Figure, which is a 3˚ slice, shows that the galaxies resides on filaments rather to be distributed in a uniform 2D way. A first extrapolation allows to state that the galaxies are disposed on the surface of bubbles rather than to fill uniformly the 3D space. The statistics of the voids is a topic of research and an important parameter is the average radius of the voids, \( \langle R \rangle \): [6] quotes \( \langle R \rangle = 18.23 h^{-1} \) Mpc and Table 1 in [7] quotes a variable average radius ranging from \( \langle R \rangle = 15.964 h^{-1} \) Mpc to \( \langle R \rangle = 63.347 h^{-1} \) Mpc according to the selected sample. As second example is represented by the overall Cosmicflows-2 catalog, data available at http://vizier.u-strasbg.fr/viz-bin/VizieR, and Figure 2 reports the sky distribution of such a catalog in Galactic coordinates.

Also in this case the voids are can be visualized selecting a thin (20 Mpc) spherical shell, see Figure 3.

3. Luminosity Function for Galaxies

The distance modulus is

\[
m - M = 5 \log(d) - 5,
\]

where \( d \) is the distance to the galaxy. Figure 1. Cone-diagram of the galaxies in the 2dFGRS with distance \( d \in [4 \, \text{Mpc}, 470 \, \text{Mpc}] \). This plot contains 117,293 galaxies.
where \( m \) is the apparent magnitude, \( M \) is the absolute magnitude and \( d \) is the distance in pc.

Let \( L \), the luminosity of a galaxy, be defined in \([0, \infty)\). The Schechter LF of galaxies, \( \Phi \), see [8], is

\[
\Phi(L; \Phi^*, \alpha, L^*) = \frac{\Phi^*}{\Gamma(\alpha)} \frac{L^*}{L}^{-\alpha} \exp\left(-\frac{L}{L^*}\right) dL,
\]

where \( \alpha \) sets the slope for low values of \( L \), \( L^* \) is the characteristic luminosity, and \( \Phi^* \) represents the number of galaxies per Mpc\(^3\). The normalization is

\[
\int_0^\infty \Phi(L; \Phi^*, \alpha, L^*) dL = \Phi^* \Gamma(\alpha + 1),
\]

where

\[
\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt,
\]

is the Gamma function. The average luminosity, \( \langle L \rangle \), is
An equivalent form in absolute magnitude of the Schechter LF is

$$\Phi(M; M^*, \alpha, M^*)\,dM = 0.921\Phi^*10^{0.4\alpha}\exp(-10^{0.4(M-M^*)})\,dM,$$

where $M^*$ is the characteristic magnitude.

A typical result of the Schechter LF in the case of Cosmicflows-2 is reported in Figure 4.

4. The Photometric Maximum

The flux, $f$, is

$$f = \frac{L}{4\pi r^2},$$

where $r$ is the distance and $L$ the luminosity of the galaxy. The joint distribution in distance, $r$, and flux, $f$, for the number of galaxies is

$$\frac{dN}{d\Omega dr df} = \frac{1}{4\pi}4\pi r^2 dr \Phi\left(\frac{L}{L^*}\right)\delta\left(f - \frac{L}{4\pi r^2}\right),$$

were the factor $\left(\frac{1}{4\pi}\right)$ converts the number density into density for solid angle and the Dirac delta function selects the required flux. In the case of Schechter LF of galaxies the number of galaxies as function of the distance is

$$\frac{dN}{d\Omega dr df} = \frac{1}{L^*}4\pi r^2 \Phi^* \left(4\frac{\pi r^2}{L^*}\right) e^{-4\pi r^2/L^*}$$

We now introduce the critical radius $r_{\text{crit}}$
\[ r_{\text{crit}} = \frac{1}{2} \frac{\sqrt{L}}{\sqrt{\pi} f}. \]  

(12)

Therefore the joint distribution in distance and flux becomes

\[ \frac{dN}{d\Omega dr df} = \frac{1}{L'} 4\pi r^2 \Phi^* \left( \frac{r^2}{r_{\text{crit}}^2} \right)^\alpha e^{-\frac{r}{r_{\text{crit}}}}. \]  

(13)

The above number of galaxies has a maximum at

\[ r_{\text{max}} = \sqrt{2 + \alpha} r_{\text{crit}}, \]  

(14)

and the average distance of the galaxies, \( \langle r \rangle \), is

\[ \langle r \rangle = r_{\text{crit}} \Gamma \left( 3 + \alpha \right) \]  

\[ \frac{\Gamma \left( \frac{5}{2} + \alpha \right)}{\Gamma \left( \frac{3}{2} + \alpha \right)}. \]  

(15)

**Figure 5** presents the number of galaxies observed in Cosmicflows-2 as a function of the distance for a given window in flux, as well as the theoretical curve. The luminosity of a galaxy is produced and attenuated according to the electromagnetism. We now deal with the mass of a galaxy. A transformation of the luminosity of a galaxy, \( L \), by the mass, \( M \), is given by the nonlinear Formula (9) in [9].

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**Figure 5.** The galaxies of Cosmicflows-2 with \( 12498.8 L_{\odot} / \text{Mpc}^2 \leq f \leq 488727.8 L_{\odot} / \text{Mpc}^2 \) are organized by frequency versus distance, (empty circles); the error bar is given by the square root of the frequency. The maximum frequency of the observed galaxies is at \( d = 73.8 \) Mpc. The full line is the theoretical curve generated by \( \frac{dN}{d\Omega dr df} \) as given by the application of the Schechter LF which is Equation (11) and the theoretical maximum is at \( d = 81.69 \) Mpc. The parameters are \( L' = 1.8 \times 10^9 L_{\odot} \) and \( \alpha = -0.83 \).
We pay careful attention to the interval of existence in absolute magnitude versus distance for Cosmicflows-2 catalog, see Figure 6. The progressive decrease of the interval of existence for the absolute magnitude is known as Malmquist bias [10]. Conversely the mass of a galaxy does not disappear with distance and therefore produce the gravitational field even if the galaxy is not visible due to the instrumental limitations.

5. Gravitational Forces

The shell theorem, according to proposition LXX, theorem XXX, in Principia [11] states that “If to every point of a spherical surface there tend equal centripetal forces decreasing in the square of the distances from those points, I say, that a corpuscle placed within that superficies will not be attracted by those forces any way” or more simply “The gravitational field inside a uniform spherical shell is zero” [12] [13] [14]. The galaxies are thought to reside on irregular shells which are approximated by a given radius. Due to the fact that the shapes of the above shells are neither exactly spherical nor uniformly populated by galaxies we search for a low value of the gravitational field rather than the theoretical zero. We therefore analyze a 2D box with size of 30 Mpc and masses as given by formula 16. The forces in every point of the considered box are evaluated according to the Newtonian force where $G$, the Newtonian constant of gravitation, is

$$G = 4.49975 \times 10^{-6} \text{Mpc}^3/M_{\odot} \cdot \text{yr}^{-2},$$

(17)

Figure 6. The absolute magnitude $M$ of 4970 galaxies belonging to Cosmicflows-2 when the absolute bolometric magnitude is 3.39 and $H_0 = 74.4 \text{ km s}^{-1} \cdot \text{Mpc}^{-1}$ (green points). The upper theoretical curve, Equation (3), is reported as a red thick line when $m = 15.39$. 

$$\frac{M}{L} = 2.35 \left( \frac{L}{10^{10} L_{\odot}} \right)^{0.32}. \quad (16)$$
where the length is in Mpc, mass is in $M_{gal}$ which are $10^{11} M_{\odot}$ and yr 8 are $10^4$ yr, see more details in [15]. The masses do not disappear with distance and the above value of the box allows processing a complete sample of galaxies. Figure 7 reports the values of the gravitational field as a cut in the middle of the 2D box. Figure 8 reports the values of the gravitational field organized as a two color map.

**Figure 7.** Cut-line of the 2D gravitational forces expressed in $\text{Mpc} \cdot M_{gal} / \text{yr}^8$ (decimal logarithm).

**Figure 8.** Color scheme for the values of the gravitational field evaluated in $500 \times 500$ points. The red zone has values of gravitational field in the interval $\left[4.27 \text{ Mpc} \cdot M_{gal} / \text{yr}^8, 0.00017 \text{ Mpc} \cdot M_{gal} / \text{yr}^8\right]$ (the zones near the galaxies) and the green zone has values of gravitational field in the interval $\left[0.00017 \text{ Mpc} \cdot M_{gal} / \text{yr}^8, 1.7 \times 10^{-8} \text{ Mpc} \cdot M_{gal} / \text{yr}^8\right]$ (the zones of the voids).
6. Conclusion

The observed “great wall” is theoretically explained by a maximum in the number of galaxies as function of the distance in Mpc, see Figure 5. The two concepts of repeller and attractor, see [16], are not used in our analysis. The presence of voids approximated by spheres in the spatial distribution of galaxies allows analyzing the gravitational field at the light of the shell theorem. Due to the facts that the number of galaxies is finite and the masses of galaxies are distributed according to a gamma probability density function the gravitational field at the center of the voids turns out to be \( \approx 2 \times 10^{-8} \text{Mpc} \cdot M_{\text{gal}} / \text{yr}^2 \) rather than zero.

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References


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