

Polynomial-Based Evaluation of the Impact of Aperture Phase Taper on the Gain of Rectangular Horns

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ABSTRACT

The aperture phase taper due to quadratic phase errors in the principal planes of a rectangular horn imposes significant constraints on the on-axis far-field gain of the horn. The precise calculation of gain reduction involves Fresnel integrals; therefore, exact results are obtained only from numerical methods. However, in horns' analysis and design, simple closed-form expressions are often required for the description of horn-gain. This paper provides a set of simple polynomial approximations that adequately describe the gain reduction factors of pyramidal and sectoral horns. The proposed formulas are derived using least-squares polynomial regression analysis and they are valid for a broad range of quadratic phase error values. Numerical results verify the accuracy of the derived expressions. Application examples and comparisons with methods in the literature demonstrate the efficacy of the approach.

Keywords: Microwave Antennas, Rectangular Horn, Gain, Quadratic Phase Error, Linear Regression

1. Introduction

Horns are among the simplest and most widely used microwave antennas. They occur in a variety of shapes and sizes and find application in areas such as wireless communications, electromagnetic sensing, radio frequency heating, and biomedicine. They are commonly used as feed elements for reflector and lens antennas in microwave systems and as high gain elements in phased arrays. Moreover, they serve as a universal standard for calibration and gain measurements of other antennas [1].

Among the microwave horns, the rectangular horn is the simplest and most reliable one. This is a hollow pipe of a rectangular cross section that is flared to a larger opening in the E - or H -plane direction (sectoral horn) or in both directions (pyramidal horn). Rectangular horns are useful tools in science and engineering due to their simplicity in construction, ease of excitation, versatility, and high gain.

A classical expression for the gain of a pyramidal horn is the Schelkunoff's horn-gain formula. This formula calculates the on-axis far-field gain of the horn as the product of the directivity of a uniform dominant mode rectangular waveguide without flares and the gain reduction due to the amplitude and phase taper across the horn aperture [2]. Its main assumptions are

that the horn operates at the dominant TE_{10} waveguide mode and it is well-matched to the feeding waveguide; moreover, it neglects the contribution of the fringe currents caused by the discontinuity of the aperture and the mutual interaction between the aperture edges [2,3]. The formula includes the geometrical optics of the radiated field and the singly diffracted fields of the aperture edges and represents the monotonic gain component. However, it omits multiple diffraction and diffracted fields reflected from horn interior; therefore, it is adequate for pyramidal horns but calculates erroneously the gain of sectoral ones [4]. In [5], Schelkunoff's formula was extended by involving an additional term that accounts for the influence of the edge effect on the on-axis gain and included sectoral horns and open-ended rectangular waveguides. In general, the expressions presented in [2,5] give adequate results and are commonly used in the literature [6-11]. Comparisons between calculated results and measured data showed an uncertainty ± 0.5 dB for frequencies below 2.6 GHz and ± 0.3 dB for higher ones [12]. Several solutions with increased accuracy can be found in the published literature, e.g. [13-16]. However, their complexity and computational cost are worthwhile only if we require very accurate results.

In both horn-gain formulas [2,5], the calculation of the gain reduction factors involves Fresnel integrals and it is made numerically. However, approximate but simple closed-form expressions are often required [11, 17,18]. In this paper, we extend the analysis in [17] to include a broader range of aperture phase error values. The approximate formulas in [17] are valid for aperture phase errors up to the optimum gain condition ones. Here, we provide improved approximate polynomial expressions for the gain reduction factors of a rectangular horn. These formulas were obtained from the application of least squares polynomial fitting over the range of aperture phase error parameters from zero to one (typical values for practical applications [19,20]). We further investigate the impact of the polynomial order on the approximation error and give representative examples that show the merits of our proposal. Comparisons with methods in the literature and results derived from professional antenna design software [21] validate the formulation.

The rest of the paper is organized as follows: Section 2 discusses some theoretical background. Section 3 presents and evaluates the proposed formulation. In Section 4, representative examples show the merits of our proposal. Finally, Section 5 concludes the paper.

2. The Schelkunoff's Classical and Improved Horn-Gain Formulas

Figure 1 shows the geometry of a pyramidal horn with throat-to-aperture length P and aperture sizes A and B . The inner dimensions of the feeding rectangular waveguide are a and b . When $A = a$ or $B = b$, we get the E - or the H -plane sectoral horn, respectively. Next, in **Figure 2**, we give the cross-sectional views of the horn in the two principal planes.

We assume a lossless pyramidal horn that it is well-matched to the rectangular waveguide and operates in the dominant TE_{10} mode. In this case, the on-axis far-field gain of the horn is¹ [2,18]

$$G = \frac{32AB}{\pi\lambda^2} L_E L_H \quad (1)$$

where λ is the free-space wavelength and L_E and L_H are the gain reduction factors that represent the impact of the aperture phase taper due to the quadratic phase errors in the principal planes calculated [22] from:

$$L_E = \left| \frac{2}{B} \int_0^{B/2} \exp \left(-jkR_1 \left[\sqrt{1 + \left(\frac{y}{R_1} \right)^2} - 1 \right] \right) dy \right|^2 \quad (2)$$

¹Usually, it is assumed that the overall efficiency (*i.e.* the product of the reflection, conduction, and dielectric efficiencies) of a rectangular horn is one. In this case, the on-axis far-field gain and the directivity of the horn are identical [1].

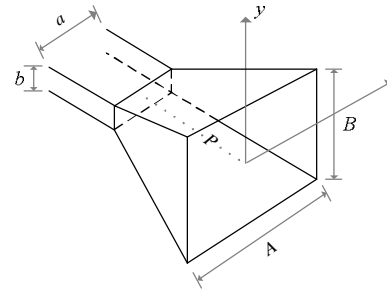


Figure 1. Pyramidal horn geometry

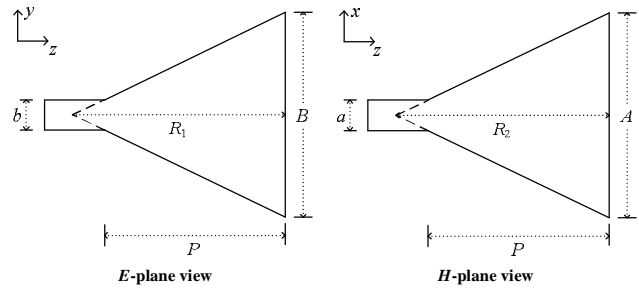


Figure 2. Cross-sectional views of a pyramidal horn antenna

$$L_H = \left| \frac{\pi}{A} \int_0^{A/2} \cos \left(\frac{\pi x}{A} \right) \exp \left(-jkR_2 \left[\sqrt{1 + \left(\frac{x}{R_2} \right)^2} - 1 \right] \right) dx \right|^2 \quad (3)$$

with $k = 2\pi/\lambda$ and $j = \sqrt{-1}$. Notice that (2) and (3) do not include the path long error approximation increasing the accuracy of the results. The gain reduction factors can also be written as functions of the aperture phase error parameters in the E - and H -plane that are given by

$$s = \frac{B(B-b)}{8\lambda P} \quad (4)$$

and

$$t = \frac{A(A-a)}{8\lambda P} \quad (5)$$

respectively, as [6,17]

$$L_E = \frac{C^2(2\sqrt{s}) + S^2(2\sqrt{s})}{4s} \quad (6)$$

$$L_H = \frac{\pi^2}{64t} \left\{ \left[C \left(\frac{1+8t}{4\sqrt{t}} \right) - C \left(\frac{1-8t}{4\sqrt{t}} \right) \right]^2 + \left[S \left(\frac{1+8t}{4\sqrt{t}} \right) - S \left(\frac{1-8t}{4\sqrt{t}} \right) \right]^2 \right\} \quad (7)$$

where $C(\cdot)$ and $S(\cdot)$ are the cosine and sine Fresnel integrals [23], respectively.

Equation (1) can be extended by incorporating the edge effect and the impact of the fringe currents at the aperture edges. In this case, the gain-formula becomes [5,7]

$$G = \frac{32AB}{\pi\lambda^2} \left[1 + \frac{1}{\sqrt{2}} \left(\frac{k}{\beta} - 1 \right) \right]^2 L_E L_H \quad (8)$$

where $\beta/k = \sqrt{1 - (\lambda/2a)^2}$ is the TE₁₀ mode propagation constant [24]. The improved formula is valid for both pyramidal and sectoral horns. It reduces to (1) for large apertures ($\beta/k \approx 1$) and calculates the gain values of the E - and H -plane sectoral horns by setting $L_H \approx 1$ and $L_E \approx 1$, respectively.

3. Polynomial Description of the Gain Reduction Factors

In [17], Aurand provided the following first- and second-order approximations for the gain reduction factors:

$$L_E^{(1)} = 1.032462 - 0.813696s \quad (9)$$

$$L_H^{(1)} = 1.033320 - 0.567302t$$

$$L_E^{(2)} = 1.001633 - 0.07082s - 2.97150s^2 \quad (10)$$

$$L_H^{(2)} = 1.002535 - 0.07341t - 1.31704t^2$$

that are valid for $s \leq 0.25$ and $t \leq 0.375$. Despite their accuracy in the given range of values, these approximations can not describe the gain reduction factors for phase errors far from the optimum gain condition ones (see next Section).

In this paper, we extend Aurand's proposal and produce closed-form expressions for the gain reduction factors L_E and L_H by polynomial regression curve fitting [25-28] of (6) and (7). The fitting curves are linear polynomials calculated with the least squares method [25-27]. In this method, curve-fitting involves the minimization of the sum of the squared residuals, *i.e.* the squared differences between the exact L_E (L_H) value and the L_E (L_H) value that is computed from the curve-fit equation for the same aperture phase error. In order to get the best fit, we use the R^2 goodness-of-fit statistics metric (this is the square of the sample correlation coefficient between the data values and the calculated ones from the fitting polynomial). The fit improves as R^2 values approach unity.

In practice, we approximate L_E and L_H with n th-order polynomials, *i.e.* it is

$$L_E \approx l_E^{(n)} = \sum_{i=0}^n e_{n,i} s^i, \quad s \leq 1 \quad (11)$$

$$L_H \approx l_H^{(n)} = \sum_{i=0}^n h_{n,i} t^i, \quad t \leq 1$$

Let \mathbf{e}_n and \mathbf{h}_n be vectors with elements the polynomial coefficients $e_{n,i}$ and $h_{n,i}$, $i = 0, 1, \dots, n$, respectively. In this case, we formulate the least squares problem as:

$$\text{find } \mathbf{e}_n : \text{ Minimize } \sum_{j=0}^N \left(l_{E,j}^{(n)} - L_{E,j} \right)^2 \quad (12)$$

and

$$\text{find } \mathbf{h}_n : \text{ Minimize } \sum_{j=0}^N \left(l_{H,j}^{(n)} - L_{H,j} \right)^2 \quad (13)$$

The subscript j in (12) and (13) denotes that the specific values are calculated at s or t equal to j/N (the maximum value of the two aperture phase error parameters is one, see (11)). Each curve is evaluated at 10001 points in steps of 10^{-4} in the range $[0, 1]$, *i.e.* $N = 10^4$. Derivation of \mathbf{e}_n and \mathbf{h}_n gives the n th-order polynomial approximation of L_E and L_H , respectively².

Recall that the choice of the best fit approximation uses the R^2 goodness-of-fit statistics metric. **Table 1** gives the calculated R^2 values for the best fit n th-order polynomial approximation of (6) and (7) for $n = 1, 2, \dots, 10$. The corresponding polynomial coefficients, $e_{n,i}$ and $h_{n,i}$, are given in **Tables 2** and **3**. In Table 1, we also give the F-statistic values [25-27] of each approximation (F-statistic value goes toward infinity as the fit becomes more ideal). Notice that L_H is adequately approximated from a polynomial with order lower than the one that is required for L_E . All the results were checked and validated using Matlab R2008a curve fitting routines [29].

Table 1. Goodness-of-fit values

n	E -plane		H -plane	
	R^2	F-statistic	R^2	F-statistic
1	0.95754249	225507.05	0.99012258	1002310.4
2	0.97988824	243562.02	0.99051231	521894.17
3	0.99871152	2582915.7	0.99988122	28051368
4	0.99993194	36715144	0.99998119	$1.328863 \cdot 10^8$
5	0.99999478	$3.8304487 \cdot 10^8$	0.99999961	$5.1801471 \cdot 10^9$
6	0.99999993	$2.2592440 \cdot 10^{10}$	0.99999999	$1.2452233 \cdot 10^{11}$
7	0.99999999	$2.3209471 \cdot 10^{11}$	1	$4.1892924 \cdot 10^{12}$
8	1	$3.7006211 \cdot 10^{13}$	1	$2.8864097 \cdot 10^{14}$
9	1	$3.7613197 \cdot 10^{14}$	1	$8.9753437 \cdot 10^{15}$
10	1	$1.2875745 \cdot 10^{17}$	1	$1.1346748 \cdot 10^{18}$

²A further discussion on this issue is beyond the scope of the paper; the interested reader can find additional information about the development and implementation of least squares algorithms in the proposed literature [25-27].

Table 2. Polynomial coefficients $e_{n,i}$

i	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
0	1.0336239	1.1457623	1.0240209	0.9888856	0.9976955	1.0004342	1.0000973	0.9999895	0.9999976	1.0000002
1	-1.1374395	-1.8103371	-0.3490752	0.3539478	0.0894666	-0.0256761	-0.0067831	0.0009942	0.0002653	-2.414×10^{-5}
2	—	0.6728975	-2.9804397	-6.1444123	-4.2926737	-3.1409013	-3.3960639	-3.5322412	-3.5161931	-3.5083720
3	—	—	2.4355582	7.3574573	2.4191175	-2.1885480	-0.7707180	0.2281648	0.0783339	-0.0120796
4	—	—	—	-2.4609496	3.0948216	11.734669	7.8352862	4.0889891	4.8195421	5.3734495
5	—	—	—	—	-2.2223085	-9.8255266	-4.2101716	3.5826714	1.5369186	-0.4574140
6	—	—	—	—	—	2.5344060	-1.5211957	-10.613170	-7.2033875	-2.7711778
7	—	—	—	—	—	—	1.1587434	6.7253379	3.3850412	-2.7660781
8	—	—	—	—	—	—	—	-1.3916486	0.3829062	5.5730509
9	—	—	—	—	—	—	—	—	-0.3943455	-2.8292557
10	—	—	—	—	—	—	—	—	—	0.48698211

Table 3. Polynomial coefficients $h_{n,i}$

i	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
0	1.0744003	1.0639483	1.0033308	0.9962335	0.9995997	1.0001198	1.0000154	0.9999976	0.9999997	1.0000000
1	-0.8163131	-0.7535950	-0.0260029	0.1160082	0.0149499	-0.0069168	-0.0010603	0.0002270	3.6932×10^{-5}	-4.7892×10^{-6}
2	—	-0.0627181	-1.8817891	-2.5209139	-1.8133644	-1.5946318	-1.6737272	-1.6962670	-1.6920827	-1.6909553
3	—	—	1.2127140	2.2069413	0.3200012	-0.5550385	-0.1155393	0.0497937	0.0107279	-0.0023056
4	—	—	—	-0.4971136	1.6257471	3.2665367	2.0578052	1.4377260	1.6282047	1.7080531
5	—	—	—	—	-0.8491443	-2.2930680	-0.5524193	0.7374358	0.2040421	-0.0834507
6	—	—	—	—	—	0.4813079	-0.7758482	-2.2807329	-1.3916927	-0.7527678
7	—	—	—	—	—	—	0.3591875	1.2805586	0.4096356	-0.4770785
8	—	—	—	—	—	—	—	-0.2303428	0.2323408	0.9805257
9	—	—	—	—	—	—	—	—	-0.1028186	-0.4538229
10	—	—	—	—	—	—	—	—	—	0.0702009

In order to describe simple but accurately the gain reduction factor, we have to estimate the minimum required polynomial order. In practical terms, the goodness-of-fit statistics may not provide an efficient way to estimate the degree of error. In this case, a graphical inspection of the

fitting curves ensures the suitability of the proposed approximations. **Figures 3** and **4** show the exact and the (approximate) fitting curves of the gain reduction factors. Notice that the curves that describe $l_E^{(4)}$ and $l_H^{(3)}$ are almost similar to the exact solution.

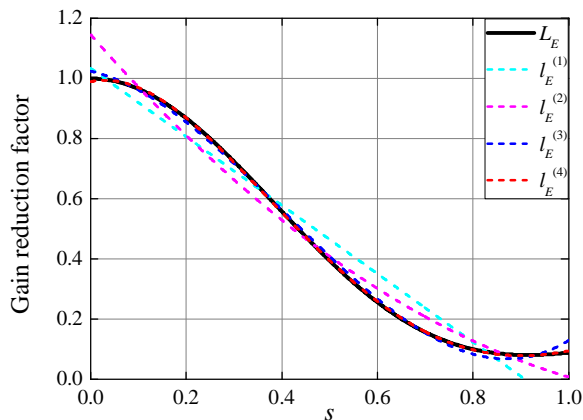


Figure 3. E-plane gain reduction factor: Exact and approximate curves

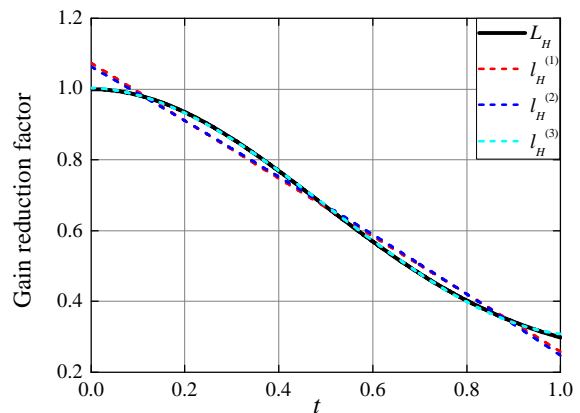


Figure 4. H-plane gain reduction factor: Exact and approximate curves

In order to further investigate the relation between the approximation error and the polynomial order, we used two well-known error metrics, the average absolute error and the rms error. **Figure 5** shows the variation of the two metrics as a function of the polynomial order of the approximate formulas. We see that L_H is adequately approximated with fewer terms than L_E ; for example, $l_H^{(3)}$ and $l_E^{(5)}$ yield errors less than 1%. We also notice that the average absolute error is always slightly smaller than the rms error.

4. Application Examples

In order to show the efficacy of our approach, we give three representative examples.

Example 1: Let us consider some typical X-band pyramidal horns, see **Table 4**. Horns operate at 10 GHz and they are fed from WR-90 waveguide. Their aperture phase errors are calculated from (4) and (5). **Figure 6** illustrates the gain values of the horns. With $G_{0,1}$ and $G_{0,2}$ we present the values that are calculated from (1) using Aurand's first- and second-order approximations, respectively; $G_1, G_2, G_4,$ and G_6 are the results obtained from (1) and the proposed first-, second-, fourth-, and sixth-order approximation. G_0 denotes the gain values that are calculated from (1) with adaptive quadrature integration of (2) and (3). Finally, G_{acc} are the exact gain values calculated with the professional antenna design software ORAMA [21].

As it was expected, (9) and (10) are adequate only for small values of s and t (moreover, $G_{0,2}$ takes complex values for great values of s ; these are not shown in **Figure 6**). In any case, the fourth-order approximations give results almost identical to the numerically calculated ones. The results are also in good consistency with the exact values calculated with ORAMA. The small gain values in cases 3 and 5 are due to the fact that the far-field gain is not maximized at the horn's axis.

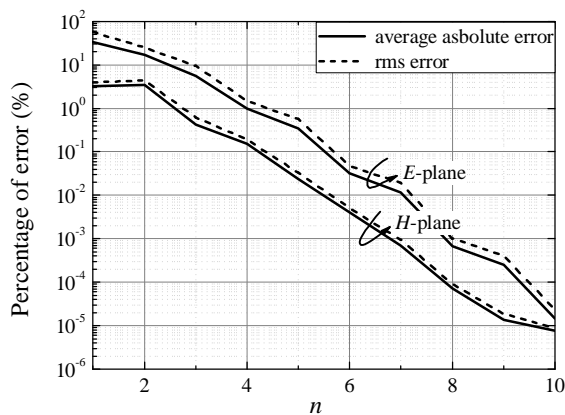


Figure 5. Average absolute and rms error versus the order of the fitting polynomials

Table 4. Horns specifications

ID	A(cm)	B(cm)	P(cm)	s	t
1	14.6	11.48	20	0.250	0.375
2	10	10	20	0.187	0.161
3	10	20	20	0.792	0.161
4	20	10	20	0.187	0.739
5	20	20	20	0.792	0.739

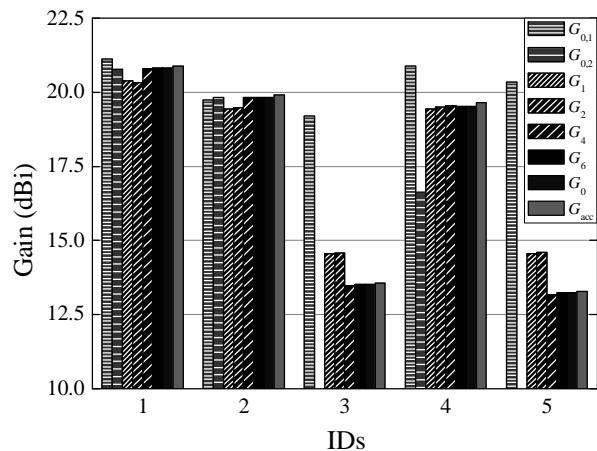


Figure 6. Calculated gain values (in dBi)

Example 2: We consider an E - and an H -plane sectoral horn that operate at 10 GHz and are fed from WR-90 waveguide. In the first case, $B = 20$ cm; in the second one, it is $B = 20$ cm. In both cases, the throat-to-aperture length is 20 cm.

First, we calculate the gain values from (8) with adaptive quadrature integration of (2) and (3). The exact horns' gains are 9.1 dB (E -plane sectoral horn) and 10.15 dB (H -plane sectoral horn). The gain values that are obtained from (1) and (9) are 14.83 and 11.52 dB, respectively; Aurand's second-order approximation gives worst results. On the other hand, our formulation gives (the subscript denotes the order of the fitting polynomial) that $G_1 = 10.19$ dB, $G_2 = 10.22$ dB, $G_4 = 9.03$ dB and $G_6 = 9.09$ dB (E -plane sectoral horn). In the case of the H -plane sectoral horn, the approximation error is smaller. The calculated gains are $G_1 = 10.37$ dB, $G_2 = 10.39$ dB, $G_4 = 10.15$ dB, and $G_6 = 10.15$ dB. Again, the fourth-order approximations give adequate results.

Example 3: In [11], Selvan proposed a design method for pyramidal horns of any desired gain and aperture phase error. However, the accuracy of his method is strictly related to the accuracy of the approximations of (6) and (7).

Let us consider the design examples in **Table 5**.

Table 6 gives the calculated horns dimensions with the method proposed in [11] using (10) (Aurand's app-

roximation) and the proposed second-, fourth-, and sixth-order approximations. Next, we used this data and calculated the exact gain values with ORAMA, see **Table 7**. Finally, **Figure 7** shows the absolute relative error between the computed and the desired gain for each case. We notice that Aurand's approximation is adequate at small values of s and t (IDs 1 and 3). However, as the aperture phase error parameters increase (IDs 2 and 4) these approximations lead to erroneous results. In any case, the proposal in [11] is an accurate design method when a polynomial approximation with polynomial order at least equal to four is used for the description of the gain reduction factors.

Table 5. Horns design examples

ID	f (GHz)	Waveguide type	G_{des} (dBi)	s	t
1	1	WR-975	15.45	0.2	0.3
2	1	WR-975	15.45	0.4	0.6
3	34	WR-28	24.58	0.25	0.375
4	34	WR-28	24.58	0.5	0.75

Table 6. Calculated horns dimensions

ID	Aurand's appr.			2 nd order appr.		
	A (cm)	B (cm)	P (cm)	A (cm)	B (cm)	P (cm)
1	74.046	55.901	50.681	77.829	59.027	57.359
2	128.095	100.331	91.916	112.992	87.949	69.228
3	6.601	5.273	14.690	6.951	5.558	16.384
4	24.200	19.646	107.371	11.980	9.667	25.500
ID	4 th order appr.			6 th order appr.		
	A (cm)	B (cm)	P (cm)	A (cm)	B (cm)	P (cm)
1	74.130	55.971	50.826	74.122	55.964	50.812
2	112.13	87.242	68.029	111.989	87.126	67.834
3	6.591	5.264	14.642	6.580	5.255	14.587
4	12.510	10.099	27.880	12.525	10.112	27.951

Table 7. Desired and calculated gain values (in dBi)

ID	Desired	Aurand's appr.	2 nd order appr.	4 th order appr.	6 th order appr.
1	15.45	15.62	16.09	15.63	15.63
2	15.45	16.75	15.64	15.57	15.56
3	24.58	24.67	25.12	24.66	24.64
4	24.58	30.34	24.22	24.60	24.61

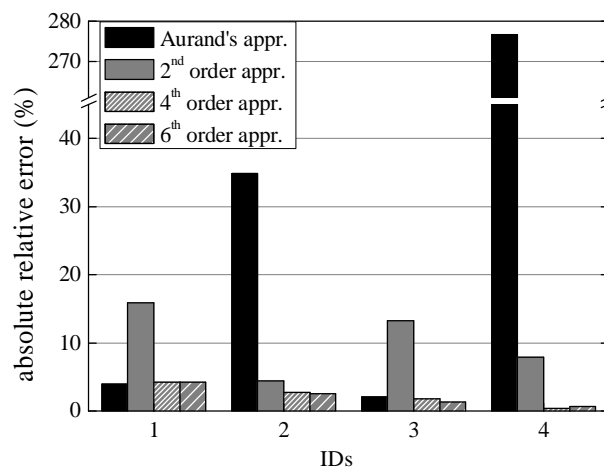


Figure 7. Relative gain errors

5. Conclusions

In this paper, we presented a set of n th-order polynomial approximate expressions for the gain reduction factors of pyramidal and sectoral microwave horns. The formulas were derived with polynomial regression curve fitting techniques. Comparisons with methods in the published literature and results calculated with commercial antenna design software verified the accuracy of the proposed formulation and demonstrated the benefits of the approach. We also explored the relation between the polynomial order of the derived formulas and the approximation error. It was found that that a third-order polynomial approximation of the gain reduction factor in the H -plane is adequate; in order to obtain accurate results in the E -plane, a fourth-order approximation is required. This paper extends previous work in the literature and applies to horns with large values of aperture phase errors. The proposed formulation is a useful tool in the analysis and design of rectangular horns, especially when simple closed-form expressions are required.

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