

Precise Measurements of the Gravitational Constant: Revaluation by the Information Approach

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Abstract

The gravitational constant discovered by Newton is still measured with a relative uncertainty that is several orders of magnitude larger than the relative uncertainty of other fundamental constants. Numerous methods are used to measure it. This article discusses the information-oriented approach for analyzing the achievable relative measurement uncertainty, in which the magnitude of the gravitational constant can be considered as plausible. A comparison is made and the advantages and disadvantages of various methods are discussed in terms of the possibility of achieving higher accuracy using a new metric called comparative uncertainty, which was proposed by Brillouin.

Keywords

Gravitational Constant, Comparative Uncertainty, Information Theory, Modeling, Relative Uncertainty

1. Introduction

Over the past three decades, many experimental physicists have devoted themselves to measuring the gravitational constant G with various methods and obtained dozens of accurate values. The Task Group from CODATA (The Committee on Data for Science and Technology) decided to use a weighted mean and gave the final G -value of $6.67435(13) \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ with the relative standard uncertainty of 4.7×10^{-5} , according to the 2014 least squares adjustment performed by CODATA [1]. This is a relatively large uncertainty when compared with the uncertainties of other fundamental constants [2]. In addition, the results obtained still do not agree well with each other. This gives reason to suspect hidden systematic errors in some experiments. Uncertainty budgets can include

only what the experimenters know, not what they do not know. It can be assumed that each experimenter provided an uncertainty budget calculation for the experiment, which is as detailed as possible, and excluded possible systematic effects. However, it should be noted that the researcher formulates his approach based on intuition, accumulated knowledge, and his life experience (personal philosophical convictions [3]). As an example, we can present the CODATA method: to determine the recommended value of the relative uncertainty of the fundamental physical constant, a detailed discussion of the input data is conducted, as well as the justification and construction of tables of values sufficient to use directly the relative uncertainty using modern advanced statistical methods and powerful computers. This, in turn, allows you to check the self-consistency of the input data and the output set of values. At the same time, an expert opinion is also used at each stage of data processing. However, the statistics are similar to experts who are witnesses in court—they will testify in favor of any party. In this case, one cannot exclude the possibility of the presence of a preconceived opinion motivated by personal convictions or preferences. That is why, if all the experiments of the same type coincide, a reasonable way to check the systematic uncertainties is to repeat the measurement with a different experimental approach and to apply a novel approach to analyze the data. Many different experiments were carried out, and different, sometimes contradictory results, were published.

A possible reasonable explanation for the discrepancy of G measurements is that there is still some unknown physics [4] including possible sinusoidal changes of G [5] and the sun's dragging effect [6]. However, it is difficult to confirm or refute such ideas due to the low accuracy of the measurement of G . Obviously, additional studies with new approaches and greater accuracy will be required in the future.

At this moment, there are several methods of G measurement taken into account in the latest CODATA-2014 adjustment: time-of-swing, angular acceleration feedback, free deflection, and electrostatic compensation, Fabry-Perot cavity, beam balance, atom interferometry [7].

Further, only such methods and the results obtained with their use (with data on the relative uncertainty of measurement and standard uncertainty), which are presented in the scientific literature and agreed by CODATA, will be considered. Analysis of publications and all necessary calculations were carried out in the office of Mechanical & Refrigeration Consultation expert (Beer-Sheba, Israel).

2. The Testable Hypothesis

The idea of the proposed method is as follows. Using the theory of similarity and information theory, it is possible to calculate the amount of information in any physical and mathematical model. This was made possible by counting the total number of quantities used in the International system of units (SI). The amount of information contained in the model can be correlated with the comparative

uncertainty (the ratio of the absolute uncertainty to the observation interval) of the measured target quantity. This, in turn, allows calculating the recommended value of the relative uncertainty in the measurement of the quantity under study.

The following presents the main provisions of the method (full evidence is introduced in [8]) with the aim of their further use in analyzing the results of measurements of the gravitational constant.

Absolutely all physicists and engineers try to describe the observed phenomena with the help of concepts inculcated by everyday experience, acquired knowledge and, not infrequently, intuition. At the same time, despite 90 years of effort, it has not been possible until now to combine classical determinism with the probabilistic laws of quantum mechanics. The only characteristic that unites all modern physics so far is that scientists use the SI to realize their ideas. SI, in its essence, is some new element in scientific knowledge, completely alien to classical concepts. It exists only because of the consensus of the researchers, although SI is absent in nature.

The SI includes the base and derived quantities used for descriptions of different classes of phenomena (*CoP*). In other words, the limits of the description of the studied material object are defined by the choice of *CoP* and the number of derived quantities taken into account in the mathematical model [9]. For example, in mechanics, SI uses the basis {*L*—length, *M*—mass, *T*—time}, *i.e.*, $CoP_{SI} \equiv LMT$. Basic accounts of electromagnetism here add the magnitude of electric current *I*. Thermodynamics requires the inclusion of thermodynamic temperature Θ . For photometry, it needs to add *J*—luminous intensity. The final base quantity of SI is an amount of substance *F* [10].

The dimension of any derived quantity *q* can be expressed as a unique combination of dimensions of the main base quantities to different powers [10]:

$$q \ni L^l \cdot M^m \cdot T^t \cdot I^i \cdot \Theta^\theta \cdot J^j \cdot F^f, \quad (1)$$

where

1) condition (1) is a very strong constraint. The presentation of experimental results as a formula, where the main quantity is represented as a correlation function of one-parameter power functions, has many limitations. However, in this study, condition (1) can be successfully applied to a system that is not in nature; for example, SI. In this system, the derived quantities are always represented as the product of the base quantities to different degrees;

2) l, m, \dots, f are exponents of the base quantities and the range of each has a maximum and minimum value; according to [11], integers are the following:

$$\begin{aligned} -3 \leq l \leq +3, -1 \leq m \leq +1, -4 \leq t \leq +4, -2 \leq i \leq +2 \\ -4 \leq \theta \leq +4, -1 \leq j \leq +1, -1 \leq f \leq +1 \end{aligned} \quad (2)$$

3) because the exponents of the base quantities can only take integer values, the number of choices of dimensions for each quantity e_l, \dots, e_f , according to (2), is the following:

$$e_l = 7; e_m = 3; e_t = 9; e_i = 5; e_\theta = 9; e_j = 3; e_f = 3. \quad (3)$$

where, for example, L^{-3} is used in a formula for density, and Θ^4 in the Stefan-Boltzmann law;

4) the total number of dimension options of physical quantities equals $\Psi^\circ = \prod_l e_l - 1$

$$\Psi^\circ = (e_l \cdot e_m \cdot e_i \cdot e_i \cdot e_\theta \cdot e_j \cdot e_f - 1) = 76544 \quad (4)$$

where “-1” corresponds to the case where all exponents of the base quantities in formula (1) are treated as zero dimension, Π is a product of elements e_l, \dots, e_f ;

5) the value Ψ° includes both required and inverse quantities (for example, L^1 is the length, L^{-1} is the running length). The object can be judged knowing only one of its symmetrical parts, while others structurally duplicating this part may be regarded as information empty. Therefore, the number of options for dimensions may be halved. This means that the total number of dimension options of physical quantities without inverse quantities equals $\Psi = \Psi^\circ / 2 = 38,272$.

According to the π -theorem [12], the number μ_{SI} of possible dimensionless criteria with $\xi = 7$ base quantities for SI will be:

$$\mu_{SI} = \Psi - \xi = 38265 \quad (5)$$

Then, let there be a situation wherein all quantities μ_{SI} of SI can be taken into account, provided the choice of these quantities is considered, *a priori*, equally probable. In this case, μ_{SI} corresponds to a certain value of entropy and may be calculated using the following formula [13] [14]:

$$H = k \cdot \ln \mu_{SI}, \quad (6)$$

where H is the entropy of SI including μ_{SI} , equally probable accounted quantities, and k_B is the Boltzmann constant.

When a researcher chooses the influencing factors (the conscious limitation of the number of quantities that describe an object, in comparison with the total number μ_{SI}), the entropy of the mathematical model changes *a priori*. The entropy change is generally measured as follows [13]:

$$\Delta H = H_{pr} - H_{ps}, \quad (7)$$

where ΔH is the entropy difference between the two cases, and pr is “*a priori*” and ps is “*a posteriori*”.

“The efficiency Q of the experimental observation method can be defined as the ratio of the information obtained to the entropy change accompanying the observation.” [13] During a thought experiment, no distortion is brought into the real system, which is why $Q = 1$. Then, one can write it according to (7):

$$\Delta A = Q \cdot \Delta H = (H_{pr} - H_{ps}), \quad (8)$$

where ΔA is the *a priori* information quantity pertaining to the observed object.

Using equations (7) and (8) and imposing symbols where z' is the number of physical quantities in the selected *CoP* and β' is the number of base quantities in the selected *CoP* leads to the following equation:

$$\Delta A' = Q \cdot (H_{pr} - H_{ps}) = k \cdot \ln \left[\mu_{SI} / (z' - \beta') \right], \quad (9)$$

where $\Delta A'$ is the *a priori* amount of information pertaining to the observed object due to the choice of the *CoP*.

The value $\Delta A'$ is linked to the *a priori* absolute uncertainty of the model, caused only by the choice of the *CoP*, Δ'_{pmm} , and S , the dimensionless interval of the observation of the main researched dimensionless quantity u , through the following dependence [13]:

$$\Delta'_{\text{pmm}} = S \cdot \exp(-\Delta A'/k_b). \quad (10)$$

Substitution of (9) into (10) gives the following dependence:

$$\Delta'_{\text{pmm}} = S \cdot (z' - \beta') / \mu_{\text{SI}}. \quad (11)$$

Following the same reasoning, it can be shown that the *a priori* absolute uncertainty of a model of the observed object, caused by the number of recorded dimensionless criteria chosen in the model, Δ''_{pmm} takes the following form:

$$\Delta''_{\text{pmm}} = S \cdot (z'' - \beta'') / (z' - \beta'), \quad (12)$$

where z'' is the number of physical dimensional quantities recorded in a mathematical model, β'' is the number of base physical dimensional quantities recorded in a model, and Δ''_{pmm} cannot be defined without declaring the chosen *CoP* (Δ'_{pmm}).

Taking into account [11] and [12], all the above derivations can be summarized in the form of the μ_{SI} -hypothesis: *In model formulation, let the system of base quantities with a total number of dimensional physical quantities be denoted by Ψ , where ξ of which are chosen and are independent of dimension. In the framework of the phenomena class (z' is the total number of dimensional quantities and β' is the number of base quantities), there is a dimensionless main quantity u that is raised to a given range of values S . Then, the absolute uncertainty Δ_{pmm} that contains u (for a given number of physical dimensional quantities, z'' is recorded in a model where β'' of which are the number of chosen base quantities) can be determined from the relationship:*

$$\Delta u_{\text{pmm}} = S \cdot [(z' - \beta') / \mu_{\text{SI}} - (z'' - \beta'') / (z' - \beta')], \quad (13)$$

where $\varepsilon = \Delta_{\text{pmm}}/S$ is the comparative uncertainty [13].

Equation (13) is very simple and using it, one can find the recommended model's uncertainty value with the theoretical analysis of the physical phenomena. Moreover, Equation (13) can also inform a limit on the advisability of obtaining an increase of the measurement accuracy in conducting pilot studies or computer simulation. It is not a purely mathematical abstraction. Equation (13) has a physical meaning. The μ_{SI} -hypothesis lays down that, in nature, there is a fundamental limit to the accuracy of measuring any process, which cannot be surpassed by any improvement of instruments, measurement methods, or the model's computerization. The value of this limit is much higher and stronger than what the Heisenberg uncertainty relation provides.

At its core, Δ_{pmm} is an *a priori* conceptual uncertainty that is inherent to any physical-mathematical model and is independent of the measurement process.

The uncertainty determined by the proposed principle is not the result of measurement; it represents an intrinsic property of the model, and it is caused only by the number of selected quantities and the chosen *CoP*. Therefore, the overall uncertainty model including additional uncertainties associated with the structure of the model and its subsequent computerization will be much greater than Δ_{pmm} .

It is to be noted that the relative and comparative uncertainties of the dimensional quantity U and the dimensionless quantity u are equal:

$$\begin{aligned}\Delta U/S^* &= (\Delta U/a) / (S^*/a) = (\Delta u/S) \\ (r/R) &= (\Delta U/U) / (\Delta u/u) = (\Delta U/U) \cdot (a/\Delta U) \cdot (U/a) = 1\end{aligned}\quad (14)$$

where S and Δu are the dimensionless quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensionless quantity u); S^* and ΔU are the dimensional quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensional quantity U); a is the dimensional scale parameter with the same dimension as that of U and S^* ; r is the relative uncertainty of the dimensional quantity U ; and R is the relative uncertainty of the dimensionless quantity u . That is why Equation (13) can be used for analyzing any experimental results with dimensional quantities.

Equating the derivative of Δ_{pmm}/S (13) with respect to $z' - \beta'$ to zero, gives the following condition for achieving the minimum comparative uncertainty for a particular *CoP*:

$$(z' - \beta')^2 / \mu_{\text{SI}} = (z'' - \beta''). \quad (15)$$

By using (15), one can find the values for the lowest achievable comparative uncertainties for different *CoP*_{SI}; moreover, the values of the comparative uncertainties and the numbers of the chosen variables are *different* for each *CoP*_{SI}:

1) For mechanics processes (*CoP*_{SI} \equiv *LMT*), taking into account the aforementioned explanations, the lowest comparative uncertainty ε_{LMT} can be reached at the following conditions:

$$(z' - \beta')_{\text{LMT}} = (e_l \cdot e_m \cdot e_t - 1) / 2 - 3 = 91 \quad (16)$$

$$(z'' - \beta'')_{\text{LMT}} = (z' - \beta')^2 / \mu_{\text{SI}} = 0.2164 < 1 \quad (17)$$

where “-1” corresponds to the case when all the base quantity exponents are zero in formula (1); dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 3 corresponds to the three base quantities L, M, T .

According to (16) and (17) ε_{LMT} equals:

$$\varepsilon_{\text{LMT}} = \left(\Delta u_{\text{pmm}} / S \right)_{\text{LMT}} = 0.0048 \quad (18)$$

In other words, according to (17), even one dimensionless main quantity does not allow the experimenter to reach the lowest comparative uncertainty. Therefore, in the frame of the suggested approach, the original comparative uncer-

tainty (18) cannot be realized using any mechanistic model ($CoP_{SI} \equiv LMT$). Moreover, the greater the number of mechanical parameters, the greater the embedded uncertainty. In other words, the Cavendish method, for example, in the frame of the suggested approach is not recommended for measurements of the Newtonian gravitational constant.

2) For electromechanical processes ($CoP_{SI} \equiv LMTI$), taking into account (3) and (5), the lowest comparative uncertainty can be reached at the following conditions:

$$(z' - \beta')_{LMTI} = (e_i \cdot e_m \cdot e_i \cdot e_i - 1) / 2 - 4 = 468 \quad (19)$$

$$(z'' - \beta'')_{LMTI} = (z' - \beta')^2 / \mu_{SI} = 468 / 38265 \approx 6 \quad (20)$$

where “-1” corresponds to the case where all the base quantities exponents are zero in formula (1); 4 corresponds to the five base quantities L , M , T , and I ; and division by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length and L^{-1} is the run length. The object can be judged based on the knowledge of only one of its symmetrical parts, while the other parts that structurally duplicate this one may be regarded as information empty. Therefore, the number of options of dimensions may be reduced by a factor of two.

Then, one can calculate the minimum achievable comparative uncertainty ε_{LMTI} :

$$\varepsilon_{LMTI} = (\Delta u / S)_{LMTI} = 468 / 38265 + 6 / 468 = 0.0245 \quad (21)$$

We will apply the information-oriented approach for analyzing measurements of the gravitational constant.

3. Step-by-Step Description of the Procedure

If the range of observation S is not defined, the information obtained during the observation/measurement cannot be determined, and the entropic price becomes infinitely large [13]. Taking into account the Brillouin suggestions, there are two options for applying Equation (13) to analyze the measurement data of the fundamental physical constants.

The first, this principle dictates, is analyzing the data of the magnitude of the achievable relative uncertainty at the moment, taking into account the *latest* results of measurements. The extended range of changes in the quantity under study S indicates an imperfection of the measuring devices, which leads to a large value of the relative uncertainty. The development of measuring technology, an increase in the accuracy of measuring instruments and the improvement in existing and newly created measurement methods together lead to an increase in knowledge of the object under study and, consequently, the magnitude of the achievable relative uncertainty decreases. However, this process is not infinite and is limited by (13), which can be called the *conformity principle*. The reader should bear in mind that this conformity principle is not a shortcoming of the measurement equipment or engineering device, but of the way the human brain works. When predicting the behavior of any physical process, physicists are in

fact predicting the perceivable output of instrumentation. It is true that, according to the μ -hypothesis, observation is not a measurement, but a process that creates a unique physical world with respect to each particular observer. Thus, in this case, the range of observation (possible interval of placing) of the fundamental physical constant S is chosen as the difference between the maximum and minimum values of the physical constant measured by different scientific groups during a certain period of recent years. Only in the presence of the results of various experiments can one speak about the possible appearance of a measured value in a certain range. Thus, using the smallest attainable comparative uncertainty inherent in the selected class of phenomena during measurement of the fundamental constant, it is possible to calculate the recommended minimum relative uncertainty that is compared with the relative uncertainty of each published study. In what follows, this method is denoted as *IARU* and includes the following steps:

- 1) From the published data of each experiment, the value z , relative uncertainty r_z , and standard uncertainty u_z (possible interval of placing) of the fundamental physical constant are chosen;
- 2) The experimental absolute uncertainty Δ_z is calculated by multiplying the fundamental physical constant value z and its relative uncertainty r_z attained during the experiment, $\Delta_z = z \cdot r_z$;
- 3) The maximum z_{\max} and minimum z_{\min} values of the measured physical constant are selected from the list of measured values z_i of the fundamental physical constant mentioned in different studies;
- 4) As a possible interval for placing the observed fundamental constant S_z the difference between the maximum and minimum values is calculated, $S_z = z_{\max} - z_{\min}$;
- 5) The selected comparative uncertainty ε inherent in the model describing the measurement of the fundamental constant is multiplied by the possible interval of placement of the observed fundamental constant S_z to obtain the absolute experimental uncertainty value Δ_{IARU} in accordance with the *IARU*, $\Delta_{IARU} = \varepsilon \cdot S_z$;
- 6) To calculate the relative uncertainty r_{IARU} in accordance with the *IARU*, this absolute uncertainty Δ_{IARU} is divided by the arithmetic mean of the selected maximum and minimum values, $r_{IARU} = \Delta_{IARU} / ((z_{\max} + z_{\min})/2)$;
- 7) The relative uncertainty obtained, r_{IARU} , is compared with the experimental relative uncertainties r_i achieved in various studies;
- 8) According to *IARU*, a comparative experimental uncertainty of each study ε_{IARU_i} is calculated by dividing the experimental absolute uncertainty of each study Δ_z by the difference between the maximum and minimum values of the measured fundamental constant S_z , $\varepsilon_{IARU_i} = \Delta_z / S_z$. These calculated comparative uncertainties are also compared with the selected comparative uncertainty ε .

In the second option of applying the conformity principle to analyze the measurement data of the fundamental physical constants, S is determined by the limits of the measuring devices used [13]. This means that as the observation interval in which the expected true value of the measured fundamental physical constant is located, a *standard uncertainty* is selected when measuring the phys-

ical constant in each particular experiment. Compared with various fields of technology, experimental physics is better for the fact that in all the research the experimenters introduce the output data of the measurement with uncertainty bars. At the same time, it should be remembered that the standard uncertainty of a particular measurement is subjective because the conscious observer may not take into account various uncertainties. The experimenters calculate the standard uncertainty, taking into account all measured uncertainties that they have observed. Then, one calculates the ratio between the absolute uncertainty reached in an experiment and the standard uncertainty, acting as a possible interval for allocating a fundamental physical constant. So, in the framework of the information approach, the comparative uncertainties achieved in the studies are calculated, which in turn are compared with the theoretically achievable comparative uncertainty inherent in the chosen class of phenomena. Standard uncertainty can also be calculated for quantities that are not normally distributed. Transformation of different types of uncertainty sources into standard uncertainty is very important. In what follows, this method is denoted as *IACU* and includes the following steps:

- 1) From the published data of each experiment, the value z , relative uncertainty r_z , and standard uncertainty u_z (possible interval of placing) of the fundamental physical constant are chosen;
- 2) The experimental absolute uncertainty Δ_z is calculated by multiplying the fundamental physical constant value z and its relative uncertainty r_z attained during the experiment, $\Delta_z = z \cdot r_z$;
- 3) The achieved experimental comparative uncertainty of each published research ε_{IACU_i} is calculated by dividing the experimental absolute uncertainty Δ_z by the standard uncertainty u_z , $\varepsilon_{IACU_i} = \Delta_z / u_z$;
- 4) The experimental calculated comparative uncertainty ε_{IACU_i} is compared with the selected comparative uncertainty ε inherent in the model, which describes the measurement of the fundamental constant.

We will apply *IARU* and *IACU* to analyze data of the G measurement.

4. Analysis of Measurement Results

4.1. Mechanistic Methods

In SI units, the value of G is equal to a unit $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$. To determine G , in the opinion of many scientists, it is necessary to measure quantities with dimensions of length, L , mass, M , and time, T . Therefore, time-of-swing, angular acceleration feedback can be associated with mechanistic methods. The measurement data are summarized in **Table 1**. The noted scientific articles belong to $CoP_{SI} \equiv LMT$ [15]-[21]. The values of absolute and relative uncertainties differ by more than a factor of 10. A similar situation exists in the spread of the values of comparative uncertainties.

Following the method *IARU*, one can discuss the order of the desired value of the relative uncertainty belonging to $CoP_{SI} \equiv LMT$. An estimated observation

Table 1. Determinations of the gravitational constant and achieved relative and comparative uncertainties by using mechanistic methods.

Year	Identification [**]	Gravitational constant	Achieved relative uncertainty	Absolute uncertainty	G possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
		$G \cdot 10^{11}$ $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	$r_G \cdot 10^5$	$\Delta_G \cdot 10^{15}$ $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	$u_G \cdot 10^{15}$ $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	$\varepsilon'_G = \Delta_G / u_G$ <i>IACU</i>	$\varepsilon''_G = \Delta_G / S_G$ <i>IARU</i>	
2000	Uwash-00	6.67425592	1.4	0.934396	1.8	0.5191	0.0440	[15]
2002	UWup-02	6.67422980	15.0	10.01134	20.0	0.5108	0.4717	[16]
2005	HUST-05	6.67222870	13.0	8.673900	18.0	0.4819	0.4086	[17]
2006	Uzur-06	6.6742512	1.9	1.268108	2.4	0.5284	0.0597	[18]
2009	HUST-09	6.6734918	2.7	1.801843	3.6	0.5005	0.0849	[19]
2010	JILA-10	6.6723414	2.1	1.401192	2.8	0.5004	0.0660	[20]
2014	UCI-14	6.6743513	1.9	1.268127	2.6	0.4877	0.0597	[21]

*Data are introduced int; **[1].

interval of G is chosen as the difference in its values obtained from the experimental results of two projects: $G_{\max} = 6.6743513 \times 10^{-11} \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ [21] and $G_{\min} = 6.6722287 \times 10^{-11} \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ [17]. In this case, the possible observed range S_G of G placing is equal to:

$$S_G = G_{\max} - G_{\min} = 2.1226 \times 10^{-14} \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}. \tag{22}$$

Taking into account (18) and (22), the lowest possible absolute uncertainty for $CoP_{SI} \equiv LMT$ is given by the following:

$$\Delta_{LMT} = \varepsilon_{LMT} \cdot S_G = 0.0048 \times 2.1226 \times 10^{-14} = 1.0 \times 10^{-16} \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}. \tag{23}$$

In this case, the lowest possible relative uncertainty $(r_{\min})_{LMT}$ is the following:

$$r_{LMT} = \Delta_{LMT} / ((G_{\max} + G_{\min}) / 2) = 1.5 \times 10^{-6} \tag{24}$$

This value is much smaller than $1.9 \cdot 10^{-5}$ cited in [21]. This situation confirms the main principle of the information approach, meaning that any experimental values of the relative uncertainty must be greater than the relative uncertainty corresponding to the mechanistic method ($CoP_{SI} \equiv LMT$), i.e., 1.5×10^{-6} . However, according to (16), even one dimensionless main quantity does not allow the researcher to reach the lowest comparative uncertainty. Therefore, in the frame of the suggested approach, the original comparative uncertainty cannot be realized using any mechanistic model ($CoP_{SI} \equiv LMT$). Moreover, the greater the number of mechanical parameters, the greater the embedded uncertainty. In other words, the Cavendish method, for example, in the frame of the suggested approach is not recommended for measurements of the gravitational constant.

Guided by the *IACU* and *IARU* methods, one can calculate the achieved comparative uncertainty in each experiment (Table 1). There is a huge gap between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMT\theta I} = 0.0048$ and the experimental magnitudes achieved during

measuring G by the mechanistic methods. As was mentioned above, *progress to achieving a higher accuracy by these methods cannot be realized*.

Significant differences between the values of the comparative uncertainties achieved in the experiments and calculated in accordance with the $IACU$ can be explained as follows. Within the framework of the information approach, the concept of comparative uncertainty assumes an equally probable account of various quantities, regardless of their specific choice by scientists when formulating a model for measuring G . Based on their experience, intuition, and knowledge, the researchers build a model containing a small number of quantities, which, in their opinion, reflects the fundamental essence of the process under investigation. In this case, many phenomena, which are characterized by specific quantities, are not taken into account.

4.2. Electromechanical Methods

The measurement data are summarized in **Table 2**. The noted scientific articles belong to $CoP_{SI} \equiv LMTI$ [22] [23] [24] [25] [26]. The values of absolute and relative uncertainties differ by more than a factor of three. On the other hand, guided by the $IARU$ method, one can calculate the achieved comparative uncertainty in each experiment (**Table 2**). A look at the distribution of the values of comparative uncertainties indicates lack of any consistency, which confirms doubts that researchers have identified the majority of potential sources of measurement uncertainty. At the same time, guided by the $IACU$ method, it is possible to verify a relative consistency of the achieved results. This suggests that each group of experimenters is learning from other research teams to identify possible sources of measurement uncertainties. With all that there is a large gap between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMTI} = 0.0245$ and the experimental magnitudes achieved during measuring G . It must be mentioned, there has been real progress to achieve higher accuracy during the last 17 years.

Following the method $IARU$, one can discuss the order of the desired value of the relative uncertainty belonging to $CoP_{SI} \equiv LMTI$. An estimated observation interval of G is chosen as the difference in its values obtained from the experimental results of two projects: $G_{\max} = 6.67559270 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ [22] and $G_{\min} = 6.67387270 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ [23]. In this case, the possible observed range S_G of G placing is equal to:

$$S_G = G_{\max} - G_{\min} = 1.72 \times 10^{-14} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}. \quad (25)$$

Taking into account (21) and (25), the lowest possible absolute uncertainty for $CoP_{SI} \equiv LMTI$ is given by the following:

$$\Delta_{LMTI} = \varepsilon_{LMTI} \cdot S_G = 0.0245 \times 1.72 \times 10^{-14} = 4.2073 \times 10^{-16} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}. \quad (26)$$

In this case, the lowest possible relative uncertainty $(r_{\min})_{LMTI}$ is the following:

$$r_{LMTI} = \Delta_{LMTI} / \left((G_{\max} + G_{\min}) / 2 \right) = 6.3 \times 10^{-6} \quad (27)$$

Table 2. Determinations of the gravitational constant and achieved relative and comparative uncertainties by using electromechanical methods.

Year	Identification [**]	Gravitational constant $G \cdot 10^{11}$ $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	Achieved relative uncertainty $r_G \cdot 10^5$	Absolute uncertainty $\Delta_G \cdot 10^{15}$ $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	G possible interval of placing* $u_G \cdot 10^{15}$ $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	Calculated comparative uncertainty $\varepsilon'_G = \Delta_G / u_G$ <i>IACU</i>	Calculated comparative uncertainty $\varepsilon''_G = \Delta_G / S_G$ <i>IARU</i>	Ref.
2001	BIMP-01	6.67559270	4.0	2.670237	8.1	0.3297	0.1258	[22]
2003	MSL-03	6.67387270	4.0	2.669549	5.4	0.4944	0.1258	[23]
2014	BIMP-14	6.67554160	2.4	1.602130	8.3	0.1930	0.0755	[24] [25]
2018	HUST-18	6.67418478	1.2	0.800902	1.5	0.5339	0.0377	[26]
2018	HUST-18	6.67448478	1.2	0.800938	1.5	0.5340	0.0377	[26]

*Data are introduced in [7]; **[1].

This value is less than 1.2×10^{-5} cited in [26]. In the frame of the information approach, any experimental values of the relative uncertainty must be greater than the relative uncertainty corresponding to the $CoP_{SI} \equiv LMTI$, *i.e.*, 6.3×10^{-6} . In other words, there are unrecorded uncertainties in [26], which must be eliminated. Therefore, experimenters are advised to double-check the full list of sources of uncertainty taken into account.

Guided by the *IACU* and *IARU* methods, one can calculate the achieved comparative uncertainty in each experiment (Table 2). There is a large gap between the comparative uncertainty calculated according to the information-oriented approach, $\varepsilon_{LMTI} = 0.0245$, and the experimental magnitudes achieved during measuring G . At the same time, progress to achieving a higher accuracy is obvious during the last 18 years.

So, the lack of previous experience influences innovative research, although specialists are highly qualified to use the latest technologies. Thus, a paradoxical situation arises when new inconsistencies give rise to more serious vulnerabilities in new measurement and computing technologies [27]. In this situation, a new information approach can play a positive role in predicting and adopting new definitions of G for SI, as the theoretical sound method.

4.3. Analysis of Measurement Results

Under the proposed approach, for each mathematical model of a physical law, there is an uncertainty, which initially, before the full-scale experimental studies, or computer simulations, describes its proximity to the examined physical phenomenon or process. This value is called the comparative uncertainty. It depends only on the number of selected quantities and the observation interval of the selected primary quantity. One of the interesting features of the proposed hypothesis is that the minimum achievable comparative uncertainty is not constant and varies depending on the class of phenomena choice. Moreover, theory can predict its value. In particular, this means that when switching from a mechanistic model (*LMT*) to CoP_{SI} with a larger number of the base quantities, this

uncertainty grows. This change is due to the potential effects of the interaction between the increased number of quantities that can be taken into account or not taken into account by the researcher. Moreover, within the framework of the information approach, in contradiction to the concept approved by CODATA, *it is not recommended to determine and declare only one value of relative uncertainty when measuring the gravitational constant by different methods.*

In addition to the comments made in Sections IV.1 and IV.2 regarding the analysis of the measurement results of the gravitational constant based on two different methods using *IACU* and *IARU* (summarized in **Table 3**), the following should be noted.

As can be seen in **Table 3**, the recent results for the gravitational constant do not agree at the level of relative uncertainty according to CoP_{SI} (*IARU*). A disagreement of this magnitude is unacceptable. This situation occurred for many years. These are signs that the main methods have not yet reached the required level of consistency and stability in 2018.

Although a common set of comparative uncertainty data calculated in accordance with the *IACU* is consistent (each group of scientists studies from another group to identify sources of uncertainty), the set of comparative uncertainties calculated in accordance with the *IARU* is inconsistent. The difference between these results is due to certain systematic errors. That is why further and detailed research of the current mechanistic electromechanical methods should be continued.

The greatest success in implementing high accuracy G measurements was achieved using electromechanical methods, given the significant difference in comparative uncertainties between $CoP_{SI} \equiv LMT$ ($\varepsilon_{LMT} = 0.0048$) and $CoP_{SI} \equiv LMTI$ ($\varepsilon_{LMTI} = 0.0245$) and the proximity of the experimental relative uncertainties achieved. At the moment, only the electromechanical method seems very attractive (in terms of its physical acceptability for measuring G and recommendations of the information-oriented method) for the possibility of achieving higher accuracy.

5. Concluding Remarks

The goal of our work is to provide a universal metric for estimating achievable

Table 3. Summarized data.

Variable	Mechanistic methods	Electromechanical methods
CoP	LMT	$LMTI$
Comparative uncertainty according to CoP_{SI}	0.0048	0.0245
$S_G = G_{\max} - G_{\min}, \text{m}^2 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$	2.12×10^{-14}	1.72×10^{-14}
Relative uncertainty according to CoP_{SI} (<i>IARU</i>)	1.5×10^{-6}	6.3×10^{-6}
Achieved experimental lowest relative uncertainty	1.9×10^{-5}	1.2×10^{-5}

accuracy when measuring the gravitational constant. This metric imposes a serious limitation (besides the Heisenberg inequality) on the measurement accuracy of all fundamental constants. For classical physics, quantum mechanics, and technical applications, this statement is not trivial. Any theorist or experimenter, based on his experience, knowledge, and intuition, chooses a test bench design or theoretical model, thereby limiting (reducing) the number of quantities that reflect the observed phenomenon, compared with the total number of quantities contained in SI. Thus, this intangible perturbation of the system is, it is possible to say, primeval, due only to the experimenter's motivation and is independent of the measuring instruments and the design of the test bench. Therefore, it can be argued that the blurriness of the object exists initially, before a specific implementation of the experiment.

We can also ask whether it is possible to reach this limit in a physically correctly formulated model. Because our estimate is given by optimization compared with the achieved comparative uncertainty and the observation interval, it is clear that in the practical case the limit cannot be reached. This is because there is an inevitable model uncertainty. This implies the initial preferences of the researcher, based on his intuition, knowledge, and experience, in the process of the model's formulation. The magnitude of this uncertainty is an indication of how likely your personal philosophical tendencies will affect the outcome of this process. When a person mentally builds a model, at each stage of its construction, there is some probability that the model will not correspond to this phenomenon with a high degree of accuracy.

To date, all experiments on the measurement of the gravitational constant were obtained in accordance with CODATA methods using advanced statistical methods, with different weights coefficients and many consistent values [28]. However, having more statistics used in CODATA does not mean that this situation is better. The future ideal method should be:

understandably; *transparent*, with clearly defined criteria, limitations and degrees of uncertainty; *adaptive*, with updating when bias, subjectivity or other weaknesses become apparent; *reproducible*: those who want to use this method should be able to reproduce it.

These criteria correspond to the proposed method.

The information approach is a broader, transparent and understandable metric for determining the best value of relative uncertainty. Using the proposed tool, experimenters will be able to approach the possible limit of measurement accuracy at lower financial costs and in a shorter time.

The μ_{SI} hypothesis does not contradict the theory of measurements and its concepts, which is an indispensable science today, in the XXI century, and will remain true forever (paraphrase by Professor L. B. Okun [29]). The scope of the μ_{SI} only limits the application of measurement theory to the magnitude of uncertainties that exceed the model uncertainty due to the finite number of quantities taken into account. The basic principles of the theory of measurement can and should be carried out separately at the subsequent stages of solving the

model and its computer implementation.

Thus, the reduction of the current uncertainty assigned to the recommended G value is possible using *only electromechanical measurement methods* in the near future.

In the future, research work will be directed to clarify the feasibility of analyzing the measurements of physical constants by the information method in order to confirm the following statements:

- the use of models with a class of phenomena $CoP_{St} \equiv LMT$ is not recommended due to the very small number of quantities that must be taken into account in order to achieve minimal comparative uncertainty;
- declaring only one value of relative uncertainty when measuring physical constants by different methods, within the framework of the information-oriented method, is inappropriate, since different comparative uncertainties and a different number of quantities that are recommended to choose are inherent in models corresponding to different classes of phenomena.

It is hoped that the search for a more accurate G will be crowned with success, and a reason will be found for the divergence of G measurements when using the concept of an information-oriented method. As a tool, this method has extraordinary potential.

The author hopes that his works will inspire others to revise the value of the statistical expert approach that exists when analyzing measurements of the gravitational constant, even if it has already been materialized in the corresponding CODATA solutions.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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