

# MHD Effects on Mixed Convective Nanofluid Flow with Viscous Dissipation in Surrounding Porous Medium

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## Abstract

In existence of concerning magnetic field, heat together with mass transfer features on mixed convective copper-water nanofluid flow through inclined plate is investigated in surrounding porous medium together with viscous dissipation. A proper set of useful similarity transforms is considered as to transform the desired governing equations into a system as ordinary differential equations which are nonlinear. The transformed equations for nanofluid flow include interrelated boundary conditions which are resolved numerically applying Runge-Kutta integration process of sixth-order together with Nachtsheim and Swigert technique. The numerical consequences are compared together with literature which was published previously and acceptable comparisons are found. The influence of significant parameters like as magnetic parameter, angle for inclination, Eckert number, fluid suction parameter, nanoparticles volume fraction, Schmidt number and permeability parameter on concerning velocity, temperature along with concentration boundary layers remains examined and calculated. Numerical consequences are presented graphically. Moreover, the impact regarding these physical parameters for engineering significance in expressions of local skin friction coefficient in addition to local Nusselt together with Sherwood numbers is correspondingly examined.

## Keywords

MHD, Mixed Convection, Nanofluid, Porous Medium, Viscous Dissipation

## 1. Introduction

Magnetic fluids which are types of particular nanofluids took advantage of mag-

netic material goods of nanoparticles inside as for example liquid rotary seals functioning with no maintenance along with tremendously low leakage in very extensive variety of applications. Magneto hydrodynamics (MHD) nanofluid flows have broadly uses of MHD generators, tunable optical fiber, optical grating, optical modulators and switches, polymer, petroleum technologies and then metallurgical industries.

Heat transfer enhancement concerning boundary layer fluid flow for different nanofluid flow passes through a vertical plate that has been studied with steady case via Rana and Bhargava [1] including with the effect such as temperature dependent heat source/sink. For different nanofluids, average of Nusselt number was found to diminish by them. The influence regarding viscous dissipation, chemical reaction together with Soret in existence of magnetic field into nanofluids flow has been considered pass through porous media through Yohannes and Shankar [2]. The Keller box process was used to solve governing equation of the concerning fluid flow, and numerical outcomes were presented for various parameters of convective heat together with mass transfer properties.

Magnetic field including thermal radiation effects for nanofluid flow has been analyzed along stretching surface through Khan *et al.* [3]. The flow field was discussed by them with dissimilar time steps and reported that average shear stress reductions with the development of magnetic field are observed. MHD boundary layer nanofluid flow regarding heat with mass transfer has been stated through porous media by Haile and Shankar [4] with considering thermal radiation including viscous dissipation with chemical reaction effects. They were considered copper (Cu)-water and  $\text{Al}_2\text{O}_3$ -water nanofluids and noted out that velocity field decreases with increase of magnetic field.

For considering the steady case, MHD mixed convective nanofluid flow through porous medium which has been deliberated past along a stretching sheet by Ferdows *et al.* [5]. They concluded that velocity together with temperature increases while concentration decreases gradually with increase of Eckert number. Heat transfer physical characteristics of flow field with three dissimilar categories of nanofluid pass through permeable stretching/shrinking surface has been considered and observed through porous medium via Pal *et al.* [6]. They found with the increment of suction/injection parameters as local Nusselt number rises for stretching sheet while decreasing for shrinking sheet.

Through inclined porous plate, magneto hydrodynamic mixed convective flow including Joule heating together with viscous dissipation on the field has been studied via Das *et al.* [7]. The velocity along with temperature of flow fields rise due to rise of particular magnetic field which was obtained by them. Within a porous medium, MHD mixed convection happening on fluid flow has been inspected and analyzed toward a vertical plate through Hari *et al.* [8] with the radiation, including heat generation with presence of chemical reaction effect. On increasing particular magnetic field, velocity profile overshoots adjacent the plate surface and then convergence to the boundary which was found by them.

On boundary layer flow, MHD effects toward exponentially shrinking sheet have been analyzed by Jain and Choudhary [9]. The numerical outcomes were observed graphically and then explored through them. For MHD Williamson nanofluid flow, effects regarding chemical reaction, melting and radiation through porous medium have been examined and discussed with specific physical parameters by Krishnamurthya *et al.* [10]. Considering Brinkman nature nanofluid in unsteady magnetohydrodynamic flow towards on vertical plate has been discussed into porous medium via Ali *et al.* [11]. Fluid velocity diminishes with the enhancement of such nanoparticle volume fraction which is noticed by them. Through porous media, MHD flow regarding nanofluid with characteristic of heat including mass transfer has been reported along stretching sheet via Reddy and Chamkha [12]. The velocity reduces while temperature together with concentration enhances as per magnetic parameter increases which were found by them. For unsteady case, MHD along with free convective nanofluid flow pass through a flat plate has been stated with radiation absorption by Prasad *et al.* [13] and influence of regarding several physical factors on flow field was studied by them.

Double diffusive magnetohydrodynamic nanofluids flow together with effects of thermal radiation including viscous-Ohmic dissipation has been discussed along nonlinear stretching/shrinking sheet via Pal and Mandal [14]. In existence of magnetohydrodynamics, unsteady natural convective regarding nanofluid flow through above a porous vertical plate has been studied for three dissimilar categories of nanofluids by Geetha *et al.* [15]. The numerical outcomes were analyzed and then presented for numerous physical factors which are concerned by them. A mathematical relation for two dimensional flow of magnetic Maxwell nanofluid subject to heat sink/source influenced by a stretched cylinder has been attained by Irfan *et al.* [16]. As stated aforementioned, it remains more practical to include MHD effects to study the impact regarding momentum including heat with mass transfer flow, and to the best of author's knowledge; no investigation is considered as MHD effects on mixed convective through nanofluid flow along viscous force in surrounding porous medium.

Therefore, in the light of above literatures, the purpose of the current work is to observe MHD effects on mixed convective nanofluid flow along with viscous dissipation in surrounding porous medium. The governing equations for considered nanofluid flow are converted into a combination as nonlinear ordinary differential equations via introducing similarity variables and solved numerically. Moreover, for cogency of numerical results, a comparison is prepared with the literature which is published and comparatively satisfactory comparison is achieved. The terms of engineering interests such as wall shear stress including rate of heat transfer together with rate of mass transfer are shown into tabular form. The influence for various physical features as magnetic parameter, angle of inclination, Eckert number and fluid suction parameter are presented on the field of flow and analyzed thereafter.

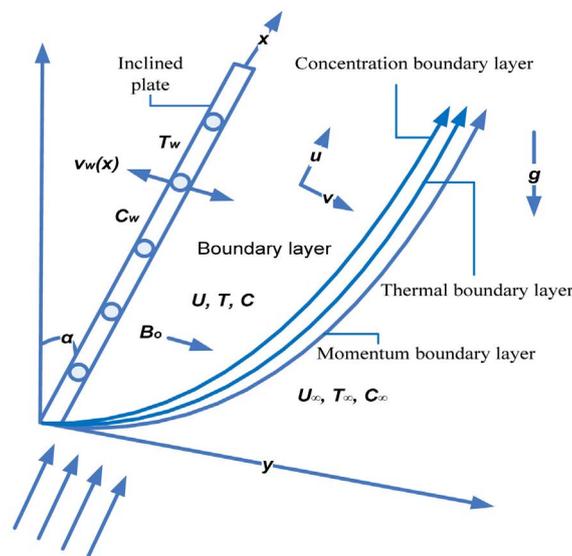
## 2. Mathematical Model

The two dimensional incompressible nanofluid flows which is steady and viscous is considered for analysis. The nanofluid flow is passing inclined porous plate in surrounding porous medium. In addition to consider magnetic field  $B_0$ , this is introduced normally to direction of nanofluid flow. For two dimensional coordinate system which is shown as below in **Figure 1**,  $(x, y)$  is (direction along of flow, direction normal of flow) and  $(u, v)$  is (velocity components of  $x$ , velocity components of  $y$ ). It is supposed that  $U_\infty$  denotes free stream velocity of flow field;  $g$  denotes the gravity by virtue of acceleration and  $\alpha$  denotes angle to vertical porous plate. Moreover, the temperature  $T_w$  at the wall is larger than ambient temperature  $T_\infty$  while concentration  $C_w$  at the wall is larger than ambient concentration  $C_\infty$ .

Consequently under the above flow field consideration, conservation law for mass is obeyed automatically as given below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The common fluid as water is considered such as base fluid while copper (Cu) is considered such as nanoparticles for flow field together with both are locally thermal equilibrium. The thermophysical properties which are used for nanofluid are specified into **Table 1**.



**Figure 1.** The diagram of flow configuration.

**Table 1.** Thermo physical properties of base fluid and nanoparticle.

Particles	Thermo Physical Properties				
	$\rho$ (kg/m <sup>3</sup> )	$C_p$ (J/kg K)	$k$ (W/m K)	$\beta \times 10^{-5}$ (1/K)	$\sigma$ (S/m)
Water (H <sub>2</sub> O)	997.1	4179	0.613	21	$5.5 \times 10^{-6}$
Copper (Cu)	8933	385	401	1.67	$59.6 \times 10^{-6}$

Using the present physical features, the modified dimensional boundary layer equations of Pal and Mandal [17] as well as Bachok *et al.* [18] for describing nanofluid flow field in expressions of conservation equations such as momentum, energy together with concentration equations can be expressed such as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} + \frac{g}{\rho_{nf}} \left\{ (\rho\beta_t)_{nf} (T - T_\infty) + (\rho\beta_c)_{nf} (C - C_\infty) \right\} \cos(\alpha) - \left( \frac{\sigma_{nf} B_0^2}{\rho_{nf}} + \frac{\nu_{nf}}{k_{pp}} \right) (u - U_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho C_p)_{nf}} \left[ \mu_{nf} \left\{ u \frac{\partial^2 u}{\partial y^2} + \left( \frac{\partial u}{\partial y} \right)^2 \right\} + \sigma_{nf} B_0^2 (u - U_\infty)^2 \right] \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

where for the nanofluid,  $\nu_{nf}$  is the kinematic viscosity,  $(\beta_t)_{nf}$  and  $(\beta_c)_{nf}$  are the coefficient for thermal and concentration expansions,  $g$  is the acceleration owing to gravity,  $\alpha$  is the angle for inclination,  $\sigma_{nf}$  is the electrical thermal conductivity,  $\rho_{nf}$  is the density,  $\alpha_{nf}$  is the thermal diffusivity and  $(\rho C_p)_{nf}$  is the heat capacitance. Furthermore,  $k_{pp}$  is permeability regarding porous medium whereas  $D$  is mass diffusivity.

The operational dynamic viscosity for nanofluid was given via Brinkman [19] while relation between physical quantities regarding nanoparticle and base fluid which were familiarized by Abu-Nada [20] as:

$$\begin{aligned} \mu_{nf} &= \frac{\mu_{bf}}{(1-\phi)^{2.5}}, \rho_{nf} = (1-\phi)\rho_{bf} + \phi\rho_{np}, \\ (\rho\beta_t)_{nf} &= (1-\phi)(\rho\beta_t)_{bf} + \phi(\rho\beta_t)_{np}, \\ (\rho\beta_c)_{nf} &= (1-\phi)(\rho\beta_c)_{bf} + \phi(\rho\beta_c)_{np}, \\ (\rho C_p)_{nf} &= (1-\phi)(\rho C_p)_{bf} + \phi(\rho C_p)_{np}, \\ \nu_{nf} &= \frac{\mu_{nf}}{\rho_{nf}}, \frac{K_{nf}}{K_{bf}} = \frac{(K_{np} + 2K_{bf}) - 2\phi(K_{bf} - K_{np})}{(K_{np} + 2K_{bf}) + \phi(K_{bf} - K_{np})}, \alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}}, \\ \frac{\sigma_{nf}}{\sigma_{bf}} &= 1 + \frac{3 \left( \frac{\sigma_{np}}{\sigma_{bf}} - 1 \right) \phi}{\left( \frac{\sigma_{np}}{\sigma_{bf}} + 2 \right) - \left( \frac{\sigma_{np}}{\sigma_{bf}} - 1 \right) \phi} \end{aligned} \quad (5)$$

where,  $\mu_{bf}$  is considered for dynamic viscosity while  $\phi$  is considered for nanoparticle solid volume fraction. The subscripts in aforementioned equations *bf* and *np* symbolize namely base fluid and nanoparticle respectively.

The accompanying boundary conditions for the existing nanofluid flow field are as follows:

$$u = 0, v = \pm v_w(x), T = T_w \text{ and } C = C_w \text{ at } y = 0 \quad (6)$$

$$u = U_\infty, T = T_\infty \text{ and } C = C_\infty \text{ at } y \rightarrow \infty \quad (7)$$

where permeability of porous plate is as  $v_w(x)$  which is for suction ( $<0$ ) or blowing ( $>0$ ) whereas the subscripts in aforementioned equations  $w$  and  $\infty$  are denoted wall and boundary layer edges respectively.

The well-known stream function is convenient to consider as equation of continuity is satisfied identically through following relations:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (8)$$

In the above,  $\psi$  is the stream function. To reduce complexity of nanofluid flow field, the succeeding dimensionless transformations which were introduced via Cebeci *et al.* [21] are used:

$$\eta = y \sqrt{\frac{U_\infty}{v_{bf} x}}, \quad \psi = \sqrt{v_{bf} x U_\infty} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

and  $s(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$  (9)

In the above,  $\eta$  is the similarity transform. The components for velocity of Equation (8) using Equation (9) can be re-written such as

$$u = U_\infty f'(\eta) \text{ and } v = \frac{1}{2} \sqrt{\frac{v_{bf} U_\infty}{x}} [\eta f'(\eta) - f(\eta)] \quad (10)$$

wherever the prime of aforementioned equation indicates as differentiation with respect to  $\eta$ .

Using similarity transformations in Equations (2) and (4) including the accompanying boundary conditions Equations (6) and (7), the transformed equations in which momentum, energy together with concentration equations are as follows:

$$f''' + \phi_1 \left\{ \frac{1}{2} \phi_2 f f'' + (\phi_3 Ri_t \theta + \phi_4 Ri_c s) \cos(\alpha) \right\} + (\phi_5 \phi_6 M + K)(1 - f') = 0 \quad (11)$$

$$\theta'' + \frac{1}{2} \left( \frac{Pr}{\phi_6} \right) \left\{ \phi_7 f \theta' + \frac{2}{\phi_1} Ec (f' f''' + f''^2) + 2 \phi_6 MEc (f' - 1)^2 \right\} = 0 \quad (12)$$

$$s'' + \frac{1}{2} Sc f s' = 0 \quad (13)$$

and corresponding boundary conditions are supposed as below:

$$f = f_w, \quad f' = 0, \quad \theta = 1, \quad s = 1, \quad \text{at } \eta = 0 \quad (14)$$

$$f' \rightarrow 1, \quad \theta \rightarrow 0 \text{ and } s \rightarrow 0 \text{ at } \eta \rightarrow \infty \quad (15)$$

In the aforementioned Equation (14), the coefficient for wall mass transfer is

$$f_w = -v_w(x) \sqrt{\frac{x}{v_{bf} U_\infty}} \quad (16)$$

such as  $f_w > 0$  for suction and  $f_w < 0$  for injection or blowing while physical parameters are defined as under:

$$\begin{aligned}
Gr_t &= \frac{g(\beta_t)_{bf} (T_w - T_\infty) x^3}{\nu_{bf}^2}, Gr_c = \frac{g(\beta_c)_{bf} (C_w - C_\infty) x^3}{\nu_{bf}^2}, Re_x = \frac{xU_\infty}{\nu_{bf}}, \\
Ri_t &= \frac{Gr_t}{(Re_x)^2}, Ri_c = \frac{Gr_c}{(Re_x)^2}, M = \frac{\sigma_{bf} B_0^2 x}{\rho_{bf} U_\infty}, K = \frac{x\nu_{bf}}{k_{pp} U_\infty}, Pr = \frac{\nu_{bf} (\rho C_p)_{bf}}{K_{bf}}, \\
Ec &= \frac{U_\infty^2}{(C_p)_{bf} (T_w - T_\infty)}, Sc = \frac{\nu_{bf}}{D}, \phi_1 = (1-\phi)^{2.5}, \phi_2 = (1-\phi) + \phi \frac{\rho_{np}}{\rho_{bf}}, \\
\phi_3 &= (1-\phi) + \phi \frac{(\rho\beta_t)_{np}}{(\rho\beta_t)_{bf}}, \phi_4 = (1-\phi) + \phi \frac{(\rho\beta_c)_{np}}{(\rho\beta_c)_{bf}}, \phi_5 = \frac{\sigma_{nf}}{\sigma_{bf}}, \phi_6 = \frac{K_{nf}}{K_{bf}} \\
\text{and } \phi_7 &= (1-\phi) + \phi \frac{(\rho C_p)_{np}}{(\rho C_p)_{bf}}
\end{aligned} \tag{17}$$

In the aforementioned Equations (11)-(13),  $Gr_t$  is the local thermal Grashof number,  $Gr_c$  is the local mass Grashof number,  $Re_x$  the local Reynolds number,  $Ri_t$  is represented as the local Richardson number of thermal,  $Ri_c$  is denoted as the local Richardson number of mass,  $K$  is indicated as the parameter of permeability,  $M$  is designated as magnetic parameter,  $Ec$  is denoted as Eckert number,  $Sc$  is indicated as the Schmidt number whereas  $\phi_i$  ( $i = 1, 2, \dots, 7$ ) are constants.

The parameters which are the engineering interest are skin-friction coefficient, local Nusselt together with Sherwood numbers. The nondimensional form of local skin-friction coefficient  $C_f$  is

$$C_f = \frac{2}{\phi_1} (Re_x)^{-\frac{1}{2}} f''(0) \tag{18}$$

Furthermore, using the thermophysical property of nanofluid, the local Nusselt number is converted through the resulting form

$$Nu_x = -\phi_6 (Re_x)^{\frac{1}{2}} \theta'(0) \tag{19}$$

However, local Sherwood number is transformed into the succeeding form as:

$$Sh = -(Re_x)^{\frac{1}{2}} s'(0) \tag{20}$$

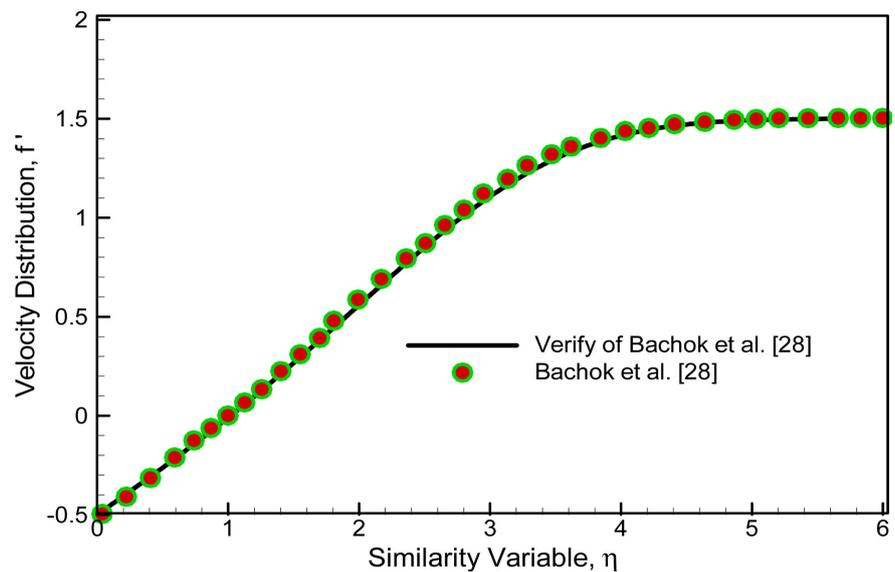
### 3. Procedures of Numerical Solutions Using Nachtsheim and Swigert Technique

The system regarding nonlinear boundary value problem which was represented by Equations (11)-(13), together with corresponding boundary conditions Equations (14) and (15) is solved using Nachtsheim and Swigert [22] technique which is used to find unspecific initial conditions. The transformed boundary value system is re-transformed to initial value system and solved numerically employing sixth order Runge-Kutta initial value solver. The useful numerical methods were described by Nachtsheim and Swigert [22] and sixth order Runge-Kutta initial value solver by Al-Shimmary [23].

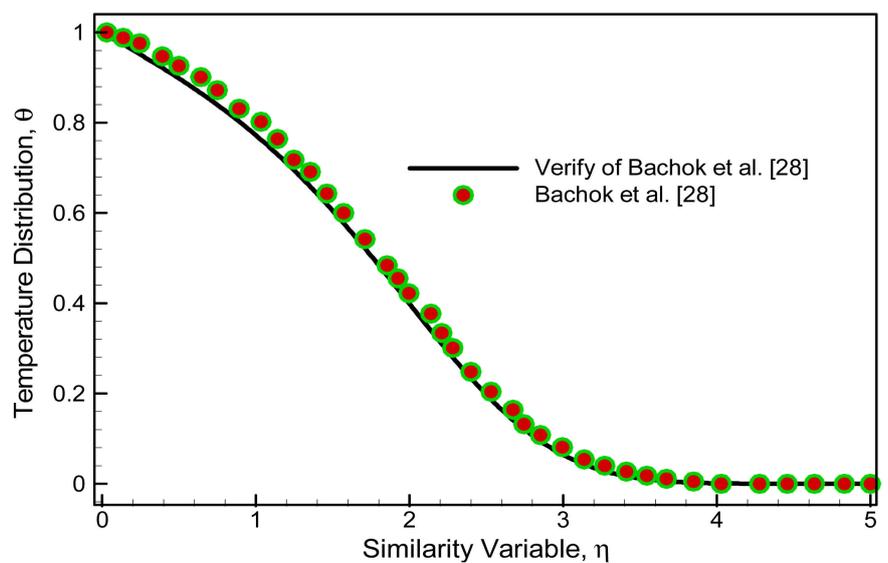
#### 4. Comparison of Numerical Results

For the accurateness of numerical results, comparisons are prepared considering the special effects regarding velocity ratio parameter  $\lambda$  on velocity and temperature profiles. The results for Bachok *et al.* [18] which was published is compared with verify of Bachok *et al.* [18] work by authors for copper (Cu)-water nanofluid flow along with Prandtl number as  $Pr = 6.2$ , nanoparticle volume fraction as  $\phi = 0.1$  and velocity ratio parameter as  $\lambda = -0.5$  as shown into **Figure 2**.

It is detected that, verified corresponding numerical results of first solution are found excellent agreement. This favorable acceptable comparison indication



(a)



(b)

**Figure 2.** Comparison of first solution of (a) velocity distribution and (b) temperature distribution for copper (Cu)-water nanofluid flow while  $Pr = 6.2$ ,  $\phi = 0.1$ ,  $\lambda = -0.5$  and  $M = 0$ .

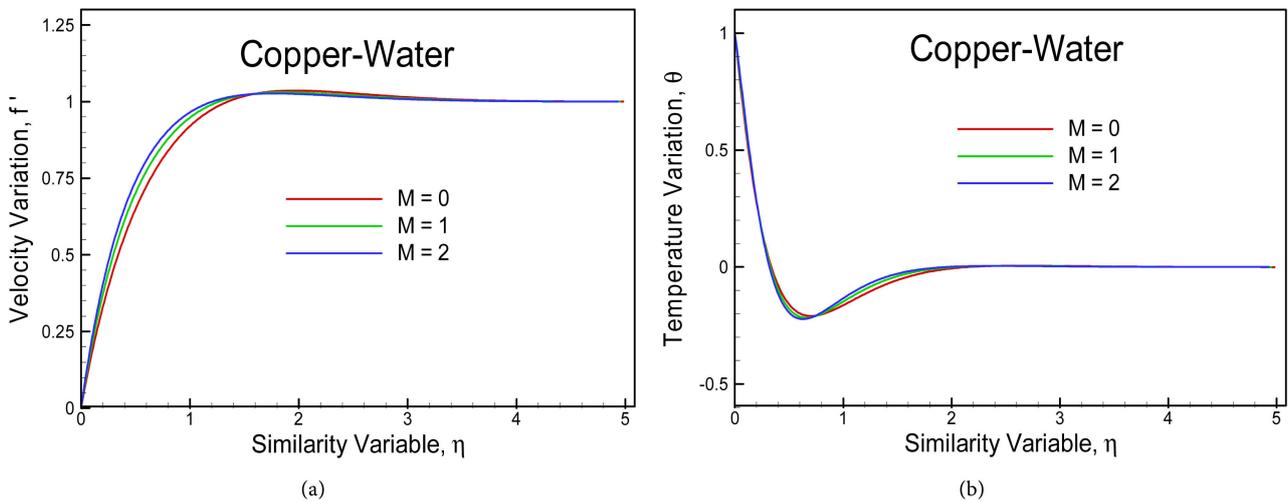
is to improve numerical approach of Nachtsheim and Swigert [22] for present work. As well, the obtained numerical results for present work will report graphically into next subsequent section and analyze thereafter.

## 5. Results and Discussion of Significant Parameters

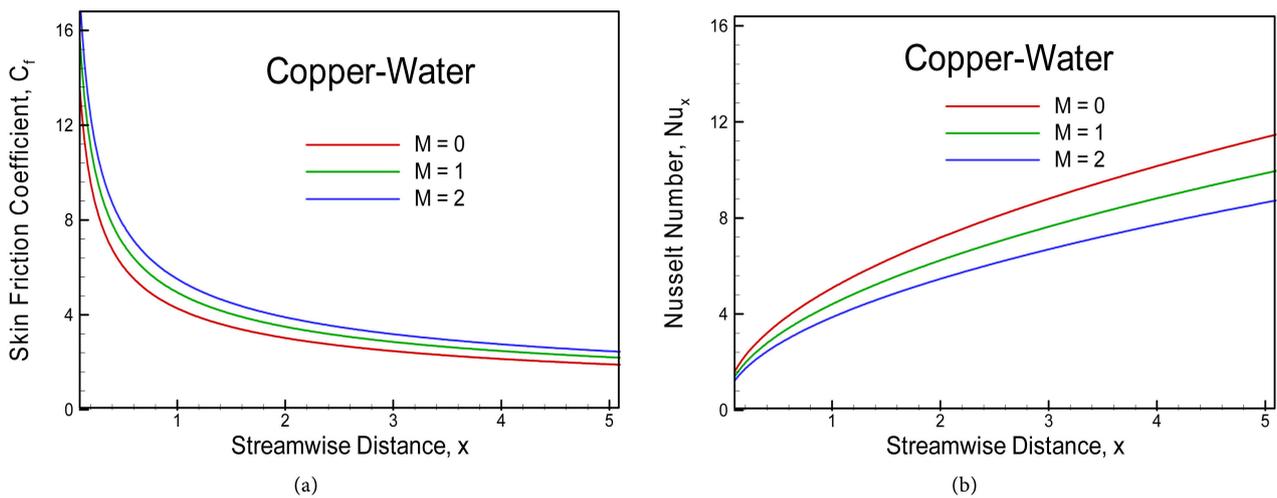
The comprehensive numerical results are computed for water (H<sub>2</sub>O)-copper (Cu) nanofluid flow including dissimilar values of nondimensional parameters that described flow characteristics of the nanofluid. The influences of namely magnetic parameter  $M$ , angle for inclination  $\alpha$ , Eckert number  $Ec$ , fluid suction parameter  $f_w$ , nanoparticles volume fraction  $\phi$ , Schmidt number  $Sc$  and permeability parameter  $K$  are analyzed to describe flow characteristics. The values for nondimensional parameters  $R_{it} = 1$ ,  $R_{ic} = 1$ ,  $M = 0.5$ ,  $K = 0.5$ ,  $\alpha = 30^\circ$ ,  $Pr = 6.2$ ,  $Ec = 0.5$ ,  $Sc = 0.60$ ,  $f_w = 1.5$ ,  $\phi = 0.03$  and  $U_\infty/\nu = 1.0$  are considered except otherwise specified. The selective results are shown graphically with velocity, temperature together with concentration flow fields. Moreover, the interest of engineering terms as local skin friction coefficient  $C_f$  and local Nusselt number  $Nu_x$  together with local Sherwood number  $Sh$  are presented graphically.

The impacts for magnetic parameter namely  $M$  ( $M = 0, 1$  and  $2$ ) on the copper (Cu) and water (H<sub>2</sub>O) nanofluid flow fields are shown in **Figure 3(a)** and **Figure 3(b)** and **Figure 4(a)** and **Figure 4(b)**. The fluid velocity for nanofluid flow rises because of rise of concerning magnetic field as given away in **Figure 3(a)** and alike result was established through Das *et al.* [7]. Because, as magnetic field strength increases; the Lorentz force regarding magnetic field creates boundary layer for nanofluid flow as thinner. On free stream velocity, magnetic lines of forces move pass through the plate. The fluid which is decelerated by the viscous force, receives an impulsion from magnetic field which counteracts a viscous effects. As results, velocity of considered nanofluid flow rises and converges to the boundary together with rising magnetic field parameter. Additionally as detected in **Figure 3(b)**, the thickness of thermal boundary layer of considered nanofluid flow rises owing to rise magnetic field because functional magnetic field which has a tendency to heat the fluid owing to electromagnetic work reduces heat transfer to the wall. Furthermore, skin friction coefficient namely  $C_f$  is established to increases with increase magnetic field strength as given away in **Figure 4(a)**. The reason for this is that, functioning magnetic field regarding nanofluid flow tends to improve flow motion and thus to improve surface friction force. In existence of growing magnetic field strength, the temperature gradient decreases at the wall which in turn leads to a reduction in rate of heat transfer. Subsequently,  $Nu_x$  namely local Nusselt number which is schemed in **Figure 4(b)** decreases together with rise magnetic field strength.

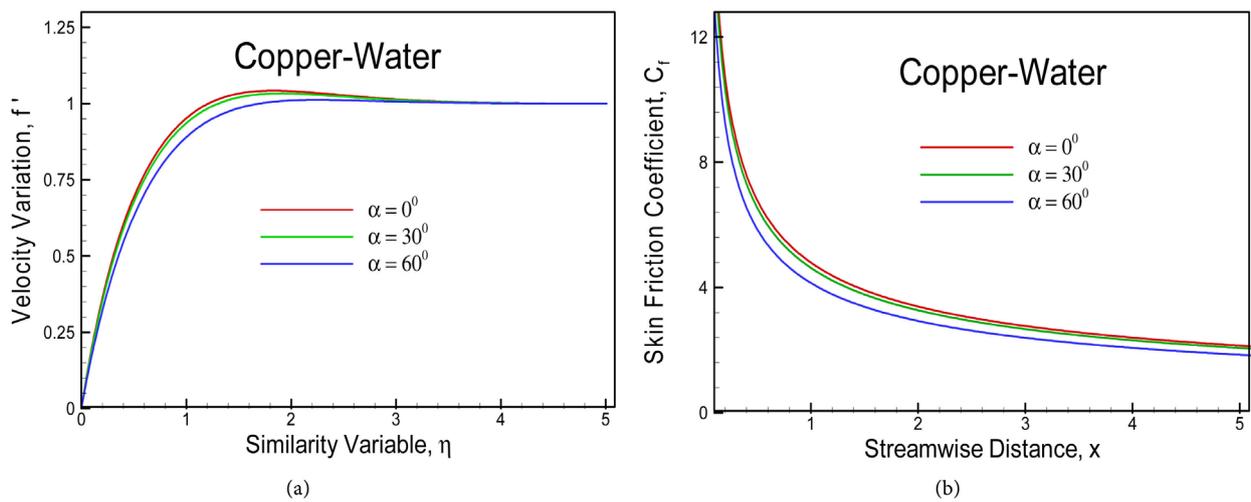
The influence for  $\alpha$  ( $\alpha = 0^\circ, 30^\circ$  and  $60^\circ$ ) which is angle for inclination for the copper (Cu) and water nanofluid flow fields is revealed in **Figure 5(a)** and **Figure 5(b)**. The angle regarding inclination is only in momentum Equation (12) by  $\cos\alpha$ . For  $\alpha = 0^\circ$ , plate assumes a vertical position whereas plate is horizontal



**Figure 3.** Variation of (a) velocity and (b) temperature for effect of magnetic parameter  $M$ .



**Figure 4.** Effects of magnetic parameter  $M$  on (a) local skin friction coefficient and (b) local Nusselt number against the streamwise distance.

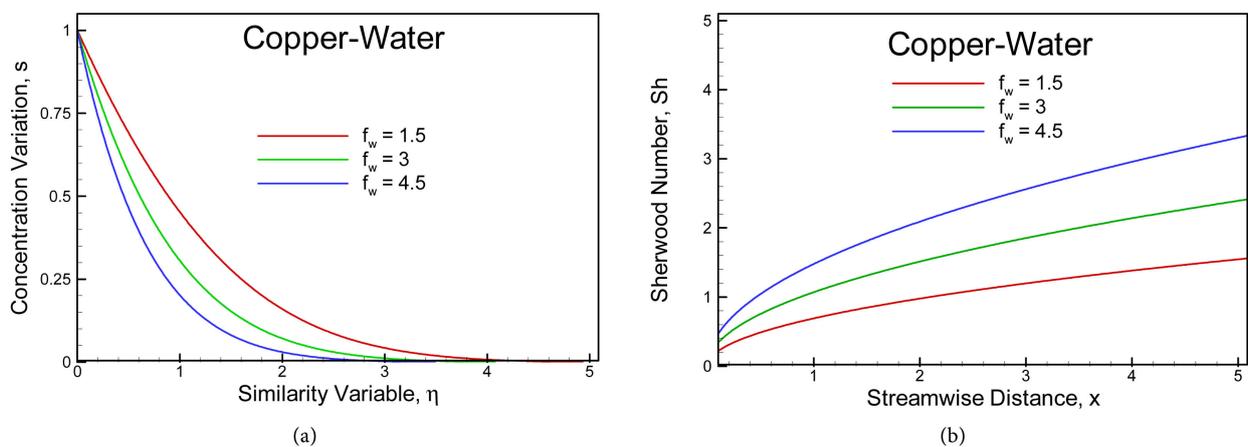


**Figure 5.** Effects of angle of inclination  $\alpha$  on (a) velocity and (b) local skin friction coefficient against the streamwise distance.

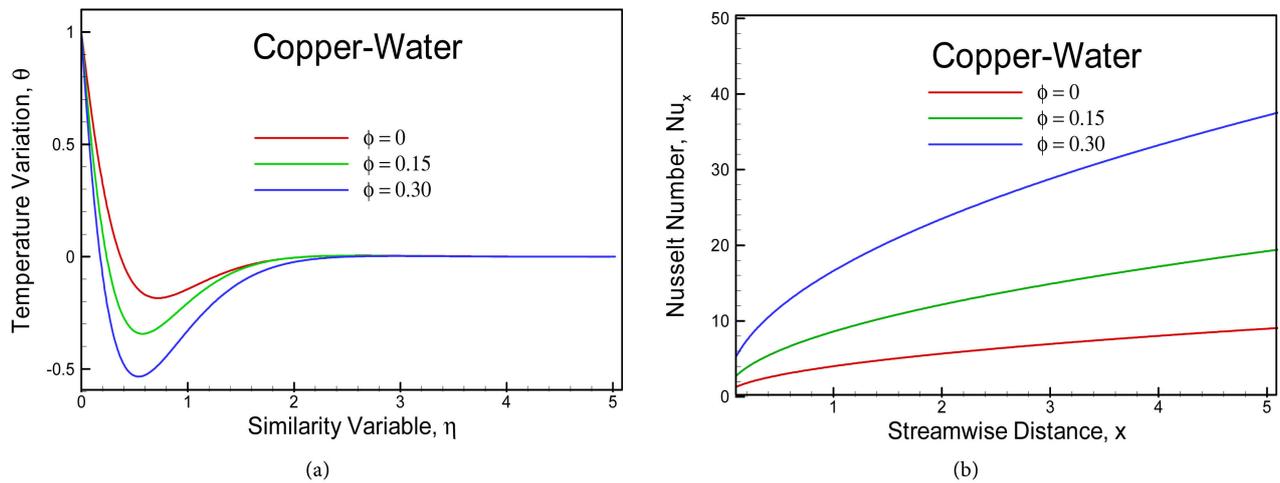
for  $\alpha = 90^\circ$ . The gravitational effects is maximum at a vertical position whereas minimum at horizontal position. Consequently, momentum boundary layer nanofluid flow decreases to increase of  $\alpha$ . As a consequence this is observed into **Figure 5(a)**, maximum velocity of nanofluid flow is found at vertical position while minimum velocity is found at horizontal position. Moreover, when, the angle regarding inclination  $\alpha$  increases, velocity of nanofluid flow decreases and so its shear stress decreases. As per significance, local skin friction coefficient namely  $C_f$  of nanofluid declines which is detected in **Figure 5(b)**.

The variation of velocity, temperature together with concentration of water-copper nanofluid flow for various values regarding fluid suction parameter  $f_w$  ( $f_w = 1.5, 3.0$  and  $4.5$ ) are given away in **Figure 6(a)** and **Figure 6(b)**. Simultaneously, as fluid suction of nanofluid increases then much number of nanofluid passes through plate and subsequently concentration regarding nanofluid flow field decreases at all points whereas local Sherwood number  $Sh$  increases which are finding out in **Figure 6(a)** and **Figure 6(b)**. The effects of nanoparticles volume fraction  $\phi$  ( $\phi = 0, 0.15$  and  $0.30$ ) for water-Cu nanofluid flow are analyzed in **Figure 7(a)** and **Figure 7(b)**. With the increasing of nanoparticles volume fraction  $\phi$ , the thermal boundary layer breadth of nanofluid flow decreases as seen in **Figure 7(a)**. It is also observed from **Figure 7(b)** that with increase of  $\phi$ , local Nusselt number  $Nu_x$  increases for as much the temperature gradient at the wall increased and as a result  $Nu_x$  increases.

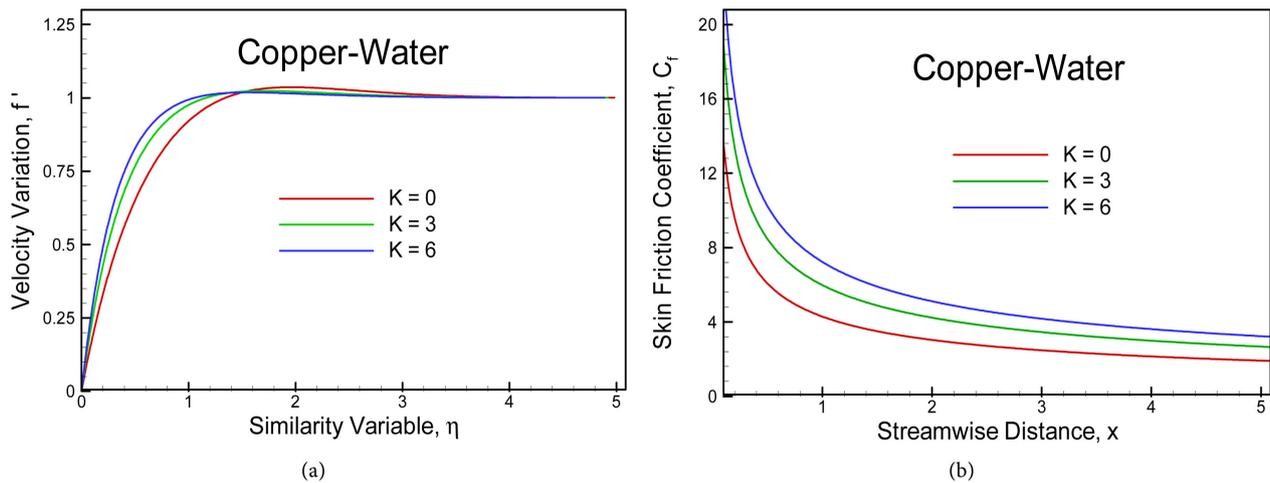
On the other hand, the effect of the permeability parameter  $K$  ( $K = 0, 3$  and  $6$ ) on the momentum boundary layer as well as local skin friction coefficient  $C_f$  are observed in **Figure 8(a)** and **Figure 8(b)**. On the basis of the velocity variation in **Figure 8(a)**, it is seen that the velocity of the flow field within the velocity boundary layer increases near the plate while decreases in the next and finally converges to the boundary condition as the permeability of the porous medium increases. However, **Figure 8(b)** shows the influence of permeability of the porous medium on the local skin friction coefficient against the streamwise distance  $x$ . In **Figure 8(b)**, it is found that the entire local skin friction coefficient increases with the increase of permeability of the porous medium parameter.



**Figure 6.** Effects of fluid suction  $f_w$  on (a) concentration and (b) local Sherwood number  $Sh$  against the streamwise distance.



**Figure 7.** Effects of nanoparticles volume fraction  $\phi$  on (a) temperature and (b) local Nusselt number  $Nu_x$  against the streamwise distance.



**Figure 8.** Effects of permeability parameter  $K$  on (a) velocity and (b) skin friction coefficient  $C_f$  against the streamwise distance.

## 6. Conclusions

The effect of magnetohydrodynamic on mixed convective nanofluid flow with viscous dissipation has been examined in surrounding porous medium. Consequently, the concerning governing equations regarding nanofluid flow are converted into nonlinear boundary layer equations by proper similarity transformations and solved numerically through sixth-order Runge-Kutta method with Nachtsheim and Swigert [22] approaches. For validity of numerical results, comparisons of numerical results remain prepared with Bachok *et al.* [18] and a comparatively acceptable comparison is reached. The concerning numerical results mainly calculated for influence of magnetic parameter  $M_i$ , angle for inclination  $\alpha$ , Eckert number  $Ec$ , fluid suction parameter  $f_w$ , nanoparticles volume fraction  $\phi$ , Schmidt number  $Sc$  and permeability parameter  $K$  on flow field.

The following results on velocity, temperature together with concentration of nanofluid flow fields including local skin friction coefficient  $C_f$ , local Nusselt

number  $Nu_x$  together with local Sherwood number  $Sh$  is deduced from the present analysis:

- The skin friction coefficient increases but local Nusselt number decreases due to increasing values for magnetic parameter.
- The local skin friction coefficient shows diminishing behavior for rising values angle of inclination.
- The local Sherwood number of nanofluid flow displays raising behavior for raising values of fluid suction parameter.
- The local Nusselt number of nanofluid flow shows raising behavior for raising values of nanoparticles volume fraction.
- The skin friction coefficient of nanofluid flow increases for increasing value of permeability parameter.

### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

### References

- [1] Rana, P. and Bhargava, R. (2011) Numerical Study of Heat Transfer Enhancement in Mixed Convection Flow along a Vertical Plate with Heat Source/Sink Utilizing Nanofluids. *Communications in Nonlinear Science and Numerical Simulation*, **16**, 4318-4334. <https://doi.org/10.1016/j.cnsns.2011.03.014>
- [2] Yohannes, K.Y. and Shankar, B. (2013) Heat and Mass Transfer in MHD Flow of Nanofluids through a Porous Media Due to a Stretching Sheet with Viscous Dissipation and Chemical Reaction Effects. *Caribbean Journal of Science and Technology*, **1**, 1-17.
- [3] Khan, M.S., Alam, M.M. and Ferdows, M. (2013) Effects of Magnetic Field on Radiative Flow of a Nanofluid Past a Stretching Sheet. *Procedia Engineering*, **56**, 316-322. <https://doi.org/10.1016/j.proeng.2013.03.125>
- [4] Haile, E. and Shankar, B. (2014) Heat and Mass Transfer through a Porous Media of MHD Flow of Nanofluids with Thermal Radiation, Viscous Dissipation and Chemical Reaction Effects. *American Chemical Science Journal*, **4**, 828-846. <https://doi.org/10.9734/ACSJ/2014/11082>
- [5] Ferdows, M., Khan, M.S., Alam, M.M. and Sun, S. (2012) MHD Mixed Convective Boundary Layer Flow of a Nanofluid through Porous Medium Due to an Exponentially Stretching Sheet. *Mathematical Problems in Engineering*, **2012**, Article ID: 408528. <https://doi.org/10.1155/2012/408528>
- [6] Pal, D., Mandal, G. and Vajravelu, K. (2014) Flow and Heat Transfer of Nanofluids at a Stagnation Point Flow over a Stretching/Shrinking Surface in a Porous Medium with Thermal Radiation. *Applied Mathematics and Computation*, **238**, 208-224. <https://doi.org/10.1016/j.amc.2014.03.145>
- [7] Das, S., Jana, R.N. and Makinde, O.D. (2015) Magnetohydrodynamic Mixed Convective Slip Flow over an Inclined Porous Plate with Viscous Dissipation and Joule Heating. *Alexandria Engineering Journal*, **54**, 251-261. <https://doi.org/10.1016/j.aej.2015.03.003>
- [8] Hari, N., Sivasankaran, S., Bhuvanewari, M. and Siri, Z. (2015) Effects of Chemical Reaction on MHD Mixed Convection Stagnation Point Flow toward a Vertical Plate

- in a Porous Medium with Radiation and Heat Generation. *Journal of Physics: Conference Series*, **662**, 12-24. <https://doi.org/10.1088/1742-6596/662/1/012014>
- [9] Jain, S. and Choudhary, R. (2015) Effects of MHD on Boundary Layer Flow in Porous Medium Due to Exponentially Shrinking Sheet with Slip. *Procedia Engineering*, **127**, 1203-1210. <https://doi.org/10.1016/j.proeng.2015.11.464>
- [10] Krishnamurthya, M.R., Gireesha, B.C. and Gorla, R.S.R. (2016) Effects of Chemical Reaction on MHD Boundary Layer Flow and Melting Heat Transfer of Williamson Nanofluid in Porous Medium. *Engineering Science and Technology: An International Journal*, **19**, 53-61. <https://doi.org/10.1016/j.jestch.2015.06.010>
- [11] Ali, F., Gohar, M. and Khan, I. (2016) MHD Flow of Water-Based Brinkman Type Nanofluid over a Vertical Plate Embedded in a Porous Medium with Variable Surface Velocity, Temperature and Concentration. *Journal of Molecular Liquids*, **223**, 412-419. <https://doi.org/10.1016/j.molliq.2016.08.068>
- [12] Reddy, P.S. and Chamkha, A.J. (2016) Soret and Dufour Effects on MHD Convective Flow of  $\text{Al}_2\text{O}_3$ -Water and  $\text{TiO}_2$ -Water Nanofluids Past a Stretching Sheet in Porous Media with Heat Generation/Absorption. *Advanced Power Technology*, **27**, 1207-1218. <https://doi.org/10.1016/j.appt.2016.04.005>
- [13] Prasad, P.D., Kumar, R.V.M.S.S.K. and Varma, S.V.K. (2016) Heat and Mass Transfer Analysis for the MHD Flow of Nanofluid with Radiation Absorption. *Ain Shams Engineering Journal*, **9**, 801-813.
- [14] Pal, D. and Mandal, G. (2017) Double Diffusive Magnetohydrodynamic Heat and Mass Transfer of Nanofluids over a Nonlinear Stretching/Shrinking Sheet with Viscous-Ohmic Dissipation and Thermal Radiation. *Propulsion and Power Research*, **6**, 58-69. <https://doi.org/10.1016/j.jprr.2017.01.003>
- [15] Geetha, E., Muthucumaraswamy, R. and Kothandapani, M. (2017) Effects of Magneto Hydrodynamics of Unsteady Natural Free Convective Flow of Nanofluid on a Vertical Porous Plate. *International Journal of Pure and Applied Mathematics*, **116**, 79-86.
- [16] Irfan, M., Khan, M., Khan, W.A. and Ayaz, M. (2018) Modern Development on the Features of Magnetic Field on Heat Sink/Source in Maxwell Nanofluid Subject to Convective Heat Transport. *Physics Letters A*, **382**, 1992-2002. <https://doi.org/10.1016/j.physleta.2018.05.008>
- [17] Pal, D. and Mandal, G. (2014) Influence of Thermal Radiation on Mixed Convection Heat and Mass Transfer Stagnation-Point Flow in Nanofluids over Stretching/Shrinking Sheet in a Porous Medium with Chemical Reaction. *Nuclear Engineering and Design*, **273**, 644-652. <https://doi.org/10.1016/j.nucengdes.2014.01.032>
- [18] Bachok, N., Ishak, A. and Pop, I. (2012) Flow and Heat Transfer Characteristic on a Moving Plate in a Nanofluid. *International Journal of Heat and Mass Transfer*, **55**, 642-648. <https://doi.org/10.1016/j.ijheatmasstransfer.2011.10.047>
- [19] Brinkman, H.C. (1952) The Viscosity of Concentrated Suspension and Solution. *The Journal of Chemical Physics*, **20**, 571-581. <https://doi.org/10.1063/1.1700493>
- [20] Abu-Nada, E. (2008) Application of Nanofluids for Heat Transfer Enhancement of Separated Floes Encountered in a Backward Facing Step. *International Journal of Heat and Fluid Flow*, **29**, 242-249. <https://doi.org/10.1016/j.ijheatfluidflow.2007.07.001>
- [21] Cebeci, T. and Bradshaw, P. (1984) Physical and Computational Aspects of Convective Heat Transfer. Springer, New York. <https://doi.org/10.1007/978-3-662-02411-9>
- [22] Nachtsheim, P.R. and Swigert, P. (1965) Satisfaction of Asymptotic Boundary Con-

ditions in Numerical Solution of Systems of Nonlinear Equations of Boundary-Layer Type. NASA TND 3004.

- [23] Al-Shimmary, A.F.A. (2017) Solving Initial Value Problem Using Runge-Kutta 6th Order Method. *ARPN Journal of Engineering and Applied Sciences*, **12**, 3953-3961.