Addendum to “On an Intrinsically Local Gauge Symmetric SU(3) Field Theory for Quantum Chromodynamics”

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Abstract
A much simpler and self-consistent derivation of the non-linear component \( G_\mu \times G_\nu \) of the quantum chromodynamic SU(3) field tensor is given which does not require the postulate of color confinement to complete the derivation and which mirrors SU(2)’s formal development.

Keywords
SU(3) Lagrangian, Local Gauge Invariance, Quantum Chromodynamics, Normed Division Algebras

1. Introduction
In this author’s previously published, referenced paper [1], a derivation of the non-linear component \( G_\mu \times G_\nu \) of the SU(3) field tensor for quantum chromodynamics was given which was elaborate and which required the somewhat artificial postulate of color confinement to complete the derivation. A much simpler and mathematically direct derivation which does not rely on color confinement and which mirrors SU(2)’s development exists and is given herein. The mathematical methodology used is taken from the subject original paper, which is covered in detail therein [1].

2. The Derivation
The gauge field “cross product” for the non-linear term of the SU(3) field tensor has the form [1]

\[
(B \times C)_i = \sum f_{ijk} B_j C_k
\]  

(1)

See Ref. [1], Sec. 3.i.
where $i = 0 - 7$ and the $f_{ijk}$ are the structure constants of the Gell-Mann commutation relation $[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k$. A bijective relation between the Gell-Mann generators $\{\lambda_a\}$ and the octonion basis elements $\{e_a\}$ was given with structure constants existing for the terms [1]

$$
f_{ijk} \quad \forall ijk = 123, 147, 246, 257, 345, 165, 376;$$

$$
f_{ijk} \quad \forall ijk = 450, 670. $$

Using the formalism’s unique division-algebraic coupling equation [1]

$$
(v_a, v)(w_a, w) = \left(v_a w_o - v \cdot w, v_o w + w v_o + v \times w\right),
$$

we now consider the coupled operator $\eta \eta$ (where $\eta$ defines the involution $\eta = \gamma_0 - \gamma_{SU(3)}$ of $\eta = \gamma_0 + \gamma_{SU(3)}$) instead of the coupled operator $\eta \eta$ as was considered in the original paper. Setting $\gamma = \gamma_{SU(3)} = \gamma_a e_a; a = 1 - 7$, we have for the applicable vector portion $\eta \eta_{SU(3)} = v_o w + w v_o + v \times w$ of the coupled operator

$$
\eta \eta_{SU(3)} = \gamma_0 \gamma - \gamma_0 + (\gamma \times \gamma) = 2(\gamma_0 \times \gamma) + (\gamma \times \gamma),
$$

in which we have used $a \times b = \frac{1}{2} [a, b]$. As we are using the $\mathbb{O}$-based coupling equation, both terms of Equation (4) are 7-dimensional cross products.

The term $(\gamma \times \gamma)$ has components $f'_{ijk} \gamma \gamma'$. Since the 7-dim cross product only sums from $i = 1 - 7$, setting $f'_{ijk} = f_{ijk}$ only covers the structure constants $f_{ijk} \quad \forall ijk = 123, 147, 246, 257, 345, 165, 376$.

To cover the remaining $f_{ijk} \quad \forall ijk = 450, 670$ we look to the term $(\gamma_0 \times \gamma)$, which has components $c'_{iak} \gamma_0 \gamma'_k$. Recalling the total asymmetry of $f_{ijk}$, we simply set $c'_{iak} = -\frac{1}{2} f_{oak}$ for $ik = \{45, 67\}$ and $c'_{iak} = \frac{1}{2} f_{oak}$ for $ik = \{54, 76\}$, with $c'_{iak} = 0$ for all other $ik$ and the $\frac{1}{2}$ being required due to the 2 in $2(\gamma_0 \times \gamma)$.

The bijective mapping between eight Clifford fields $\hat{G}_j \hat{G}_k$ and the eight $SU(3)$ gauge fields $G_j G_k$ follows as in the original paper, with

$$
\sum d_{ijk} \gamma_0 \gamma_k \hat{G}_j \hat{G}_k \Leftrightarrow \sum f_{ijk} G_j G_k,
$$

$$
d_{ijk} = c'_{ijk} \quad \forall ijk = 450, 670;
$$

$$
d_{ijk} = f'_{ijk} \quad \forall ijk = 123, 147, 246, 257, 345, 165, 376;
$$

$$
d_{ijk} = 0 \quad \text{otherwise},
$$

thus generating the non-linear component $G_\mu \times G_\nu$.

3. Results and Discussion

The derivation herein of the non-linear portion of $SU(3)$’s field tensor is more direct and mathematically straightforward than the original paper’s derivation. Further, it mirrors the $SU(2)$ formalism’s use of $\eta \eta_{SU(3)}$ in generating the
$W_\mu \times W_\nu$ portion of the $SU(2)$ field tensor and does not require the somewhat artificial postulate of color confinement for the mathematical derivation. Lastly, given this derivation the previously established bijective relation between the octonion basis $\{e_a\}$ and the Gell-Mann generators $\{\lambda_\alpha\}$ [1] is now seen to be unnecessary and superfluous to the octonionic development of $SU(3)$ gauge theory, since the vector section $\eta_{SU(3)} = v_0 w + \nu \nu_0 + v \times w$ of Equation (3) generates the entirety of $SU(3)$'s Lie algebra structure constants while residing solely within the $\{e_a\}$ basis in doing so.

**Conflicts of Interest**

The authors declare no conflicts of interest regarding the publication of this paper.

**References**