

Designs of Two-Element Optical Refracting System to Achieve Uniform Laser Beam Profile

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Abstract

Various specific laser irradiance distribution outputs are needed in many applications. To fulfill this need, a detailed step-by-step design procedure for split refracting system is proposed for three types of laser beams transformation: 1) Annular-uniform-to-uniform; 2) Annular-Gaussian-to-uniform; and 3) Gaussian-to-uniform to obtain the required laser irradiance distributions. Mathematical expressions of the two Plano-aspheric surfaces are derived for each type. The proposed designs take into account few important parameters such as the system length, the surfaces radii of curvature, the annular beam starting cone angle, and the beams power ratio. Further, the proposed designs are much better than the ones, which were previously reported.

Keywords

Annular Laser Beam, Refracting System, Lens Design

1. Introduction

The TEM₀₀ laser type produces output beam with either Gaussian or near-Gaussian intensity profiles [1] [2]. It is one of the majorities of laser types in current use for many applications in which the laser beam is being focused to a small spot. However, there are many other applications where uniform intensity distributions are needed. For example, in coherent image processing, pattern recognition, Fourier transforms based correlation, and materials processing tasks, a uniform intensity distribution is required to illuminate evenly the entire processed area. In addition, uniform illumination benefits a wide range of other applications such as in machine vision, industrial inspection, microphotolithography, and in medicine [3] [4] [5] [6] [7]. Thus, the conversion of Gaussian and non-Gaussian laser beams to uniform beam profiles appears to be very beneficial to many applications. Refractive optical system method is among many successful

and efficient methods that can make this conversion. The method relies on geometric optics for designing laser beam shaping systems [8]-[19]. Over the years, one-element and two-element refracting systems have been proposed to achieve beam shaping where Gaussian and annular-Gaussian beam profiles were transformed to uniform beams, as well as to Bessel beam profiles [20]-[25].

In this paper, two-element (instead of one-element) refracting beam shaping systems are designed. Two separate lenses (input and output) with aspheric surfaces of revolution are designed to convert: 1) annular-uniform beam profile to uniform profiles; 2) annular-Gaussian beam to uniform profiles; and 3) Gaussian beam to uniform beam profiles. Detailed step-by-step design is outlined to derive the mathematical expressions that represent the curvature and the asphericity of the input and output lenses. In addition, discussions of the various parameters that are affecting the proposed design are presented such the overall length of the system, the radii of curvature of the surfaces, and the power ratio.

2. Design Procedure

The two-element design refracting optical system for beam shaping is governed by two main conditions that are derived from the geometrical optics of ray tracing, namely:

- 1) The input rays that enter the first lens and leave the second lens must have the same optical path length.
- 2) The input rays to the first lens and the corresponding output rays from the second lens must be parallel to each other.

In addition, a third condition which is related to the beams profiles that are being converted is imposed by the energy conservation, *i.e.*:

- 3) The ratio of the input beam power (through the first lens) to the output beam power (from the second lens) must equal to a constant.

The geometrical configuration of the two-lens refracting system is shown in **Figure 1** where the incident beam has annular-uniform profile while the exit beam will have uniform profile. Consequently, there is no radiation passing through the central aperture of the input lens. The two aspheric lenses (made from glass with index of refraction $n = 1.5172$) are separated by air (index of refraction $n = 1$) by a distance D . θ_i and θ_r are the incident and refracted angles of the rays that enter and exit the first aspheric lens while θ_{r_o} and θ_{i_o} are the incident and refracted angles of the rays that reached the second aspheric lens. It is worth mentioning that the surface of the input convex aspheric lens is designed to map the rays that enter the first half section of convex aspheric lens to the half section of the second aspheric lens after traveling in air. Further, the surface of the output concave aspheric lens is designed to receive the rays reaching the lens and refract them so that they exit parallel to the first incident rays. The two designed surfaces will be designated mathematically as $y_i(r_i)$ and $y_o(r_o)$ as functions of the radial distances r_i and r_o of the input and output lenses, respectively.

From the geometry of the set-up in **Figure 1**, one can deduce that:

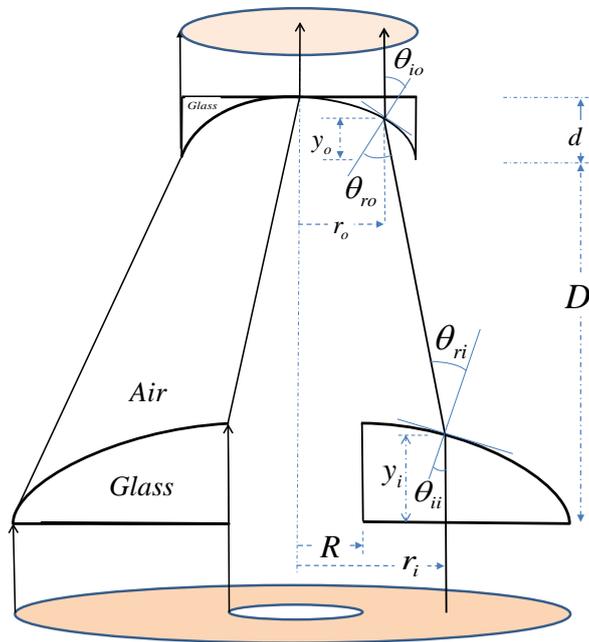


Figure 1. The geometric set-up of the two-lens refracting system beam transformation.

$$\tan(\theta_{ri} - \theta_{ii}) = \tan(\theta_{ro} - \theta_{io}) = \frac{r_i - r_o}{D - y_i + y_o} \quad (1)$$

Next, the above first condition is translated into the following equation:

$$ny_i + \sqrt{(r_i - r_o)^2 + (D - y_i + y_o)^2} + n(d - y_o) = f \quad (2)$$

where f is a constant. Adding $(-nD)$ term to both sides of Equation (2), yields:

$$-nD + ny_i - ny_o + \sqrt{(r_i - r_o)^2 + (D - y_i + y_o)^2} = f - nd - nD$$

Let $f' = f - nd - nD$ and use Equation (1), we obtain:

$$\frac{-n}{\tan(\theta_{ri} - \theta_{ii})} + \frac{\sec(\theta_{ri} - \theta_{ii})}{\tan(\theta_{ri} - \theta_{ii})} = \frac{f'}{r_i - r_o} \quad (3)$$

Further, the second condition of input and output ray parallelism dictates that both input and output surfaces slopes must be equal, namely:

$$\frac{dy_i}{dr_i} = \frac{dy_o}{dr_o} = -\tan \theta_{ii} = -\tan \theta_{io} \quad (4)$$

Furthermore, by applying Snell's law ($n \sin \theta_{ii} = \sin \theta_{ri}$) at the input lens and using trigonometric identities, one can obtain the following important relationship from Equation (3) and Equation (4):

$$\frac{dy_i}{dr_i} = \frac{dy_o}{dr_o} = \frac{-1}{\sqrt{\left(\frac{f'}{r_i - r_o}\right)^2 + n^2 - 1}} \quad (5)$$

Equation (5) is the fundamental equation that will be used to achieve the main

objective of the two-element design, which is to obtain the mathematical expressions for the input and the output surfaces of the lenses $y_i(r_i)$ and $y_o(r_o)$. The following is a step-by-step design procedure to achieve this objective:

- 1) Specify the type of glass used for lenses: obtain the index of refraction n .
- 2) Specify the length of the system by selecting the constant f' .
- 3) Specify the power transfer ratio k between the input and the output beams.
- 4) Obtain the relationship between the radial distances r_i and r_o from the power ratio k .
- 5) Obtain the numerical values for the surfaces' slopes dy_i/dr_i and dy_o/dr_o from Equation (5).
- 6) Obtain the numerical values of the surfaces $y_i(r_i)$ and $y_o(r_o)$ by numerical integration.
- 7) Apply appropriate initial values on the surfaces $y_i(r_i)$ and $y_o(r_o)$ to obtain the constants of integrations.

Note that the least-squared polynomial curve-fitting routine will be used to find the mathematical functions of the surfaces $y_i(r_i)$ and $y_o(r_o)$. Moreover, in the above design procedure one needs to consider other important parameters such as:

- 1) The overall system length, which is preferred to be as small as possible to be practical and to account for power absorption.
- 2) A small value for the starting surface slope dy/dr is desirable to provide for less diffraction and to make the fabrication of the lens much easier. Further the starting cone angle is critical for annular beams.
- 3) Large radii of curvature $\rho(r)$ for the surfaces are preferable to help managing the fabrication process. $\rho(r)$ of any surface is given by:

$$\rho(r) = \frac{\left[1 + (dy/dr)^2\right]^{3/2}}{d^2y/dr^2} \quad (6)$$

Consequently, the second derivative of the starting surface slope d^2y/dr^2 needs to be small too.

3. Laser Beam Transformation

In this section, the above-mentioned design procedure will be applied to transform three different types of beams and they are compared to three previously reported one-element refracting systems [20] [21].

3.1. Annular-Uniform to Uniform Beam Transformation

As shown in **Figure 1**, uniform input annular beam irradiance will be redistributed by the output lens to form a circular uniform output beam. The input surface is designed in such a way to refract the beam forward and inward onto the second surface. The second surface is designed to reorient the refracted beam upward and parallel to the original beam. The energy balance condition implies that we must define a constant ratio of intensities between the input and the

output beams. These intensities, in turn, define the respective cross-sectional areas of the input and the output surfaces. Thus, a ratio k , between the cross-sectional areas can be set as:

$$\frac{\pi(r_i^2 - R^2)}{\pi r_o^2} = k^2 \quad (7)$$

where r_i and R are the outer and the inner radii of the annular beam, and r_o is the radius of the output circular uniform beam. Equation (7) is solved to obtain:

$$\begin{aligned} r_i &= \sqrt{k^2 r_o^2 + R^2} \\ r_o &= \sqrt{(r_i^2 - R^2)/k^2} \end{aligned} \quad (8)$$

Equations (7) and (8) declare that the infinitesimal annular area at each input radial position (r_i) and the infinitesimal annular area at each corresponding output radial position (r_o) form a constant ratio k . In other word, this means that the value of k is the same at each radial position (r_i, r_o). Equation (8) will be used along with Equation (5) to develop various designs for the two-lens refracting system transformation as detailed below. In addition, the proposed designs will demonstrate the effects of changing the inner radius R and the power ratio k on the design parameters. The first proposed design will utilize the same parameters of Reference [19], namely $R = 10$ cm and $k = 2$, for comparison.

The first step in the design is to choose a reasonable value of the constant f' , which leads to selecting a reasonable value for the length D . Note that the starting value of the surface slope, the initial value of the radius of curvature, and the value of D are related to each other's. These initial values are plotted in **Figure 2(a)** as function of f' . The following algorithm provides the steps needed to obtain the curves in **Figure 2(a)**:

- 1) Set $r_o = 0$; $r_i = R = 10$ cm; $y_i = y_o = 0$; and for each value of f' from 1 to 9, do the following:
- 2) Calculate dy/dr_i from Equation (5)
- 3) Calculate θ_{ii} from Equation (4)
- 4) Calculate θ_{ri} from Snell's law $n \sin \theta_{ii} = \sin \theta_{ri}$
- 5) Calculate D from Equation (1)
- 6) Calculate $\rho(r)$ from Equation (6)

From **Figure 2(a)**, a value $f' = 5.75$, which corresponds to $D = 16.54$ cm is selected as the closest value to $D = 16.59$ cm used in Reference [19]. Using this selected value of f' in Equation (5) along with Equation (8), we obtain the numerical values for the surface slopes by using polynomial curve-fitting that approximate the equations for dy/dr as:

$$\begin{aligned} \frac{dy_i}{dr_i} &= -(0.0019)r_i^4 + (0.0915)r_i^3 - (1.6273)r_i^2 + (12.8542)r_i - 38.82 \\ \frac{dy_o}{dr_o} &= (7.4819 \times 10^{-5})r_o^4 - (6.1302 \times 10^{-4})r_o^3 - (0.0014)r_o^2 + (0.0159)r_o - 0.7827 \end{aligned} \quad (9)$$

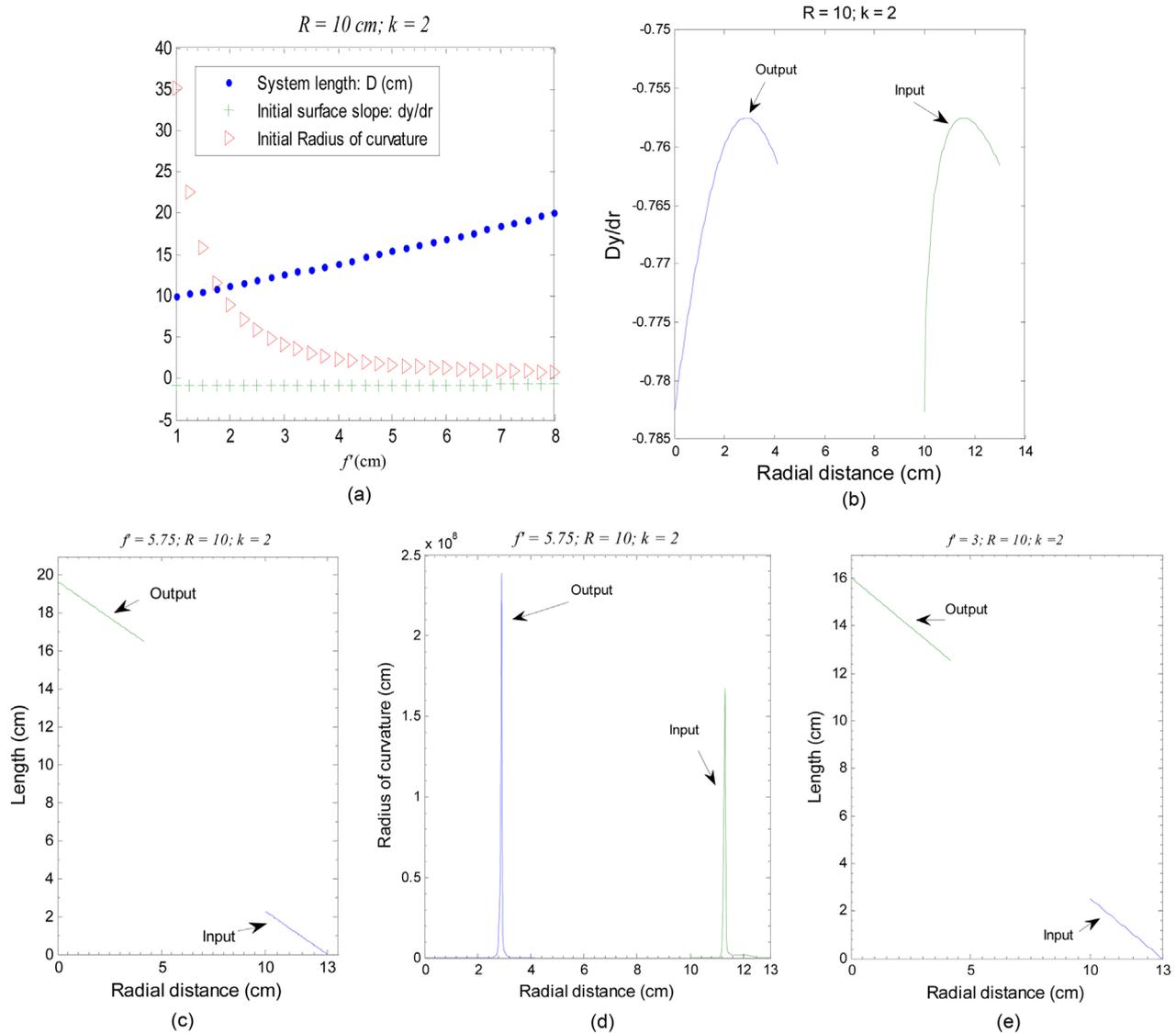


Figure 2. Two-element refracting system design for $R = 10$ cm, $k = 2$, $D = 16.54$ cm: (a) selecting f' ; (b) surface slopes dy_i/dr_i and dy_o/dr_o ; (c) the two surfaces of the lenses; (d) input and output radii of curvatures; (e) the two surfaces of the lenses for shorter system length when $f' = 3$.

Figure 2(b) shows the surfaces slopes for this design. Now, by integrating Equation (9) and using the initial conditions $y_i(r_i = 13) = 0$ and $y_o(r_o = 0) = 3.17$:

$$\begin{aligned}
 y_i(r_i) &= -(3.8611 \times 10^{-4})r_i^5 + (0.0229)r_i^4 - (0.5424)r_i^3 \\
 &\quad + (6.4271)r_i^2 - (38.82)r_i + 99.9797 \\
 y_o(r_o) &= (1.4964 \times 10^{-5})r_o^5 - (1.5325 \times 10^{-4})r_o^4 - (4.5245 \times 10^{-4})r_o^3 \\
 &\quad + (0.008)r_o^2 - (0.7827)r_o + 3.1726
 \end{aligned} \tag{10}$$

Figure 2(c) illustrates the final surfaces of the lenses. Large radii of curvatures for the surfaces are noticed as shown in **Figure 2(d)**. For the input surface, the radius of curvature starts from a value of 921.5 cm reaching a maximum of 1.673×10^8 cm; then it goes to a value of 13,840 cm; while for the output surface, the

radius of curvature starts from a minimum value of 8050 cm reaching a maximum value of 2.923×10^8 cm then falls to a minimum value of 6333 cm. With respect to the overall system length of the proposed design, it is found to be 19.69 cm compared to 24.08 cm in Reference [20]; whereas the starting cone angle $\tan^{-1}(D - y_i + y_o)/R$, is calculated to be 59.27° compared to 58.92° in Reference [19].

The previous design was introduced for comparison purposes. However, a better and shorter system length can be easily found when selecting a lower value for f' . For instance, a design for $f' = 3$ cm, which correspond to $D = 12.56$ cm, is illustrated in Figure 2(e) for which the overall length of the system design is reduced to 16.05 cm compared to 24.08 cm in Reference [19], with a starting cone angle equal to 53.53° compared to 58.92° . Next, the effects on the design parameters when the power ratio is changed to $k = 4$, ($f' = 5.75$, $R = 10$ cm) is shown in Figure 3(a). It is observed that the output beam radius r_o , the overall system length, and the starting cone angle decrease, respectively, to 2.1 cm (was 4.15), 18.15 cm (was 24.08), and 57.66° (was 58.92°). On the other hand, when the power ratio is set to a value $k = 0.707$ (lower than 1), as illustrated in Figure 3(b), then the opposite happened, *i.e.*, the circular beam radius r_o , the overall system length, and the starting cone angle increase to 11.76 cm, 23.26 cm, and 65.34° , respectively. Furthermore, the importance of the starting cone angle from the inner radius R of an annular beam is discussed. This issue will affect the overall length of the two-element lens design for a fixed power ratio k , in addition to other factors such as the initial surface slope and the initial radius of

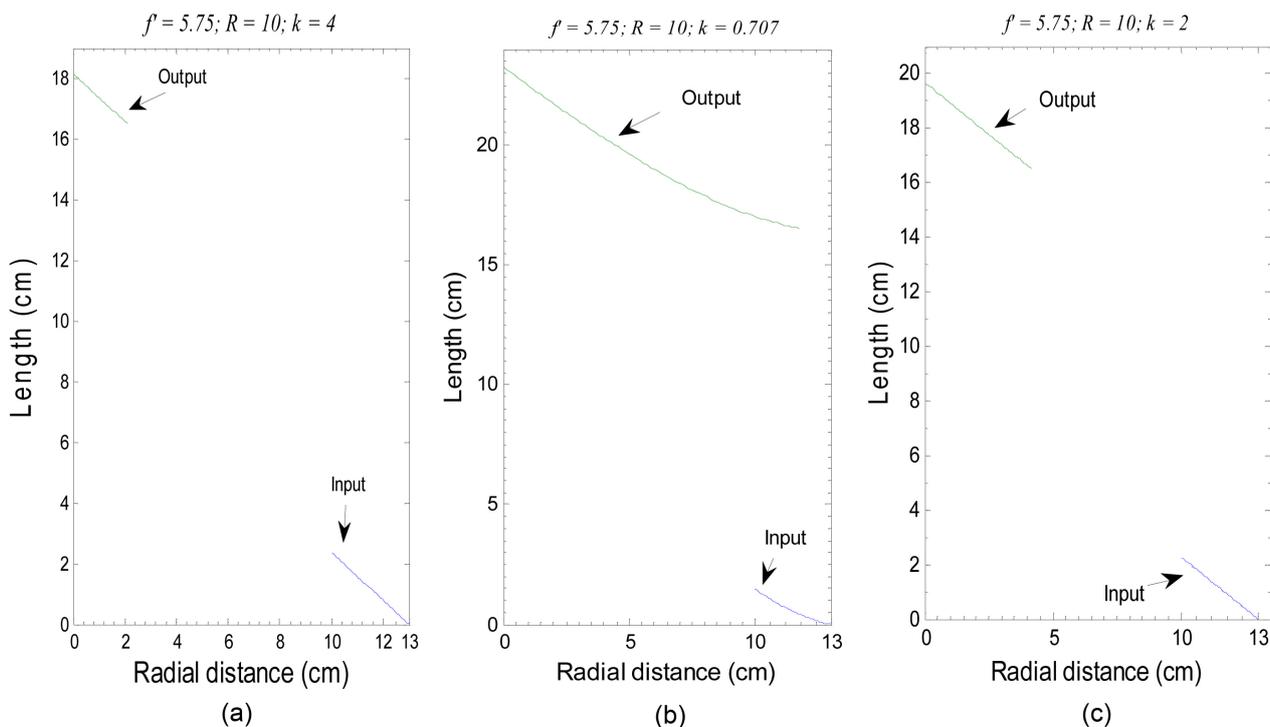


Figure 3. Effects of the power ratio on the designed surfaces for $f' = 5.75$ cm, $R = 10$ cm: (a) $k = 4$; (b) $k = 0.707$; and (c) $k = 2$.

curvature of the surface. **Figure 4(a)** and **Figure 4(b)** show the designs for $R = 10$ cm and $R = 3$ cm, respectively, both for $f' = 5.75$ cm, $k = 2$. Note that in this case, the uniform beam radius r_o is reduced to 2.6 cm from 4.15 cm, the overall system length is reduced to 12.51 cm from 19.69 cm, and the cone angle is increased to 75° from 60° . Since lowering f' leads to a desirable smaller value for D with a large starting cone angle, then one can easily shorten the overall system length as demonstrated in **Figure 4(c)** for $f' = 2$ cm, $R = 3$, $k = 2$. The overall system length is reduced to 6.778 cm with a cone angle equals to 56.51° .

3.2. Annular-Gaussian to Uniform Beam Transformation

In this sub-section, a refracting two-lens design is performed to transform an annular-Gaussian beam to uniform beam profile. The design procedure will utilize the same Equations (1) to (6) derived earlier. For the case of annular-Gaussian input beam that has input intensity expressed as [1] [2]

$I(r) = \left[1 - R_0 e^{-2r^2/w_0^2} \right] e^{-2r^2/w^2}$, one can set the ratio of the cross-sectional areas of the two beams as:

$$k^2 = \frac{2\pi I_o \int_0^R r \left[1 - R_0 e^{-2r^2/w_0^2} \right] e^{-2r^2/w^2} dr}{\pi I_o r_o^2} \tag{11}$$

where r_o is the radius of the circularly-uniform output beam; I_o is the peak input beam intensity; w_0 is the Gaussian beam radius; $w^2 = (M^2 - 1)w_0^2$ is the beam spot size in the large Fresnel number limit; $M = \sqrt{2}$ is the magnification; r_i and R are the outer and the inner radii of the annular-Gaussian beam. Note that

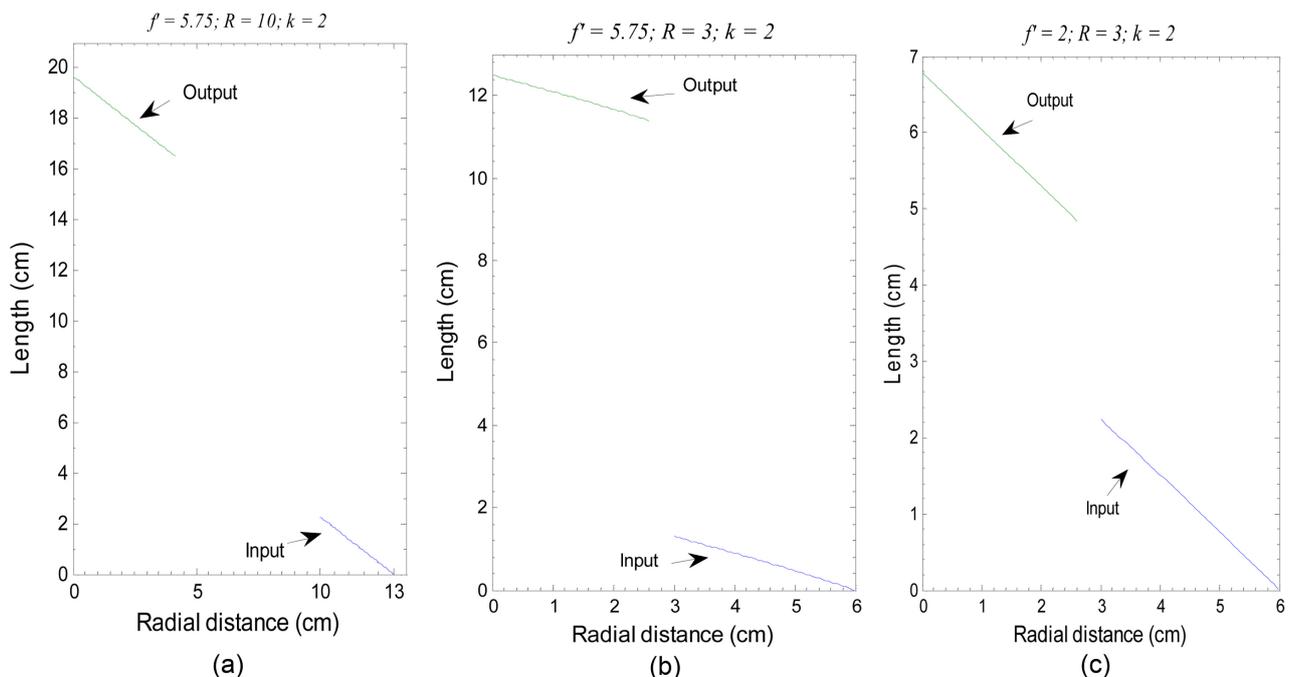


Figure 4. Effects of the inner radius R on the designed surfaces for $f' = 5.75$ cm, $k = 2$: (a) $R = 10$ cm; (b) $R = 3$ cm; (c) a shorter lens system design for $f' = 2$ cm, $k = 2$, $R = 3$ cm.

R_o is the reflectivity of a central mirror in the resonator that generates the annular-Gaussian beam [2]. Solving the integral in Equation (11) leads to:

$$r_o = \left\{ \frac{R_o/2}{\left(1/w_o^2 + 1/w^2\right)} \left[e^{-2r_i^2(1/w_o^2 + 1/w^2)} - e^{-2R^2(1/w_o^2 + 1/w^2)} \right] - w^2 \left(e^{-2r_i^2/w^2} - e^{-2R^2/w^2} \right) / 2 \right\}^{1/2} / k \tag{12}$$

Note that when $M = \sqrt{2}$, then $w = w_o$. Consequently, Equation (12) is simplified to:

$$r_o = \left(\frac{w_o}{k} \right) \sqrt{R_o \left(e^{-4r_i^2/w_o^2} - e^{-4R^2/w_o^2} \right) / 4 - \left(e^{-2r_i^2/w_o^2} - e^{-2R^2/w_o^2} \right) / 2} \tag{13}$$

Note that in Equation (13), the constant k decreases as the radial position r_i of the annular-Gaussian beam increases. Therefore, the input surface needs to be designed so that it attenuates the intense region of the annular-Gaussian beam while, at the same time, it redistributes the excess energy of the intense regions to the less intense peripheral region of the beam. Consequently, the surface of the first aspheric lens directs the incident rays so that they are uniformly distributed at the surface of the second aspheric lens, which then redirects the rays to be collimated.

It is worth discussing, for annular beam, the effect of the starting refracting angle $\theta_{ri} = \sin^{-1}(n \sin \theta_{ri})$ at the inner radius R on the design. When R increases, it is expected that the length D will increase too. This is demonstrated in **Figure 5**, where D is plotted versus the refracting angle for $R = 1$ cm and $R = 6$ cm, for the

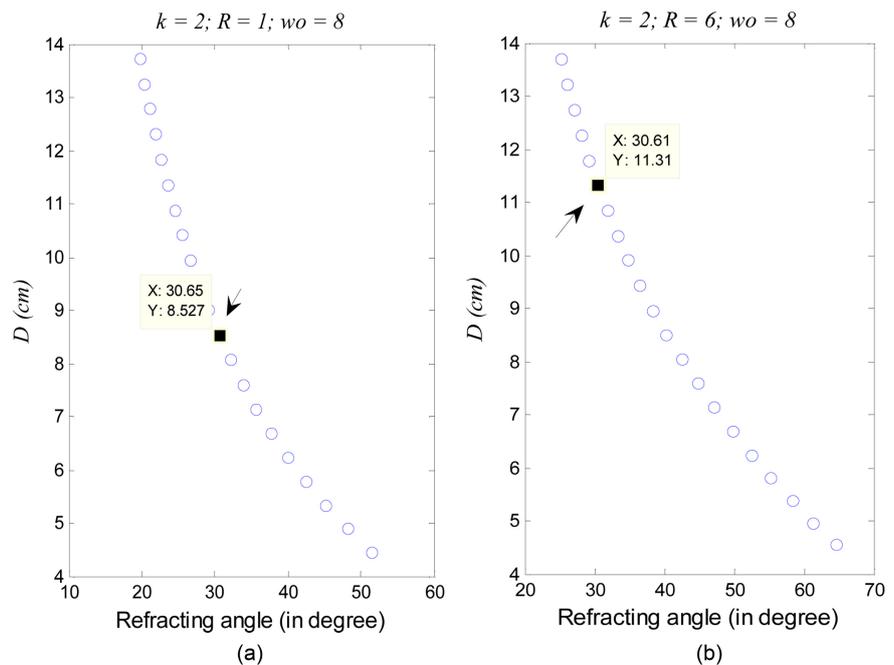


Figure 5. Effects of the inner radius R of an annular beam on the starting refracting angle when $k = 2$, $w_o = 8$: (a) $R = 1$ cm; (b) $R = 6$ cm.

same values of $k = 2$, $R_o = 0.9$, $w_o = 8$ cm. As shown in the figure, the length D increases from 8.527 cm to 11.31 cm to maintain, for instance, the same refracting angle $\theta_{ri} = 30.6^\circ$ when f' changes from 2 cm to 7 cm.

Applying the design procedure outlined earlier for $R = 1$ cm, $k = 2$, $w_o = 8$ cm, $R_o = 0.9$, then a two-element refracting system is achieved as illustrated in **Figure 6**. From **Figure 6(a)**, f' is selected to be 2.25 cm, which provides a starting angle $\theta_{ri} = 47^\circ$. The approximate equations for lens surfaces, plotted in **Figure 6(b)**, are given by:

$$\begin{aligned}
 y_i(r_i) &= -(6.896 \times 10^{-4})r_i^5 + (0.0086)r_i^4 - (0.0331)r_i^3 \\
 &\quad - (0.0266)r_i^2 - (0.2625)r_i + 2.0939 \\
 y_o(r_o) &= -(0.2776)r_o^5 + (0.7125)r_o^4 - (0.6405)r_o^3 \\
 &\quad + (0.0204)r_o^2 - (0.3893)r_o + 0.4972
 \end{aligned}
 \tag{14}$$

Figure 6(c) shows the radius of curvature for the input surface, which has a minimum value of 46.82 cm and a maximum value of 327.3 cm; while the radius of curvature for the output surface has a minimum value of 5.654 cm and a maximum value of 5580 cm.

On the other hand, for comparison purposes, **Figure 7(a)** represents a two-element design that uses the same parameters of the one-element design, as it was proposed in Reference [20], *i.e.*, $f' = 2.25$, $k = 0.4$, $R = 0.5$ cm, $w_o = 3.47$ cm, $R_o = 0.9$. Considering the overall length of system, our proposed two-element design reduces the overall length of system by more than 62% (from 13.1 cm to 4.898 cm). Note that the starting cone angle of the proposed design is slightly decreased to 83.65° compared to 87.81° . Further, in **Figure 7(b)** and **Figure 7(c)**, the radius of curvature for the input surface changes from a minimum value of 3.025 cm to a maximum value of 6.197×10^4 cm; while the

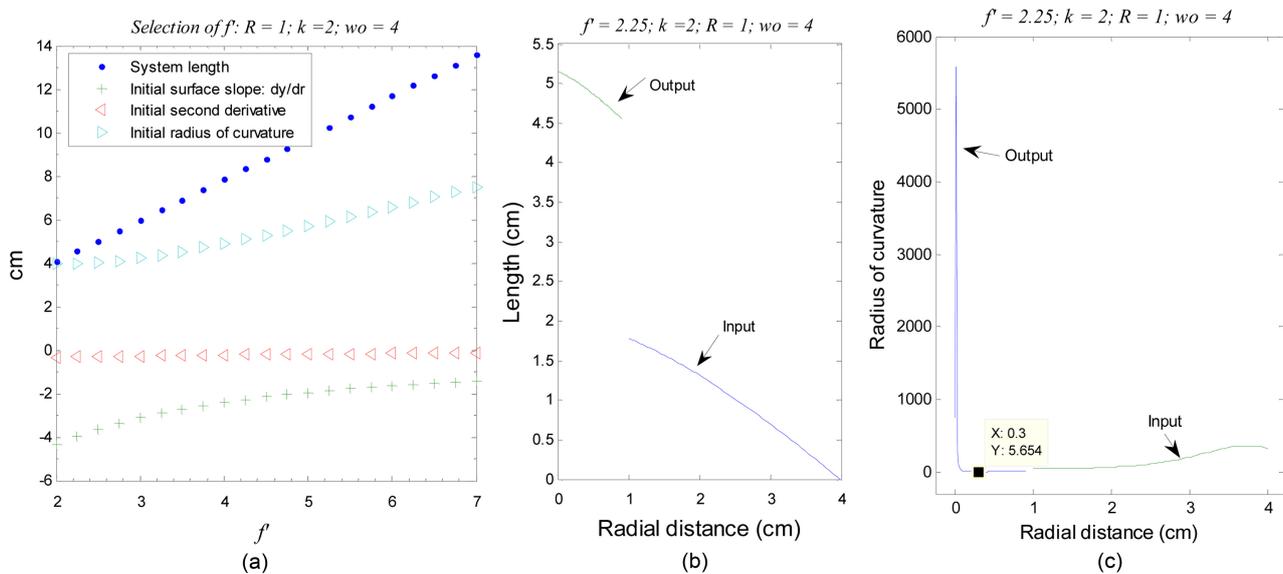


Figure 6. Two-element refracting system transformation design for annular-Gaussian to uniform with $k = 2$, $R = 1$ cm, $w_o = 4$: (a) selecting f' ; (b) the two surfaces of the lenses; (c) input and output radii of curvatures.

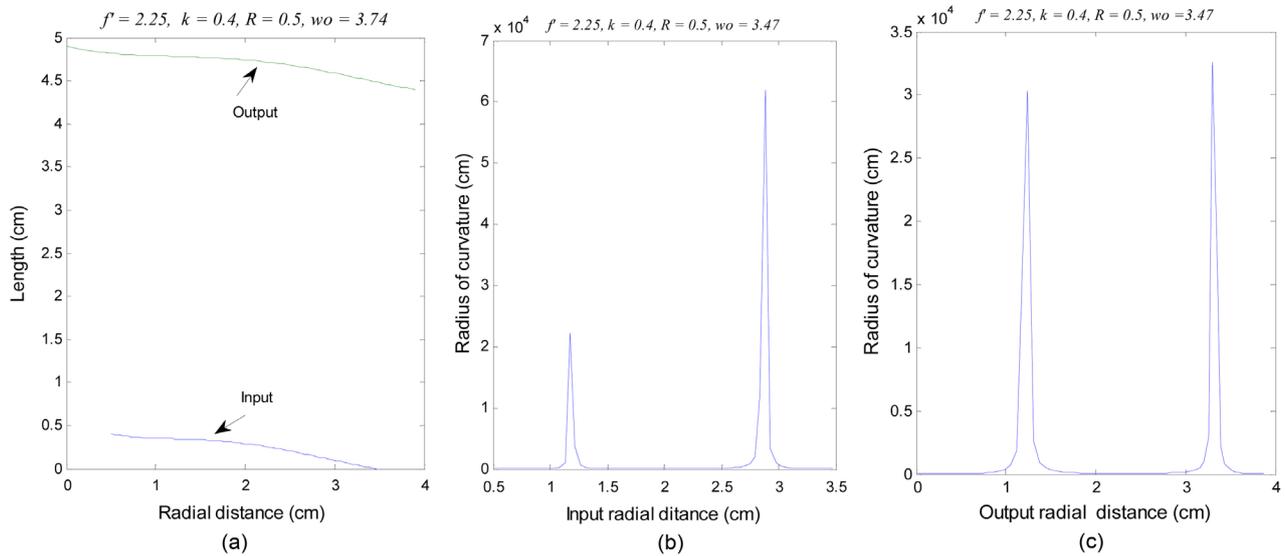


Figure 7. Two-element refracting system transformation design for annular-Gaussian to uniform with $k = 0.4$, $R = 0.5$ cm, $w_0 = 3.74$: (a) the two surfaces of the lenses; (b) input radius of curvature; (c) output radius of curvature.

radius of curvature of the output surface changes from a minimum of 9.793 to a maximum value of 3.237×10^4 cm. These values far outperform the ones in Reference [20]. It is worth mentioning that the length of the system can be further reduced by selecting a smaller value for f' .

Finally, the least-square curve-fitting approximate equations for the surfaces for this design are given by:

$$\begin{aligned}
 y_i(r_i) &= -(0.0038)r_i^5 + (0.0592)r_i^4 - (0.315)r_i^3 \\
 &\quad + (0.6849)r_i^2 - (0.6814)r_i + 0.6087 \\
 y_o(r_o) &= (9.1227 \times 10^{-4})r_o^5 - (7.6221 \times 10^{-5})r_o^4 - (0.0498)r_o^3 \\
 &\quad + (0.1657)r_o^2 - (0.2237)r_o + 0.4931
 \end{aligned} \tag{15}$$

3.3. Gaussian to Uniform Beam Transformation

In this sub-section, we have provided designs procedure to transform the irradiance distribution of a Gaussian beam into a circularly-uniform beam using split two-element refracting system. Note that the optical set-up of **Figure 1** needs to be modified to accommodate the design of the Gaussian beam profile input as illustrated in **Figure 8**. The power ratio between the two different beams is:

$$k^2 = \frac{2\pi I_o \int_0^{r_i} r e^{-r^2/w_o^2} dr}{\pi I_o r_o^2} = \frac{w_o^2 (1 - e^{-r_i^2/w_o^2})}{r_o^2} \tag{16}$$

where r_o is the radius of the circularly-uniform output beam; I_o is the input beam peak intensity; and w_o is the Gaussian beam radius, r_i is the radial position of the Gaussian beam. From Equation (16), the relationships between the input and output radii are:

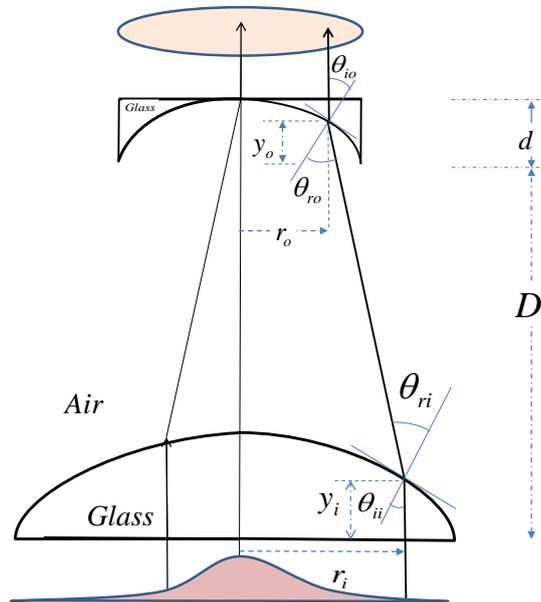


Figure 8. The optical set-up to transform Gaussian beam to uniform beam.

$$\begin{aligned}
 r_o &= (w_o/k) \sqrt{1 - e^{-r_i^2/w_o^2}} \\
 r_i &= w_o \sqrt{-\ln(1 - k^2 r_o^2/w_o^2)}
 \end{aligned}
 \tag{17}$$

Equation (17) relates the constant k to the input and output radial positions r_i and r_o of the lens surfaces. The beam transformation process can be explained as follows. The power in the input Gaussian beam varies inversely as the radial distance r_r . This power needs to be uniformly distributed as an output beam that has a particular radius r_o which is determined by a constant k . This constant, in turn, is related to the output power concentration. As a result, every input Gaussian ray, located at a radial position r_b , will be redirected to a radial position r_o , after propagating in air such that Equation (16) holds true, *i.e.* k remains constant.

The proposed two-element design shown in **Figure 9**, uses $r_i = 2.97$ cm, $r_o = 1.92$ cm, $k = 1.23$, $w_o = 3$, which are the same values utilized in the one-element design in Reference [20]. However, f is selected equal to 0.75 cm to reduce the overall system length to 2.134 cm. For this design, the radius of curvature for input surface spans a minimum value of 12.76 cm to a maximum value of 255 cm; while the radius of curvature for output surface spans a range of a minimum value of 6.392 cm to a maximum value of 19.83 cm. Comparing with the one-element design of Reference [20], our proposed design is much better. The surfaces equations of the input and output lenses are given by:

$$\begin{aligned}
 y_i(r_i) &= (2.6314 \times 10^{-4}) r_i^5 + (0.0032) r_i^4 - (0.0198) r_i^3 \\
 &\quad - (0.1099) r_i^2 - (0.003) r_i + 1.2085 \\
 y_o(r_o) &= (0.0084) r_o^5 - (0.0307) r_o^4 + (0.0149) r_o^3 \\
 &\quad - (0.1586) r_o^2 + (4.3318 \times 10^{-4}) r_o + 0.6842
 \end{aligned}
 \tag{18}$$

On the other hand, **Figure 10** is a design that transforms a Gaussian beam—with a small cross-sectional area—into a circularly-uniform beam with a larger cross-sectional area. This can be achieved when the power ratio k is less than 1. The overall length system for this design is 2.64 cm and the approximate equations for surfaces are:

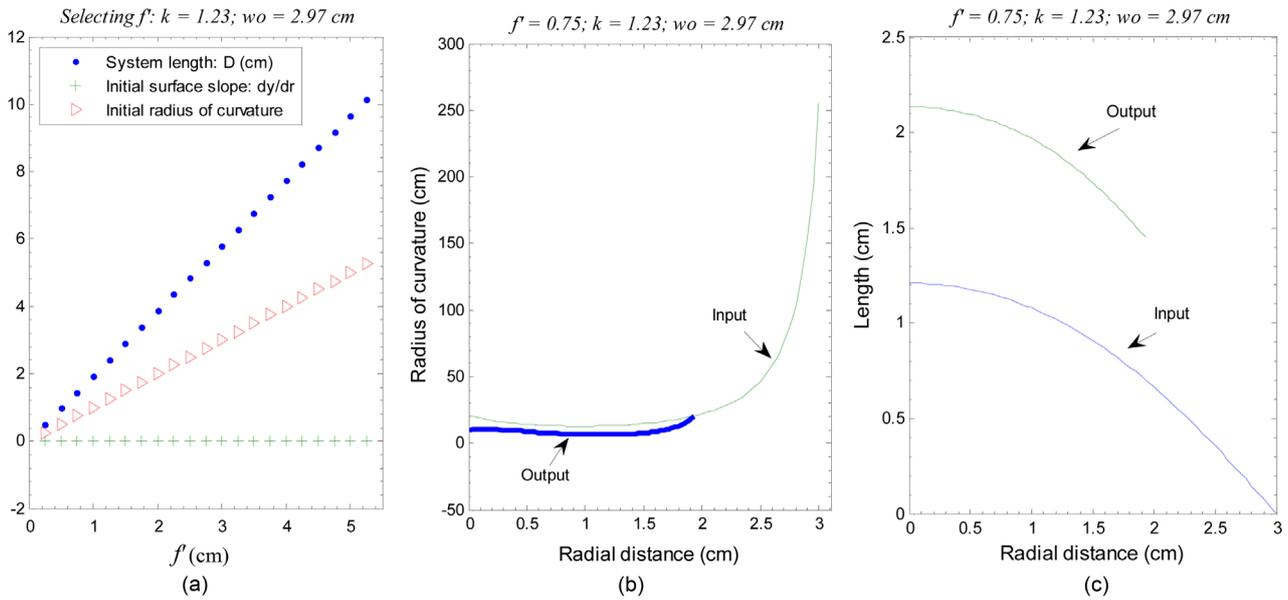


Figure 9. Two-element refracting system transformation design for Gaussian to uniform with $k = 1.23$, $w_o = 3$: (a) selecting f ; (b) the input and output radii of curvatures; (c) the two surfaces of the lenses.

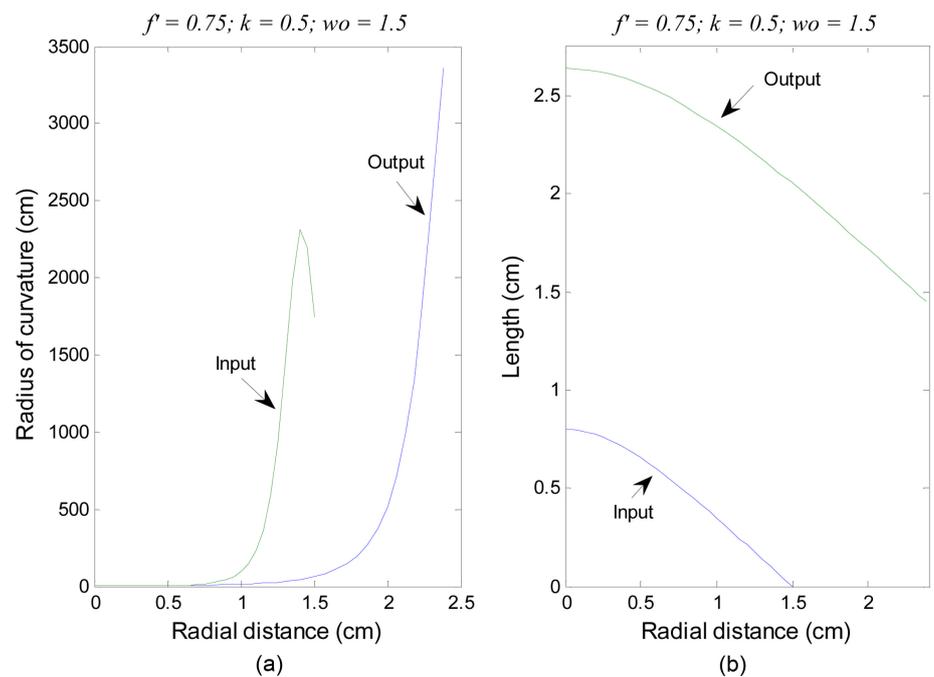


Figure 10. Two-element refracting system transformation design for Gaussian to uniform with $f = 0.75$, $k = 0.5$, $w_o = 1.5$: (a) the input and output radii of curvatures; (b) the two surfaces of the lenses.

$$\begin{aligned}
 y_i(r_i) &= (0.0061)r_i^5 - (0.0918)r_i^4 + (0.3970)r_i^3 \\
 &\quad - (0.7693)r_i^2 + (0.0066)r_i + 0.7995 \\
 y_o(r_o) &= -(9.9011 \times 10^{-4})r_o^5 - (0.0014)r_o^4 + (0.0756)r_o^3 \\
 &\quad - (0.3695)r_o^2 + (0.004)r_o + 1.1898
 \end{aligned} \tag{19}$$

4. Conclusion

We have presented various split two-element refracting systems designs to transform three types of laser beam irradiance distributions, namely: 1) annular-uniform-to-uniform, 2) annular-Gaussian-to-uniform, and 3) Gaussian-to-uniform. A detailed step-by-step design procedure is outlined. For each type of beams, we carried out rigorous analysis demonstrating the effects on the design when changing few system parameters such as the system length, the starting cone angle for annular beams, and the power ratio. In addition, the proposed two-element designs were compared to previously reported one-element designs. It was demonstrated that the proposed two-element designs are much better than the reported ones.

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