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A Frequency-Equivalent Scale-Free Derivation of the Neutron, Hydrogen Quanta, Planck Time, and a Black Hole from 2 and π; and Harmonic Fraction Power Laws

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Abstract

We find that π represents dual attributes. One is within the purely mathematical domain and can be derived for example, from infinite series, among several other methods. The other is within a 2D geometric-physical domain. This paper analyzes several physical constants from an analogous perspective where they are defined solely by mathematical and 2D geometric properties independent of any actual physical scaling data. The constants are evaluated as natural unit frequency equivalents of the neutron, electron, Bohr radius, Rydberg constant, Planck's constant, Planck time, a Black hole with a Schwarzschild radius, the distance light travels in one time unit; and the fine structure constant. These constants are defined within two inter-related harmonic domains. In the linear domain, the ratios of the frequency equivalents of the Rydberg constant, Bohr radius, electron; and the fine structure constant are related to products of 2 and π. In the power law domain, their partial harmonic fraction powers, and the integer fraction powers of the fundamental frequency for Planck time are known. All of the constants are then derived at the point where a single fundamental frequency simultaneously fulfills both domains independent of any direct physical scale data. The derived values relative errors from the known values range from 10^{-3} to 10^{-1} supporting the concept and method.

Keywords

Fundamental Constants, Neutron, Black Hole, Planck Time, Computational Physics, Mathematical Physics, Hydrogen

1. Introduction

Physics is the science that defines physical phenomena within well-defined mathematical systems. The famous quote of Galileo Galilei sums up the relationship of physics and mathematics "Mathematics is the key and door to the sciences" (physical universe). In the quantum age, others have expanded this concept to include the mathematical universe hypothesis where all of physics are defined completely by mathematics [1]. This paper demonstrates that many of the fundamental constants can be accurately derived without any physical scaling data within a combined linear and power law harmonic system. This supports a general mathematical hypothesis defining physical systems.

We demonstrate a mathematical method and a conceptual physical model to calculate, to a first approximation, the natural frequency equivalents, ν, of the neutron, n^0, ν_{n^0}, the electron, e^-, ν_{e^-}, Bohr radius, a_0, ν_{a_0}, Rydberg constant, R, ν_R, Planck time, t_P, a Black Hole, BH, ν_{BH}, with Schwarzschild radius, the distance light travels in one second, one unit of time; and the fine structure constant, α. The actual unit of time is irrelevant in this type of dimensionless system, therefore, it is equivalent to 1 divided by one unit of time or Hz for the SI units. These are evaluated within a dimensionless Hz divided by Hz or, unit frequency divided by unit frequency ratio system. These constants are chosen to evaluate a wide range of fundamental physical domains and scales. No classic direct physical scaling data such as a specific mass, distance, or frequency are utilized. This is possible since the natural unit frequency equivalents of e^-, a_0, R, in the linear domain have known ratio relationships defined mathematically by products of 2, π, and α [2] [3]. These same constants are also defined within a harmonic partial fraction power law domain defining Planck time squared using a fundamental frequency base which is related to the annihilation frequency of the neutron [4]-[11].

There is reciprocity in that the frequency equivalents of R, e^-, a_0, and α must be precisely scaled equivalently in each of their respective domains, and fulfill geometric imperatives. In either the linear or power law domain, there exist an infinite number of possibilities that can fulfill their respective geometries and ratio scales. However, there is one and only one set of values that uniquely fulfill both domains simultaneously. These derived values are closely related to their known constants. Our goal is to demonstrate an accurate and logical mathematical method to derive these frequency equivalents, and consequently the scale relationships of these fundamental physical constants to which they are associated without knowledge of any standard scaling physical data.

2. General Properties

2.1. Mathematical and Geometric-Physico Duality of π

The uniqueness of π represents an irrational number with dual mathematical attributes. One is solely within the purely mathematical domain and is derived froma variety of infinite series where n equals integers: Leibniz' formula, Equation (1) [12]; an infinite series consisting of the squares of harmonic fractions, Equation (2) [13]; and John Wallis' formula for π/2, Equation (3) [14] [15] [16]. In this purely mathematical domain there is no direct relationship to a physical meaning of π. The other attribute, which we findis within a 2D, geometric domain with the physical properties of a circle, sinusoidal and harmonic systems:

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\cdots=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} \tag{1}$$

$$\pi=\sqrt{6\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots\right)} \tag{2}$$

$$\frac{\pi}{2}=\frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\frac{8}{9}\cdots=\prod_{n=1}^{\infty}\left(\frac{2n}{2n-1}\frac{2n}{2n+1}\right) \tag{3}$$

This paper analyzes some of the most important fundamental physical constants from an analogous perspective where they are defined solely by dimensionless mathematical properties on a 2D plane, or ratios independent of any direct physical scaling data or unit system. There are numerous examples of dual physical and purely mathematical systems, some of which include the Divergence theorem [17], and the theorems of Green [18] and Stokes [19] [20], *i.e.* mathematical constructs having direct physical application.

2.2. Power Laws and Harmonic Systems

Power laws and harmonic systems are ubiquitous in Physics [21] and Mathematics [22]. In power laws the relative change in one quantity results in an exponential change in the other quantity, independent of the initial size of those quantities. Power laws are plotted on log-log plots as linear relationships between two different variables, **Figure 1**, **Figure 2**. Examples of physical power laws include: the Stefan–Boltzmann law [23]; square-cube law [24]; inverse-square laws of Newtonian gravity and electrostatics; and restorative potential in simple harmonic motion [25]; and Kepler's third law [26].

In **Figure 1** the X-axis equals the quantum fractions, *qf*s, minus 1 or −1/n. The Y-axis is the difference between the log base v_{n^0} and its partial fraction, *δ*. This geometry shows that the *bwk* and bem are almost symmetrically split. [4] Their slopes are slightly different as well. The positively sloped dashed line, defined by our wk_d equation, includes the Bohr radius and that of the electron. The negatively sloped, electromagnetic solid line, em_d, starts at 1 Hz point, (−1, 0) and intersects the Rydberg constant at its harmonic fraction point. The first three odd prime harmonic fractions −1/3, −1/5, and −1/7 are respectively associated with *R*, a_0, and e^-. The *bwk* is closely scaled to $\log_{(v_{n^0})}(2)/(128/35)$. This point is related to Planck time squared. This relationship is used in the derivation.

In **Figure 2** the X-axis equals the quantum fractions, *qf*s, minus 1 or −1/n. The Y-axis is the difference between the derived log base v_F constant, and their power fractions, δ_d. This is a simplified geometry and not identical to the known, as seen in **Figure 1**. [4] The positively sloped dashed line, defined by our wk_d

equation, includes the derived values of the Bohr radius and that of the electron. The negatively sloped derived electromagnetic solid line, em_d, starts at 1 Hz point, (−1, 0) and intersects the Rydberg constant at its harmonic fraction point. The harmonic fractions −1/3, −1/5, and −1/7 are respectively associated with R, a_0, and e^-. The bwk_d, awk_d, and $-bem_d$ all equal $\log_{(v_F)}(2)/(128/35)$. This point is related to Planck time squared. The exponents of our fundamental frequency sweep through those values that fulfill the ratios depicted by Equations (4-8).

Harmonic systems also exhibit power law relationships. For example in music the ratio of octave frequencies are related to the product of the fundamental frequency, and 2 raised to a consecutive integer series. Harmonic systems are associated with sinusoidal periodic functions where integer and integer harmonic fractions define inter-relationships via dimensionless ratios. The combination of power laws and harmonic systems is extremely organized, predictive, and mathematically restricted.

2.3. Physical Coupling Ratios of the Frequency Equivalents of R, a_0, e^-, and α with 2 and π

Though the properties of R, a_0, e^-, and α are quantum in nature, they are not mathematically independent variables. It has been demonstrated that when transformed to their frequency equivalents, v; the electron, v_{e^-}, Bohr radius, v_{a_0}, and the ionization energy of hydrogen as the Rydberg constant, v_R, are all inter-related by factors of 2, π, and the fine structure constant, α, in a dimensionless ratio system [2] [3]. These relationships are in the linear frequency domain and harmonic in character since they are related to 2π, Equations (4)-(8). Note that in Equation (4) $8\pi^2$ is embedded within the actual natural unit frequencies of these three quanta, not added. This is the Schrödinger equation

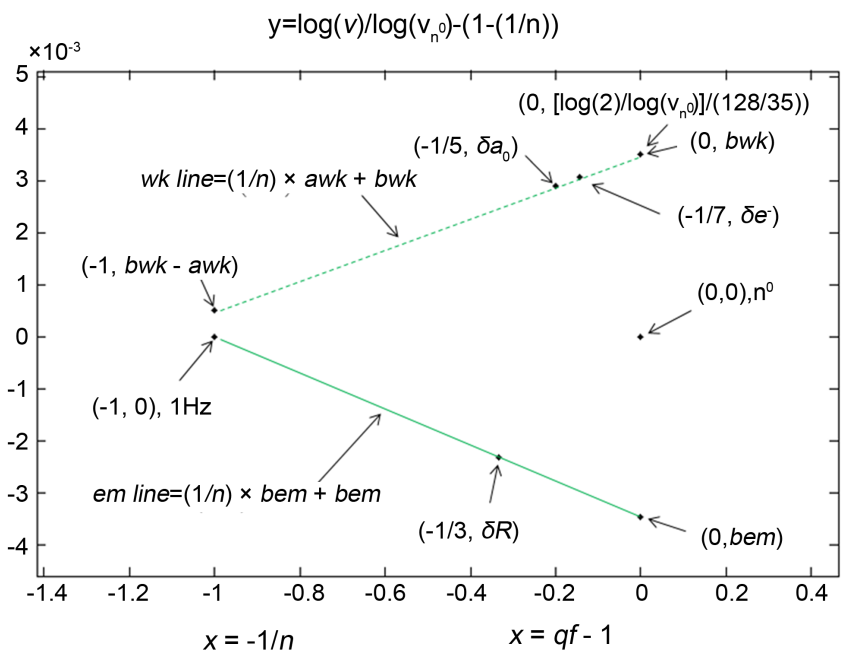

Figure 1. The v_{n^0} power law domain geometry.

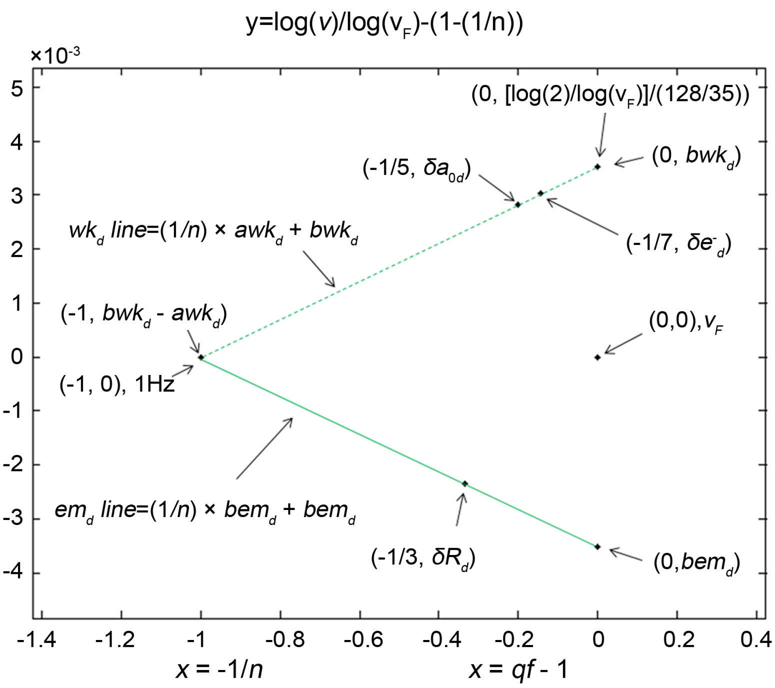

Figure 2. Simplified power law geometry used for the derivations.

geometric factor. We utilize the notation where A is a ratio. The numerator of the ratio is the upper constant symbol followed by its power in parentheses. The denominator of the ratio is lower natural frequency symbol followed by its power in parentheses. There can be more than one constant in either the numerator or denominator. The following is an example of the ratio related to the ν_R raised to the third power and ν_{n^0} raised to the second power, $A^{\nu_R(3)}_{\nu_{n^0}(2)}$.

$$8\pi^2 = \left(\frac{\nu_{a_0}{}^2}{\nu_{e^-}\nu_R}\right) = A^{\nu_{a_0}(2)}_{\nu_{e^-}\nu_R} \tag{4}$$

$$\alpha = \left(\frac{\nu_{a_0}}{2\pi\nu_{e^-}}\right) \tag{5a}$$

$$\left(\frac{\nu_{a_0}}{\nu_{e^-}}\right) = 2\pi\alpha = A^{\nu_{a_0}}_{\nu_{e^-}} \tag{5b}$$

$$\alpha = \left(\frac{4\pi\nu_R}{\nu_{a_0}}\right) \tag{6}$$

$$\left(\frac{\nu_R}{\nu_{a_0}}\right) = \frac{\alpha}{4\pi} = A^{\nu_R}_{\nu_{a_0}} \tag{6b}$$

$$\alpha^2 = \left(\frac{2\nu_R}{\nu_{e^-}}\right) \tag{7a}$$

$$\left(\frac{\nu_R}{\nu_{e^-}}\right) = \left(\frac{\alpha^2}{2}\right) = A^{\nu_R}_{\nu_{e^-}} \tag{7b}$$

$$\alpha = \left(\frac{2\nu_R}{\nu_{e^-}}\right)^{1/2} \tag{8}$$

3. The Harmonic Neutron Hypothesis

3.1. Overview

The Harmonic Neutron Hypothesis, HNH, has demonstrated that the fundamental constants are inter-related within power laws with partial harmonic fraction powers of the frequency of the neutron, ν_{n^0} related to specific constants [4]-[11]. All harmonic systems are associated with harmonic and partial harmonic fraction ratios in both the linear and power domains. Harmonic fractions are $1/n$ where n is the consecutive integer series starting at 1. For example the wavelength of a fundamental wavelength, λ, equals the product of λ and $1/n$. The frequencies equal n. In the power domain the harmonics are related to the ν_F raised to the partial harmonic fractions $1 \pm (1/n)$ for n equals the consecutive integer series starting at 1. All of the fundamental constants are analyzed as dimensionless ratios of the frequency equivalent of any constant, ν, divided by ν_{n^0}. Any other physical unit, Joules, electron volt, mass, could be utilized and the results would not change. The standard unit value can be reconstructed by multiplying by the unit value. This is equivalent to Planck's constant, the speed of light, and unit charge all equaling a dimensionless 1. It has been shown that it is possible to begin with four natural units of the ν_{n^0}, ν_{e^-}, ν_{a_0}, ν_R to derive other fundamental constants, including Planck time, t_P, Higgs' boson, H^0, the Hubble constant, H_0, the quarks, cosmic microwave background radiation peak spectral radiance, CMB, and the mass of the proton, p^+ [4]-[11]. This is equivalent to deriving the constants within integer power laws of these four frequency equivalents. This follows the same pattern as seen with the hydrogen constants above.

The power law relating many of the fundamental constants with a frequency equivalent of more than 1 Hz, but less than the neutron is related to the dimensionless ratio of the constant's frequency equivalent raised to an integer power, $n + 1$, divided by the product of ν_{n^0} raised to the power n, and the frequency near 1 Hz, $A^{\nu(n+1)}_{\nu_{n^0}(n)}$, that equals 1, Equation (9). This is true for all harmonic systems since the harmonics are defined by partial harmonic fraction powers. Note that in Equation (9b) the power of Hz for the constant is 1. This is the sum of $n/(n+1)$ plus $1/(n+1)$. Here $n/(n+1)$ and $1/(n+1)$ are partial and harmonic fractions. The unit powers for Hz are accurately calculated within these types of power laws so the log calculations remain valid and accurate. The Hz powers will not be shown since they complicate the equations unnecessarily. Other integer fraction or integer powers of these A values are valid, and fall on the same power law line. Though these A values are derived from identity equations they represent fundamental constants that bridge far beyond that constant's typical physical significance and inter-relate many other constants.

$$1 = \left(\frac{\nu^{n+1}}{\nu_{n^0}^{n} A_{\nu_{n^0}(n)}^{\nu(n+1)}} \right) \tag{9a}$$

$$\nu = \nu_{n^0}^{\;n/(n+1)} \left[A_{\nu_{n^0}(n/(n+1))}^{\nu} \right] \tag{9b}$$

$$A_{\nu_{n^0}(n)}^{\nu(n+1)} = \left(\frac{\nu^{n+1}}{\nu_{n^0}^{n}} \right) \tag{9c}$$

The n^{th} power for each constant is not arbitrary, but a natural dimensionless quantum unit. It is related to the only n power where the frequency for $A_{\nu_{n^0}(n)}^{\nu(n+1)}$ is near to 1 Hz. These values range from $A_{\nu_{n^0}(6)}^{\nu_{e^-}(7)}$, $\nu_{e^-}^{\;7} / \nu_{n^0}^{\;6}$, 3.1976599, $A_{\nu_{n^0}(4)}^{\nu_{a_0}(5)}$, $\nu_{a_0}^{\;5} / \nu_{n^0}^{\;4}$, 2.1906464, to $A_{\nu_{n^0}(2)}^{\nu_R(3)}$, $\nu_R^{\;3} / \nu_{n^0}^{\;2}$, 0.68986216 Hz for the hydrogen-quanta, the electron, Bohr radius, and the Rydberg constant. With any other power the A Hz values are very distant from 1 Hz. This power law pattern repeats when the powers are both multiplied by the consecutive integer series. Each ratio of $A_{\nu_{n^0}(n'n)}^{\nu(n'(n+1))}$ is related the $A_{\nu_{n^0}(n)}^{\nu(n+1)}$ raised to the integer, n', used to raise the powers as well. This is a classic harmonic resonance pattern as seen in music. These n and $n+1$ powers are therefore natural powers, and follow a spontaneous systematic pattern. The natural A denominator followed by the numerator powers for $\nu_{\delta R}$, $\nu_{\delta a_0}$, $\nu_{\delta e^-}$ are 2 and 3, 4 and 5, and 6 and 7. This is a consecutive integer series seen in many quantum systems. Since the power of the constant and the power of the neutron are separated by 1 the fractional power of the neutron for the constant with a power of 1 is always a partial harmonic fraction, $1 - (1/n)$ for constants with frequency equivalents of more than 1 and less than the neutron, Equation (9b). Note that this structure is similar to Equation (4) following a typical quantum constant pattern seen with the hydrogen quanta.

In such a system any ν, constant can be defined as the product of ν_{n^0} raised to partial harmonic fraction and $A_{\nu_{n^0}(1-(1/n))}^{\nu}$, Equation (9b). This approach allows for a unified definition of any constant based on raising the neutron frequency to integer fraction powers. This is analogous to Equations (4)-(8) where any constant can defined from other constants within power laws. In this case the neutron is chosen as the unifying constant since all of the hydrogen quanta arose from the neutron in the negative beta decay process. The neutron is also related to gravitational systems through neutron stars transforming into black holes. Other bases are associated with linear power law lines as well. If other quantum values are used for the fundamental frequency, ν_F the integer fractions change, but remain logically inter-related power laws.

It has also been shown that it is possible to accurately derive the properties of hydrogen from ν_{n^0} alone since it represents a ν_F, of a harmonic system [9]. This is also possible due to the mathematical characteristics of power laws and harmonic systems. These physical constants are related to the first four odd primes when used as denominators of their partial harmonic or harmonic fractions. The Rydberg constant, R is associated with the partial harmonic fraction, 2/3, the Bohr radius, a_0 with 4/5, the electron e^- with 6/7, and the reciprocal of the fine structure constant, $1/\alpha$ with the harmonic fraction 1/11. These, when used in conjunction with the only even prime, 2, represent a symmetry of the global system and the factor in kinetic energy equations scaling.

3.2. Assumptions

The assumptions enabling this derivation include: First, that all harmonic systems are defined by a fundamental frequency, ν_F. Second, the electron, Bohr radius, Rydberg constant are associated with known prime number denominators in partial harmonic fractions of the frequency of the neutron in power laws. These respectively are for the electron 6/7, for the Bohr radius 4/5, and for the Rydberg constant 2/3. Third, the weak and electromagnetic forces are scaled inversely (reciprocally in the frequency domain) across the X-axis of the power law representing quantum symmetry. This is a classic property of harmonic systems. This is an approximation of the true geometry based on the neutron as the ν_F [4]. Fourth, Planck time squared is related to a kinetic energy, and therefore related to the factor ½ in its scaling. Planck's time squared can be approximated by the fundamental frequency raised to a composite integer fraction related to the hydrogen quanta fractions. Fifth, the frequency equivalents of R, a_0, e^- and α are inter-related by known 2 and π ratio relationships in the linear domain [2] [3]. Sixth, there is a unique fundamental frequency closely related to the neutron that fulfills all of these restrictions simultaneously in both the linear and power law domains. Seventh, the values derived from that fundamental frequency are related to the known physical fundamental constants despite the absence of scaling physical data since the system is purely mathematical in character like π.

3.3. Conversion of Physical Constants to Frequency Equivalents, Exponents, δ, and Harmonic Fractions

All data for the fundamental constants were obtained from the websites: http://physics.nist.gov/cuu/Constants/ and www.wikipedia.org. The NIST site http://physics.nist.gov/cuu/Constants/energy.html has an online physical unit converter that can be used for these types of calculations so the values used in the model are all standard unit conversions. Energies in joules are divided by h for frequency equivalents. The speed of light, c, is divided by the frequency equivalent is wavelength. Masses in kg are converted to frequency equivalents by multiplying by the speed of light squared, c^2, and dividing by Planck's constant, h. The product of the Rydberg constant and c equals its frequency equivalent, ν_R.

All of the constants are evaluated as dimensionless ratios, ν Hz/ν Hz. Known physical values are denoted with subscript, "k". Derived values are labeled with subscript, "d". Floating point accuracy is based upon known quantum experimental data, of approximately 5×10^{-8}. This is related to the rest mass of the electron. The derived values are shown to five digits. Table 1 and Table 2 list the standard unit; frequency equivalent; integer power, n_{ip}; integer fraction power, n_{ifp}; partial harmonic integer fraction powers, δ; and log base ν_F, $\log_{\nu F}$; relative errors , *i.e.*, from the known experimental values of the constants evaluated in this paper.

Table 1 lists the values of the various $[(v)s]$ and the slope and Y-intercept of the *wkδ*-line, *bwk*, *awk*; and slope and Y-intercept of the *EMδ*-line used for the derivations of e^-, a_0, R, α, t_P, a unit *BH*. The derived values closely approximate the known values as seen by the small relative errors, (r.e.).

Table 2 lists the physical constants, quantum numbers, standard unit values, frequency equivalents, n_{ip}, n_{ifp}, exponents, δs, and the integer or partial harmonic fractions. Here nip stands for "n" integer power, and nif for "n" integer fraction power. The derived values closely predict the known values as seen by the small relative errors, (r.e.).

The harmonic power law domain has a set of integer or integer fraction powers applied to the base the fundamental frequency, in this case, v_{n^0} for the known values, and v_F for the generalized setting which when exponentiated are related to the frequency equivalent of that specific constant's value, Equation (12). Equation (10a) shows the natural logarithmic conversion of the frequency equivalent, v, of any known physical constant, where the annihilation frequency of the neutron, v_{n^0}, is chosen as the fundamental logarithmic base, v_F. This results in a partial harmonic quantum fraction qf plus a small variation δ. These δ values represent the log base v_F equivalents of the A values, Equation (10a).

$$\log_{v_{n^0}}(v_k) = \frac{\log_e(v_k)}{\log_e(v_{n^0})} = 1 - \left(\frac{1}{n_{ifp}}\right) + \delta_k = qf + \delta_k$$
$$= 1 - \left(\frac{1}{n_{ifp}}\right) + \left(\frac{\log\left(A^{v}_{v_{n^0}(1-(1/n))}\right)}{\log(v_{n^0})}\right) \tag{10a}$$

These A values do not represent errors, but are mathematically imperatives since any fundamental frequency raised to the known discrete integer fractions powers related to 2, (10/1155) or π, (29/1155) do not exactly equal 2 or π. These δs and *A*s "shim" these power values to exactly 2 or π from Equations (4)-(8). This is essential so that there is a single fundamental frequency for all entities. We refer to the integer fractions as quantum fractions, *qf*. Not all fractions are partial harmonic or harmonic fractions. There are composite constants such as Planck time. Equation (10a) then shows how we use $qf + \delta$ as the combined exponent of the neutron's dimensionless base, v_{n^0}, to recover the dimensionless equivalent of the physical constant. Since all of the constants are evaluated as dimensionless ratios. The calculations are dimensionless then the units can be reconstructed. What we find here is that when the neutron is used as the fundamental base, the physical constants we discuss here are readily derived. Computationally, the dimensionless base of the neutron,
$\log_e(v_{n^0}) = 53.780055612(22)$.

In Equation (10b) we depict a sequential process starting from an arbitrary fundamental base (v_F), which is used to convert any dimensionless constant to its natural logarithmic equivalent, $\log_e(v_F)$, to obtain $qf + \delta$. The resulting partial harmonic quantum fraction plus its δ then becomes the power of the chosen arbitrary fundamental frequency base.

$$\log_{v_F}(v)=\frac{\log_e(v)}{\log_e(v_F)}=qf+\delta_d=1-\left(\frac{1}{n_{ifp}}\right)+\delta_d$$
$$=qf+\delta_d=1-\left(\frac{1}{n_{ifp}}\right)+\left(\frac{\log\left(A^{v}_{v_{n^0}(1-(1/n))}\right)}{\log(v_F)}\right) \quad (10b)$$

The known or derived log base v_F minus the quantum integer fraction, *qf*, or partial harmonic fraction equals the known or derived δ. Equation (11a) uses the neutron's dimensionless constant whereas Equation (11b) does the same for an arbitrary dimensionless v_F base. The known or derived frequency equivalent of a constant v is calculated by raising (v_{n^0}) or (v_F) to the sum power.

$$y_k=\delta_k=\log_{v_{n^0}}(v)-qf=\log_{v_{n^0}}(v)-\left(1-\frac{1}{n_{ifp}}\right)=\left(\frac{\log\left(A^{v}_{v_{n^0}(1-(1/n))}\right)}{\log\left(v_{n^0}\right)}\right) \quad (11a)$$

$$y_d=\delta_d=\log_{v_F}(v)-qf=\log_{v_F}(v)-\left(1-\frac{1}{n_{ifp}}\right)=\left(\frac{\log(A^{v}_{v_F(1-(1/n))})}{\log(v_F)}\right) \quad (11b)$$

Equation (12a) demonstrates that either the base v_{n^0} or as shown in Equation (12b) the generalized form v_F, when raised to the known or derived sum power , equals the known or derived frequency equivalent.

$$v_k=\left(v_{n^0}\right)^{\left(1-\frac{1}{n_{ifp}}+y_k\right)}=\left(v_{n^0}\right)^{\left(1-\frac{1}{n_{ifp}}+\delta_k\right)}=\left(v_{n^0}\right)^{(qf+\delta_k)} \quad (12a)$$

$$v_d=(v_F)^{\left(1-\frac{1}{n_{ifp}}+y_d\right)}=(v_F)^{\left(1-\frac{1}{n_{ifp}}+\delta_d\right)}=(v_F)^{(qf+\delta_d)} \quad (12b)$$

3.4. Estimate of v_F from $8\pi^2$ and the Partial Fractions of the Electron, Bohr Radius, and Rydberg Constant

An estimate of v_F can be made directly from an integer fraction power related to the harmonic partial fractions of the electron, Bohr radius, and Rydberg constant; and $8\pi^2$. From Equation (4) if the δ values were all equal to 0, therefore, a linear geometry, v_F raised to the composite power of the sum of 4/5, 4/5, −2/3, 6/7, or 8/105 must equal $8\pi^2$. Therefore v_F must equal $8\pi^2$ raised to 105/8, or $8.002768195282 \times 10^{24}$ Hz. This value is close to the frequency equivalent of the neutron, $2.2718590(01) \times 10^{23}$ Hz. The actual geometry is more complicated.

3.5. The Neutron 2D Power Law Domain

Figure 1 is a plot of the power law relationships plotted with the v_F equal to the v_{n0} [4]. Each individual fundamental constant is plotted as a point on a power law plane. The X-axis is scaled by $\log_{v_{n^0}}(v_{n^0})$ equaling 1, or by $\log_{v_F}(v_F)$. There are two points that scale the X-axis: the point for 1 Hz which is related to Planck's constant, h at (−1, 0), and the neutron, n^0 or v_F point at (0, 0). Planck's constant in the frequency domain equals a dimensionless 1 [4]. The log value equals 0. The X-axis is related to the partial harmonic or quantum fractions minus 1. This centers the ν_F at (0, 0), and takes into account that all of the constants are divided by ν_F.

There are two fundamental lines expressed in linear form as ax + b defined by four natural units that scale the global δ or Y-axis power law. These are referred to as the δ-lines, as shown in **Table 1** and **Table 2**. Points falling on a single line represent a power law. The first power law line, is defined by the Bohr radius point, (−1/5, δ_{a_0}) and by the electron (−1/7, δ_{e^-}), where we utilize the primes 5 and 7 as harmonic fractions. This is referred to as the weak kinetic line, "wk". The Y-intercept is defined as "bwk_k", 3.51638329(18) × 10^{-3}, and its slope is "awk_k", 3.00036428(15) × 10^{-3}. Their derivations are shown in references [4] [7] [9] [11], **Table 1**.

The second δ-line is defined by the points (−1, 0), 1 Hz, and the ionization energy of hydrogen, *R*, (−1/3, δ_R), where we utilize the prime, 3. This is referred to as the electromagnetic, (*EM*) line. The Y intercept is defined as "bem_k", −3.45168347(17) × 10^{-3}, and the slope as "aem_k", −3.45168347(17) × 10^{-3}. This is referred to as "bem_k" only.

3.6. Simplified 2D Power Law Domain

The same points are plotted using a simplified power law geometry which can be derived to the first approximation from the ν_F only (**Figure 2**). The X-axis is scaled by $\log_{\nu_F}(\nu_F)$ equaling 1. There are two points that scale the X-axis: the 1 Hz point related to h at (−1, 0), and the derived ν_F point at (0, 0). The X-axis is related to the partial harmonic or quantum fractions minus 1.

3.7. The Power Law y Axis Scaling Related to ν_F and Planck Time Squared

Planck time squared, $t_P{}^2$, in the frequency domain is equivalent to the Newtonian gravitational constant [5]. The product of $t_P{}^2$ and the frequency equivalents of two masses and the distance separating them equals the gravitational binding energy in Hz. From the perspective of the gravitational binding energy of the electron in hydrogen $t_P{}^2$ equals the ratio of the frequency equivalent of the binding energy divided the product of the frequencies of the proton, electron, and Bohr radius. This is in units of seconds squared. The gravitational binding energy frequency equivalent of the electron is nearly equals to the scalar reciprocal of ν_{n^0} divided by 2. Therefore, from the integer and partial fraction perspective $t_P{}^2{}_d$ can be approximated as ν_{n^0} raised to the power −128/35, (−1−1−4/5−6/7), $-2^7/(7 \times 5)$, −3.6571428571, all divided by 2, Equation (13a). Equation (13b) is the generalized ν_F form.

$$\left(t_P{}^2\right)_k \approx \frac{1}{2\left(\nu_{n^0}\right)^{128/35}} \tag{13a}$$

$$\left(t_P{}^2\right)_d = \frac{1}{2\left(\nu_F\right)^{128/35}} \tag{13b}$$

Table 1. List of known and derived natural units.

Physical constant	Value
ν_{n^0}, known Hz	$2.2718590(01) \times 10^{23}$ Hz
ν_F, derived HZ, r.e. = 5.7×10^{-2}	$2.40132968929221 \times 10^{23}$ Hz
$\log_e(\nu_{n^0})$, known	53.780055612(22)
$\log_e(\nu_F)$, derived, r.e. = 1.031×10^{-3}	53.83547976
bwk_k, y-intercept, weak force, *wk* line $bwk_k = \left(\dfrac{\left(\dfrac{\nu_{e^-}{}^{7/2}}{\nu_{n^0}\nu_{a_0}{}^{5/2}} \right)}{\log_e(\nu_{n^0})} \right)$	$3.51638329(18) \times 10^{-3}$
$bwk_d = (\log_e(2)/\log_e(\nu_F))/(128/35)$ r.e. = 1.1952×10^{-3}	3.5206×10^{-3}
$(\nu_{n^0})^{bwk_k}$, $(\nu_{F_d})^{bwk_d}$ r.e. = 4.2123×10^{-4}	1.20817540, 1.20868432
awk_k, slope, weak force, *wk* line $awk_k = \left(\dfrac{\left(\dfrac{\nu_{e^-}}{\nu_{n^0}{}^{2/35}\nu_{a_0}} \right)^{35/2}}{\log_e(\nu_{n^0})} \right)$	$3.00036428(15) \times 10^{-3}$
$awk_d = (\log_e(2)/\log_e(\nu_F))/(128/35)$ r.e. = 1.7339×10^{-1}	3.5206×10^{-3}
$(\nu_{n^0})^{awk_k}$, $(\nu_{F_d})^{awk_d}$ r.e. = 2.8573×10^{-2}	1.175107647381330, 1.20817540
bem_k, Y-intercept, electromagnetic, *EM* line $bem_k = \left(\dfrac{\left(\dfrac{\nu_{e^-}{}^{3/2}}{\nu_{n^0}} \right)}{\log_e(\nu_{n^0})} \right)$	$-3.45168347(17) \times 10^{-3}$
$bem_d = (\log_e(1/2)/\log_e(\nu_F))/(128/35)$ r.e. = 1.9962×10^{-2}	-3.5206×10^{-3}
$(\nu_{n^0})^{bem_k}$, $(\nu_{F_d})^{bem_d}$ r.e. = 3.8931×10^{-3}	8.3057942×10^{-1}, 8.2735×10^{-1}
aem_k, slope, electromagnetic, *EM* line	$-3.45168347(17) \times 10^{-3}$
$aem_d = (\log_e(1/2)/\log_e(\nu_F))/(128/35)$ r.e. = 1.9962×10^{-2}	-3.5206×10^{-3}
$(\nu_{n^0})^{aem_k}$, $(\nu_{F_d})^{aem_d}$ r.e. = 3.8931×10^{-3}	8.3057942×10^{-1}, 8.2735×10^{-1}

Planck time squared, $t_P{}^2$, is a composite single scaling factor on the Y-axis. It scales three important constants in Physics: Planck's constant, *h*; the speed of light, *c*; and the Newtonian gravitational constant, *G*, into a time/frequency unit. The composite of these two slopes awk_k and aem_k, and their respective Y-intercepts, bwk_k and bem_k, define the line associated with the $t_P{}^2$ point [5].

The slope of the line joining the Planck time squared point $\left[(-128/35)-1, \delta\left(t_P{}^2\right)_d\right]$, to the 1 Hz point, (−1, 0) scales the entire Y-axis. Since kinetic phenomena are associated with the factor 1/2, this splits the harmonics off of the x-axis. The value for $\delta_{1/2}$ equals $\log_e(1/2)/\log_e(\nu_{n^0})$, which calculates to $-1.2888554 \times 10^{-2}$. The slope and Y-intercept of the line from the estimated derived Planck time squared $\left[(-128/35)-1, \delta\left(t_P{}^2\right)_d\right]$ through the 1 Hz point, (1, 0) for the neutron is 3.5242141(2) × 10^{-3}, which nearly equals bwk_k, and the known value in Equation (14a), and likewise in Equation (14b) for the generalized ν_F. This slope should logically represent an estimate of the scaled "*bwk*" and "*EM*" lines, slopes and intercepts of the power law and are referred as bwk_d, awk_d, and bem_d, Equations (14a, 14b), and **Table 1**, **Table 2**.

$$bwk_k \approx awk_k \approx -bem_k \approx -aem_k \approx \left(\frac{\log_e(2)}{\log_e\left(\nu_{n^0}\right)}\right) \Big/ \left(\frac{128}{35}\right) = 3.5242141(2)\times 10^{-3} \quad (14a)$$

$$bwk_d = awk_d = -bem_d = -aem_d = \left(\frac{\log_e(2)}{\log_e(\nu_F)}\right) \Big/ \left(\frac{128}{35}\right) \quad (14b)$$

3.8. Derivation of ν_{F_d}, ν_{R_d}, $\nu_{a_0 d}$, $\nu_{e^- d}$

Arbitrary powers of e that define ν_F are evaluated from 1 to 60, as shown in **Figure 3**. In **Figure 3** the X-axis is the $\log_e(\nu_F)$ of the fundamental frequency. The Y-axis is a plot of several equations as a function of 'x'. Equation (4) is the derived line $y_1 = \left(8\pi^2\right)_d - 8\pi^2$. Equation y_2 represents the difference of the derived fine structure constants, α_d from Equations (5) and (6) in the text. Equation y_3 represents the difference of the derived α_d from Equations (5) and (8). The circle is centered at the known logarithm of the neutron's frequency, $\log_e\left(\nu_{n^0}\right)$. The point where these differences converge to zero very closely approximates the actual physical value, shown enlarged in the black box (c.f. **Table 1**, **Table 2**). This convergence point is the derived exponent of ν_{F_d}, which is slightly larger than the known value. The set of ν_F ranges from 2.7182818284 to 3.069849640 × 10^{69} Hz. The derived slopes and Y-intercepts for awk_d, bwk_d, and bem_d for each ν_F were calculated from Equation (14b). The geometry was assumed to be related to a symmetric slope and intercept pattern as seen in **Figure 2** as an estimate of the true state. [4] The log(ν)/log(ν_F), and δ_d were derived for each ν_F, for each derived frequency equivalent of *R*, ν_{R_d}, a_0, $\nu_{a_0 d}$, e^-, $\nu_{e^-_d}$, and $1/\alpha$, as computed in Equations (15)-(17). The partial harmonic fractions for *R*, 2/3, a_0, 4/5, and the e^- 6/7 are utilized. The δ values are calculated by the product of the partial harmonic fraction and the derived bem_d, and bwk_d values, Equation (14).

$$\nu_{R_d} = \nu_{F_d}{}^{((2/3)\times(1+bem_d))} \quad (15)$$

$$\nu_{a_{0_d}} = \nu_{F_d}{}^{((4/5)\times(1+bwk_d))} \quad (16)$$

$$\nu_{e^-_d} = \nu_{F_d}{}^{((6/7)\times(1+bwk_d))} \quad (17)$$

The derived value for $8\pi^2{}_d$ was calculated from Equation (4). For example, the difference equals $8\pi^2$ minus $\nu_{a_{0d}}$ squared divided by the product of $\nu_{e^-_d}$ and ν_{R_d}. Three different derived α_ds were calculated based on Equations (5)-(6) and Equation (8). For example, α_d equals $\nu_{a_{0d}}$ divided the product of 2, π, and $\nu_{e^-_d}$. These include: $\alpha_{a_0 e^-}$; α_{Ra_0}; and α_{Re^-}. The arithmetic differences between

Table 2. Experimental standard units, known and derived values.

Constant unit	n_{ip} or n_{ifp}	$1 \pm 1/n_{ifp}$ or qf
Electromagnetic energy × s, h, at a Hz of 1	0	
$6.62606957(29) \times 10^{-3}$ $Js \times 1$ Hz, $\log_{(\nu n0)} = 0$	n_{ip}	0
Neutron, elemental mass, fermion, known	1	
$939.565378(21) \times 10^{6}$ MeV/c^2,		
$2.2718590(01) \times 10^{23}$ Hz, $\log_e = 53.780055612(22)$, $\log_{(\nu n0)} = 1$	n_{ip}	1
n^0 derived, 993.10×10^{6} MeV/c^2		
$2.40132968929221 \times 10^{23}$ Hz, r.e. = 5.7×10^{-2}		
$\log_e = 53.83547976$, r.e. = 1.031×10^{-3}		
Rydberg constant, R, EM energy, boson, known	−3	2/3 =
$1.09737315(5) \times 10^{7}$ m^{-1}, $3.28984196(17) \times 10^{15}$ Hz		
$\log_{(\nu n0)} = 2/3 - 2.30112231(11) \times 10^{-3} = 0.664365544(33)$	n_{ifp}	1 − (1/3)
R, derived 1.1357×10^{7} m^{-1}, 3.4048×10^{15} Hz		
r.e. = 3.4946×10^{-2}		
$\log_{(\nu Fd)} = 2/3 - 2.34706 \times 10^{-3} = 0.66431960$		
Bohr radius, a_0, distance, known	−5	4/5 =
$0.52917721092(17) \times 10^{-10}$ m, $5.66525639(28) \times 10^{18}$ Hz		
$\log_{(\nu n0)} = 4/5 + 2.9163104(2) \times 10^{-3} = 0.8029163(1)$	n_{ifp}	1 − (1/5)
Bohr radius, a_0, derived		
0.50887×10^{-10} m, 5.8913×10^{18} Hz, r.e. = 3.9901×10^{-2}		
$\log_{(\nu Fd)} = 4/5 + 2.8165 \times 10^{-3} = 8.02816469$		
Electron, e^-, mass, matter, fermion, known	7	6/7 =
$0.510998910 \times 10^{6}$ eV/c^2, $1.235589964(62) \times 10^{20}$ Hz		
$\log_{(\nu n0)} = 6/7 + 3.08775982(16) \times 10^{-3} = 0.86023061(04)$	n_{ifep}	1 − (1/7)
Electron, e^-, derived		
0.53393×10^{6} eV/c^2, 1.2910×10^{20} Hz, r.e. = 4.4881×10^{-2}		
$\log_{(\nu Fd)} = 6/7 + 3.017645 \times 10^{-3} = 0.86016050$		
Fine structure constant, coupling constant, α, known	−11	1/11 =
α, $7.29735257 \times 10^{-3}$, $1/\alpha$, 137.035999(7)	n_{ifp}	1 − (10/11)
Fine structure constant, α_d, derived		
7.2626×10^{-3}, $1/\alpha_d$, 137.69218, r.e. = 4.7656×10^{-3}		
Planck time squared, t_P^2, known		
$1.82611(11) \times 10^{-86}$ s^2,		−128/35
$\log_{(\nu n0)} = (-128/35) - 1.37371(8) \times 10^{-2} = -3.6708799$		
Derived Planck time squared, $t_P{}^2{}_d$		
1.5601×10^{-86} s^2, r.e. = 1.4534×10^{-1}		
$\log_{(\nu Fd)} = (-128/35) - 1.2888554 \times 10^{-2} = -3.67003$		
t_P known, $5.39106(32) \times 10^{-44}$ s		
t_P derived, 4.9839×10^{-44} s, r.e. = 7.5526×10^{-2}		
BH, $r_s = c \times s$ known		
2.0186×10^{35} kg, 2.7380×10^{85} Hz		128/35
BH, $r_s = c \times s$ derived , distance light travels in one unit of time		
2.3630×10^{35} kg, 3.2051×10^{85} Hz, r.e. = 1.7007×10^{-1}		128/35

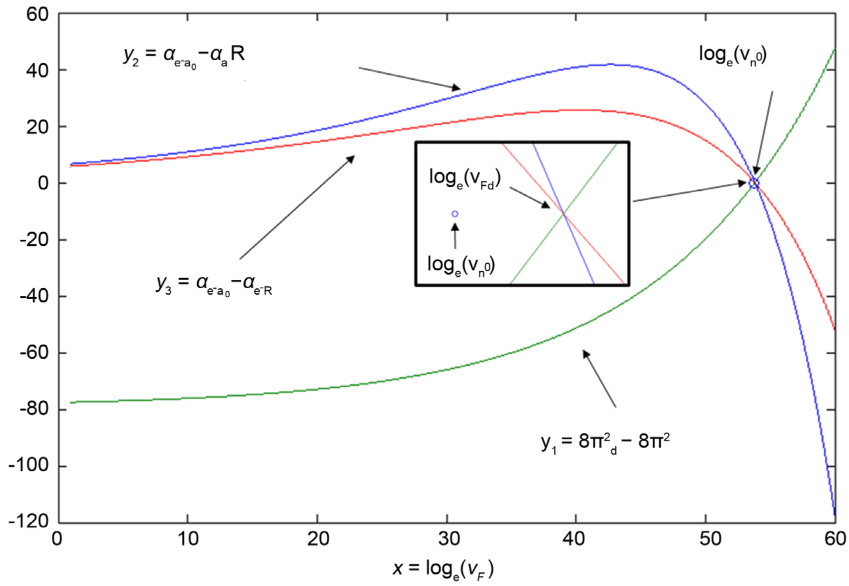

Figure 3. Differences of the Derived α_d's; and $8\pi^2{}_d$ versus $\log_e[(\nu_F)]$.

the derived values for of $8\pi^2{}_d$ and $8\pi^2$ were calculated for each ν_F. The arithmetic differences between the derived values for $\alpha_{a_0e^-}$ minus α_{Ra_0}, $\alpha_{a_0e^-}$ minus α_{Re^-}, and α_{Ra_0} minus α_{Re^-} were each individually calculated for each ν_F. The only valid values where both domains are fulfilled are those where the $8\pi^2$ and α_d differences all converge to zero at a common $\log_e(\nu_F)$ point, as shown in **Figure 3**. This derived ν_{F_d} validates our computed value for the fundamental frequency of the neutron.

These differences we depict between $8\pi^2$ and the various α (y_1, y_2, y_3) are plotted as the Y-axis values and the X-axis as the $\log_e(\nu_F)$ in **Figure 3**. The differences all converge to 0 at the $\log_e(\nu_F)$ value of 53.83547976, the relative error from the known value of the frequency of the neutron is 1.031×10^{-3}, **Table 1**, **Table 2**. The known exponential value for $\nu_{n^0}s$ is 53.780055612(22). The frequency equivalent of the derived value for ν_{F_d} is $2.40132968929221 \times 10^{23}$ Hz, with relative error of 5.7×10^{-2}. The known value for ν_{n^0} is $2.2718590(01) \times 10^{23}$ Hz.

The derived value for Rydberg's constant, R is $1.1357 \times 10^{7}\ \text{m}^{-1}$, with relative error 3.4946×10^{-2}. The known value for R is $1.09737315(5) \times 10^{7}\ \text{m}^{-1}$. The derived value for the Bohr radius is 0.50887×10^{-10} m, with relative error 3.9901×10^{-2}. The known value for a_0 is $0.52917721092(17) \times 10^{-10}$ m. The derived value for the electron is $0.53393 \times 10^{6}\ \text{eV/c}^2$, with relative error 4.4881×10^{-2}. The known value for the mass of the electron is $0.510998910 \times 10^{6}\ \text{eV/c}^2$. The accuracy of our computations based solely upon the derived fundamental frequency, when compared with known values, appears non-coincidental.

3.9. Derivation of α

The derived fine structure constant, α_d, was calculated from Equation (5). The derived value is 7.2626×10^{-3}, relative error 4.7656×10^{-3}. The known value of α is $7.29735257 \times 10^{-3}$.

3.10. Derivation of t_P

The derived no h bar Planck time squared, $t_{Pd}{}^2$ was calculated from ν_F in Equation (14b). The derived value is 1.6601 × 10^{-86} s^2, relative error 1.4534 × 10^{-1}. The known value is 1.82611(11) × 10^{-86} s^2. The derived h bar Planck time, t_{Pd} was calculated from ν_F in Equation (14b). The derived value is 4.9839 × 10^{-44} s, relative error 7.5526 × 10^{-2}. The known value is 5.39106(32) × 10^{-44} s.

3.11. Derivation of a Unit BH

Equation (18) proposes both a definition and derivation of the mass of a Unit Black Hole, m_{BH}, with a Schwarzschild radius, r_s, of one light second, one unit of time, which equates to $c \times s$ meters. This distance, $c \times s$, is associated with Compton wavelength of a wave with a frequency of 1 Hz.

$$m_{BH} = \frac{c^3 \times s}{2G} \tag{18}$$

The equivalent mass is 2.0186 × 10^{35} kg, with a frequency of 2.7380 × 10^{85} Hz, $\nu_{m_{BH}}$, equates to 1.012 × 10^5 $M_\odot$, a mass well-beyond the Chandrasekhar Limit of 1.4 $M_\odot$, Equation (19a). The derived Unit Black Hole frequency ν_{BH_d} can also be calculated from Equation (19b). The derived value for the frequency of this Black Hole with a Schwarzschild radius of one light unit of time in based ν_{F_d} is 3.2051 × 10^{85} Hz. The equivalent mass is 2.3630 × 10^{35} kg. The relative error is 1.7007 × 10^{-1}, Table 2.

$$\nu_{BH} = \frac{1}{2t_P{}^2} = 2.7380 \times 10^{85}\ \text{Hz} \tag{19a}$$

$$\nu_{BH_d} = \frac{1}{2t_P{}^2{}_d} = \left(\nu_{F_d}\right)^{(128/35)} = 2.6160 \times 10^{85}\ \text{Hz} \tag{19b}$$

4. Results

Table 1 and Table 2 demonstrate that the derived values are close approximations to the known values. The smallest relative error is the exponent of the ν_{F_d}, or 1.031 × 10^{-3}. The largest relative error is the derived mass of the Unit BH_d, 1.7 × 10^{-1}. Most of the constants are within 5 or 10 percent of the known values.

Not all possible harmonic fraction values are associated with valid difference values that converge to 0. The smallest valid consecutive integer series is {2, 3, 4}, but these are not all consecutive primes. The smallest consecutive prime number valid series is {3, 5, 7} which corresponds to the known values. The other consecutive prime series, {2, 3, 5}, {5, 7, 11}, {11, 13, 17}, and {13, 17, 19} are not valid, for use as harmonic fractions. The consecutive prime series {7, 11, 13} is valid.

5. Discussion

A fundamental question asks "is there a limit beyond which physics can no longer be defined purely based on mathematics?" The standard consensus interpretation is that there is a limit, and there is no pure mathematical foundation to physics independent of any physical reality, and there is controversy related to multiple universe theories [1]. Every well-understood aspect of physics is defined by mathematics. Then why is it not logical to assume that the whole system is purely mathematical, and independent of physical phenomena? Physics refers to physical phenomena, but in the extreme is defined solely by pure mathematics. This model is based on harmonic systems which represent self-organizing systems.

The intimate connection between describing the physical world and pure mathematical constructs have been demonstrated in recent papers: Wallis formula, alluded to above that derives $\pi/2$, is imbedded in the mathematics of the possible orbital levels of the hydrogen atom [14]. Another paper describes the relationship of prime numbers in numbers theory and the quarks [27]. And at least one author has speculated that primes can be associated with classical quantum states [28]. In this paper we show that there are two domains, one related to harmonic ratio relationships of electromagnetic constants, and the other related to a gravity power law. When both of these domains mathematical requirements are fulfilled the derived set of values accurately correspond to the known physical constants. These values are not based on any actual physical scaling data. There are known mathematical requirements, but each has no scale uniquely within its own domain. In Equations (4)-(8) it is possible to derive one constant from others in absence of any physical data. Therefore this type of derivation is common in the quantum domain rather than the exception. In essence this paper shows that both gravity and electromagnetic properties are intimately inter-related to the same constants, but in different domains. The known physical constants are therefore a unified system linking gravity, electromagnetic, kinetic, and quantum.

We have demonstrated the relationship of the first five prime numbers to the electron, Bohr radius, Rydberg constant, and the fine structure constant in derivations from the neutron. [9] We have also demonstrated that the fundamental constant organizations including the quarks are related to progressive composites of certain primes [8]. In both cases the smallest possible logical primes are those that are actually seen in physical systems. We have shown in this work that the smallest consecutive primes {2, 3, 5} cannot fulfill both the power law and linear domains of the physical constants. These primes do not represent the known physical pattern primes.

The next smallest possible consecutive prime number set is {3, 5, 7}, and these do represent the actual physical domain values. The integer associated with α need not be any specific value, but the known value is 11 again supporting our observation that the physical constants are dependent on a unique set of progressive primes. This is similar the Pauli exclusion rule and other quantum systems, but based on prime numbers. This is logical that each prime factor is associated with a physical entity in a pure mathematical system since primes are unique.

It should be possible to make more exact derivations from a power law geometry that more closely approximates the true 2D power law geometry [4]. The derivation in this paper is intentionally primitive to make the process simpler (but in Einstein's own words "not simple"). This approach does not take into account the vacuum energy or the deformity of space by gravity. In physics and in mathematics, harmonic systems are frequently "slightly" asymmetrically split from the purenumerical mathematical harmonics as is seen in this case. A good quantum example is the electron g-spin factor from 2. In music the actual frequencies that humans recognize as the most "harmonic" sounds are not exactly the true harmonic fraction values, but slightly split from those values since the overtones demonstrate beat phenomena.

6. Conclusion

It is possible to derive some of the most important fundamental constants in the absence of any actual scaling physical data. This is possible since there are well-defined known 2D geometric relationships of the frequency equivalents of the electron, Bohr radius, Rydberg constant and fine structure constant in the linear harmonic domain, and these same factors within gravity and a power law domain. One domain is in the harmonic linear domain, the other in a harmonic partial fraction power law domain. The unique sets of values, which can fulfill both domains for a single fundamental frequency, are closely related to the actual physical domain. This suggests that the fundamental constants represent a unified harmonic spectrum like other quantum spectra.

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