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Record Values from the Inverse Weibull Lifetime Model: Different Methods of Estimation

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Abstract

In this paper, we use the lower record values from the inverse Weibull distribution (IWD) to develop and discuss different methods of estimation in two different cases, 1) when the shape parameter is known and 2) when both of the shape and scale parameters are unknown. First, we derive the best linear unbiased estimate (BLUE) of the scale parameter of the IWD. To compare the different methods of estimation, we present the results of Sultan (2007) for calculating the best linear unbiased estimates (BLUEs) of the location and scale parameters of IWD. Second, we derive the maximum likelihood estimates (MLEs) of the location and scale parameters. Further, we discuss some properties of the MLEs of the location and scale parameters. To compare the different estimates we calculate the relative efficiency between the obtained estimates. Finally, we propose some numerical illustrations by using Monte Carlo simulations and apply the findings of the paper to some simulated data.

Keywords: Scale Parameter, Location Parameter, Best Linear Unbiased Estimates (BLUEs), Maximum Likelihood Estimates, Relative Efficiency and Monte Carlo Simulations

1. Introduction

Record values arise naturally in many real life applications involving data relating to weather, sport, economics and life testing studies. Many authors have studied record values and the associated statistics; see, for example, [1-7]. Reference [8] has established some recurrence relations for the moments of record values from the Gumbel distribution. Similar work has been carried out by [9,10] for the generalized extreme value and exponential distributions, respectively. Reference [11] have also discussed some inferential methods based on record values from Gumbel distribution. [12,13] have discussed inferential techniques based on Weibull and generalized Pareto distributions, respectively. Reference [14] have compared different estimates based on record values from Weibull distribution. Reference [15] has considered different loss functions to develop the Bayesian estimates of the parameters of the IWD.

Let $X_{L(1)}, X_{L(2)}, \ldots, X_{L(n)}$ be the first n lower record values from the IWD with density function (pdf)

$$f(x) = cx^{-c-1}e^{-x^{-c}}, \; x \geq 0, \;\; c > 0, \tag{1.1}$$

and cumulative distribution function (cdf)

$$F(x) = e^{-x^{-c}}, \quad x \geq 0, \;\; c > 0. \tag{1.2}$$

The scale form of the IWD has its density function given by

$$f(y) = \frac{c}{\sigma}\left(\frac{\sigma}{y}\right)^{c+1} \exp\left\{-\left(\frac{\sigma}{y}\right)^{c}\right\}, \; y \geq 0, \sigma > 0 \tag{1.3}$$

while the location-scale IWD has its density function given by

$$f(y) = \frac{c}{\sigma}\left(\frac{\sigma}{y-\theta}\right)^{c+1} \exp\left\{-\left(\frac{\sigma}{y-\theta}\right)^{c}\right\}, \; y \geq 0, \sigma > 0, \; \theta \geq 0 \tag{1.4}$$

Reference [16] calls the IWD as the complementary Weibull distribution, while [17] call it the reciprocal Weibull distribution. Reference [18] have discussed some useful measures for the IWD.

The IWD plays an important role in many applications, including the dynamic components of diesel engines and several data set such as the times to breakdown of an insulating fluid subject to the action of a constant tension. [19] provide an interpretation of the IWD in the context of the load-strength relationship for a component.

Reference [20] has fitted the IWD to the flood data. For more details on the IWD, see for example, [21].

The joint density function of the first n lower record values $X_{L(1)}, X_{L(2)}, \ldots, X_{L(n)}$ is given by [5]

$$f_{1,2,\ldots,n}(x_{L(1)}, x_{L(2)}, \ldots, x_{L(n)}) = f(x_{L(n)})\prod_{i=1}^{n-1}\frac{f(x_{L(i)})}{F(x_{L(i)})}. \quad (1.5)$$

From (1.5), the pdf. of $X_{L(m)}$ can be obtained as

$$f_m(x) = \frac{1}{\Gamma(m)}\{-\log[F(x)]\}^{m-1} f(x),\ x \geq 0,\ m = 1,2,\ldots, \quad (1.6)$$

where $f(.)$ and $F(.)$ are given in (1.1) and (1.2), respectively.

The the joint pdf of $X_{L(m)}$ and $X_{L(n)}$ is given by

$$f_{m,n}(x,y) = \frac{1}{\Gamma(m)\Gamma(n-m)}\{-\log[F(x)]\}^{m-1}\{-\log[F(y)] + \log[F(x)]\}^{n-m-1}\frac{f(x)}{F(x)}f(y),\ 0 \leq y < x < \infty, \quad (1.7)$$
$$m, n = 1,2,\ldots, m < n,$$

where $f(.)$ and $F(.)$ are given in (1.1) and (1.2), respectively.

The single and product moments of record values are (see [22])

$$\mu_m^{(i)} = \frac{\Gamma(m-\frac{i}{c})}{\Gamma(m)},\ i < mc, \quad (1.8)$$

and

$$\mu_{m,n}^{(i,j)} = \frac{\Gamma(m-\frac{i}{c})\Gamma(n-\frac{i+j}{c})}{\Gamma(m)\Gamma(n-\frac{i}{c})},\ i+j < nc,\ m < n, \quad (1.9)$$

where $\Gamma(\cdot)$ is the gamma function.

In the following section, we derive the exact form of the BLUE scale parameter and present the BLUEs of the location-scale case of IWD. Next in Section 3, we derive the maximum likelihood estimates of the parameters of IWD. Finally, in Section 4 we discuss the relative efficiency of the obtained estimates.

2. The BLUEs

In this section, we derive the BLUE of the scale parameter and present the BLUEs of the location and scale parameters of the IWD.

2.1. The Scale Case

Let $Y_{L(1)}, Y_{L(2)}, \cdots, Y_{L(n)}$ denote the first n lower record from the distribution in (1.3), and let $X_{L(i)} = Y_{L(i)}/\sigma,\ i = 1,2,\ldots,n$ be the corresponding record values from the IWD in (1.1). Assume

$$Y = \left(Y_{L(1)}, Y_{L(2)}, \cdots, Y_{L(n)}\right)^T,$$
$$\mu = \left(\mu_1, \mu_2, \cdots, \mu_n\right)^T$$
$$1 = \underbrace{(1,1,\cdots 1)}_{n}^T$$

and $\sum = ((\sigma_{i,j})), 1 \leq i,\ j \leq n$. Then, the BLUE of the scale parameter σ is given by

$$\sigma_1^* = \left\{\frac{\mu^T\Sigma^{-1}}{\mu^T\Sigma^{-1}\mu}\right\} Y = \sum_{i=1}^{n} a_i Y_{L(i)}, \quad (2.1)$$

and its variance is given by

$$Var(\sigma_1^*) = \sigma^2\left\{\frac{1}{\mu^T\Sigma^{-1}\mu}\right\}, \quad (2.2)$$

for details, refer to [23,6]. Since the double moment $\mu_{m,n}$ can be written as

$$\mu_{m,n} = p_m q_n,\ p_m = \frac{\Gamma(m+1/c)}{\Gamma(m)}\ and$$
$$q_n = \frac{\Gamma(n-2/c)}{\Gamma(n-1/c)} - \frac{\Gamma(n-1/c)}{\Gamma(n)},\ n > 2/c, \quad (2.3)$$

then the covariance matrix Σ can be inverted analytically. Then $(i,j)^{th}$ element σ^{ij} of Σ^{-1} can be derived as

$$\sigma^{ij} = \begin{cases} -\dfrac{c(ic-1)\Gamma(i+1)}{\Gamma(i-2/c)}, & j = i+1,\ i = 1,2,\ldots,n-1, \\ \dfrac{(2i^2c^2 - 2ic^2 - 4ic + c^2 + 2c + 1)\Gamma(i)}{\Gamma(i-2/c)}, & i = j = 2\ and\ n-1, \\ \dfrac{(c-1)^2}{\Gamma(1-2/c)}, & i = j = 1, \\ \dfrac{q_{n-1}}{q_n}\dfrac{c(nc-c-1)\Gamma(n)}{\Gamma(n-1-2/c)}, & i = j = n \\ 0, & j > i+1. \end{cases} \quad (2.4)$$

From (1.8), (2.1) and (2.4), we have the BLUE of the scale parameter σ as

$$\sigma_1^* = a_n y_{L(n)} = \frac{\Gamma(n)}{\Gamma(n-1/c)} Y_{L(n)}, \quad (2.5)$$

with variance is given by

$$Var(\sigma_1^*) = \left(\frac{\Gamma(n)\Gamma(n-2/c) - \Gamma^2(n-1/c)}{\Gamma^2(n-1/c)}\right)\sigma^2,\ \text{n} > 2/\text{c}. \quad (2.6)$$

It is clear from (2.5) that σ^* is an unbiased estimate of

σ.

Table 1 below shows the coefficients a_n of the BLUE of σ when $n = 4$ to 7 and $c = 3,4,5$.

The BLUE of σ given in (2.5), can be used to construct $100(1-\alpha)\%$ confidence interval for σ through the formula

$$P\left(\frac{\Gamma(n-1/c)\sigma_1^*}{\Gamma(n)T_{1-\alpha/2}} \le \sigma \le \frac{\Gamma(n-1/c)\sigma_1^*}{\Gamma(n)T_{\alpha/2}}\right) = 1-\alpha, \qquad (2.7)$$

where $T_{\alpha/2}$ and $T_{1-\alpha/2}$ are the lower and upper percentage points of the pivotal quantity

$$T = \frac{\sigma_1^*\Gamma(n-1/c)}{\sigma\Gamma(n)}.$$

The cdf of T is obtained to be $F_T(t) = \frac{\Gamma(n,t^{-c})}{\Gamma(n)}$, where $\Gamma(n,t^{-c})$ is the incomplete gamma function.

Example

Five lower record values are simulated from IWD with $c = 3$ and $\sigma = 1.0$ as follows: 3.07586, .90607, 68454, .62296, .62283, by using the coefficients of the BLUE of the scale parameter given in **Table 1**, we have $\sigma_1^* = 1.63139(.62283) = 1.016079$, and the standard error of this estimate is $S.E.(\sigma_1^*) = 1.016079\sqrt{.02692} = 0.166711$. Now, we calculate the lower and upper 5% percentage points of of T to be $T_{0.025} = 0.46048001250$ and $T_{0.975} = 0.8508449965$. Then 95% confidence interval for σ can be calculated from (2.7) to be $(0.7320149148, 1.352569185)$.

2.2. The location-Scale Case

Reference [15] has used the single and product moments of the record values from IWD. Then, he has used these moments to calculate the coefficients of BLUEs and the variances for records of size 4,5,6 and 7 and $c = 3,4,5$ by using the forms (see [23]).

$$\theta_2^* = \sum_{i=1}^{n} A_i Y_{L(i)}, \ and \ \sigma_2^* = \sum_{i=1}^{n} B_i Y_{L(i)}. \qquad (2.8)$$

Table 2 represents the coefficients of the BLUEs A_i and B_i for records of sizes 4, 5, 6, 7 and the shape parameter $c = 3,4,5$. while **Table 3** represents the variances and covariances of the BLUEs in this case.

Reference [24] has used the coefficients of BLUEs in **Table 2** to construct different confidence intervals for the location and scale parameters of IWD based on Edgeworth approximation and compare them with those based on Monte Calro simulation.

3. The MLEs

In this section, we discuss the maximum likelihood estimates of the parameters of IWD when the available data are lower record values. We consider two different cases: 1) the scale-parameter case and 2) the location-scale parameter case.

3.1. The Scale-Parameter Case

Let $Y_{L(1)}, Y_{L(2)}, \ldots, Y_{L(n)}$ represents the first n lower record values from the scale-parameter IWD in (1.3), then the log likelihood function is given by

$$\ell(c,\sigma) = nc\log(\sigma) + n\log(c) - \sigma^c y_{L(n)}^{-c} - (c+1)\sum_{i=1}^{n}\log y_{L(i)}. \qquad (3.1)$$

Now, we discuss two cases. They are:

1) When σ unknown and c known: the maximum likelihood estimate of σ can be obtained from (3.1) as

$$\hat{\sigma}_1 = n^{1/c} y_{L(n)}. \qquad (3.2)$$

2) When both of σ and c are unknown: the maximum likelihood estimates of σ and c can be obtained from (3.2) by solving the following two equations as

$$\hat{c} = n\left(\sum_{i=1}^{n}\log\left(\log(x_{L(i)})\right) - n\log\left(x_{L(n)}\right)\right)^{-1}, \qquad (3.3)$$

and

$$\hat{\hat{\sigma}}_1 = n^{1/\hat{c}} y_{L(n)}. \qquad (3.4)$$

From (3.2), we see that

Table 1. The coefficients of the BLUE of the scale parameter and the variance when $\sigma = 1$.

	$c = 3$		$c = 4$		$c = 5$	
n	a_n	$Var(\sigma_1^*)$	a_n	$Var(\sigma_1^*)$	a_n	$Var(\sigma_1^*)$
4	1.49544	03548	1.35655	.01929	1.27818	.01211
5	1.63139	.02692	1.44699	.01476	1.34545	.00931
6	1.74792	.02169	1.52314	.01194	1.40151	.00756
7	1.85073	.01815	1.58937	.01003	1.44984	.00636

Table 2. The Coefficients of the BLUEs.

	c = 3		c = 4		c = 5	
n	A_i	B_i	A_i	B_i	A_i	B_i
4	-0.2353	0.3519	-0.5294	0.7182	-0.8421	1.0764
	-1.4118	2.1112	-1.7647	2.3939	-2.1053	2.6909
	-2.1176	3.1668	-2.3529	3.1919	-2.6316	3.3636
	4.7647	-5.6299	5.6471	-6.3040	6.5789	-7.1309
5	-0.1400	0.2284	-0.3383	0.4896	-0.5591	0.7523
	-0.8400	1.3704	-1.1278	1.6319	-1.3978	1.8807
	-1.2600	2.0555	-1.5038	2.1759	-1.7473	2.3509
	-1.6200	2.6428	-1.8045	2.6111	-2.0161	2.7126
	4.8600	-6.2972	5.7744	-6.9085	6.7204	-7.6966
6	-0.0942	0.1647	-0.2395	0.3649	-0.4071	0.5706
	-0.5653	0.9881	-0.7985	1.2162	-1.0178	1.4265
	-0.8479	1.4821	-1.0646	1.6216	-1.2723	1.7831
	-1.0902	1.9055	-1.2776	1.9459	-1.4680	2.0575
	-1.3082	2.2866	-1.4601	2.2239	-1.6311	2.2861
	4.9058	-6.8270	5.8403	-7.3725	6.7964	-8.1238
7	-0.0684	0.1266	-0.1809	0.2875	-0.3143	0.4556
	-0.4104	0.7595	-0.6029	0.9582	-0.7857	1.1391
	-0.6156	1.1393	-0.8038	1.2776	-0.9821	1.4239
	-0.7915	1.4648	-0.9646	1.5331	-1.1332	1.6429
	-0.9498	1.7578	-1.1024	1.7521	-1.2591	1.8255
	-1.0959	2.0282	-1.2249	1.9468	-1.3686	1.9842
	4.9316	-7.2763	5.8794	-7.7552	6.8429	-8.4712

Table 3. The variances and covariances of the BLUEs when $\sigma = 1$.

c	n	$Var(\theta_2^*)$	$Var(\sigma_2^*)$	$Cov(\theta_2^*, \sigma_2^*)$
3	4	0.3152	0.6834	-0.4713
3	5	0.1875	0.4901	-0.3059
3	6	0.1262	0.3803	-0.2206
3	7	0.0916	0.3103	-0.1696
4	4	0.3128	0.5646	-0.4243
4	5	0.1999	0.4138	-0.2893
4	6	0.1415	0.3255	-0.2156
4	7	0.1069	0.2680	-0.1698
5	4	0.3135	0.5055	-0.4007
5	5	0.2082	0.3739	-0.2801
5	6	0.1516	0.2960	-0.2124
5	7	0.1170	0.2447	-0.1696

$$E(\hat{\sigma}_1) = n^{1/c} E(\sigma x_{L(n)}), \tag{3.5}$$

which upon using (1.8) gives

$$E(\hat{\sigma}_1) = \sigma n^{1/c} \frac{\Gamma(n-1/c)}{\Gamma(n)}. \tag{3.6}$$

This shows that

$$\tilde{\sigma}_1 = \frac{\Gamma(n)}{\Gamma(n-1/c)} y_{L(n)}, \tag{3.7}$$

is an unbiased estimate for σ in this case.

The variance of $\tilde{\sigma}_1$ is calculated to be

$$Var(\tilde{\sigma}_1) = \left(\frac{\Gamma(n)\Gamma(n-2/c) - \Gamma^2(n-1/c)}{\Gamma^2(n-1/c)} \right) \sigma^2, \text{ n} > 2/\text{c}. \tag{3.8}$$

Lemma 1

The MLE of σ given in (3.2) is asymptotically unbiased and its variance converges to zero as $n \to \infty$.

Proof The proof can be easily done by using the expansion of gamma function see [24]

$$\Gamma(z) = z^{z-1/2}\exp(-z)\sqrt{2\pi}\left(1+\frac{1}{12z}+\frac{1}{288z^2}+O(\frac{1}{z^3})\right).$$

3.2 The Location-Scale Case

When $Y_{L(1)}, Y_{L(2)}, \ldots, Y_{L(n)}$ represents the first n lower record values from the location-scale parameter IWD given in (1.4), then the log likelihood function is given by

$$l(c,\sigma,\theta) = n\log\left(\frac{c}{\sigma}\right)-\left(\frac{y_{L(n)}-\theta}{\sigma}\right)^{-c}-(c+1)\sum_{i=1}^{n}\left(\log(x_{L(i)}-\theta)-\log(\sigma)\right). \tag{3.9}$$

Now, we discuss two cases. They are:

1) When c is known: the maximum likelihood estimates of θ and σ can be obtained by solving the following two equations

$$\hat{\sigma}_2 = n^{1/c}\left(y_{L(n)}-\hat{\theta}_2\right), \tag{3.10}$$

$$(c+1)\sum_{i=1}^{n}\left(y_{L(i)}-\hat{\theta}_2\right)^{-1}-\frac{nc}{y_{L(n)}-\hat{\theta}_2}=0. \tag{3.11}$$

Table 4 below displays the bias and the estimated variance of the MLEs of the location and scale parameters in this case.

2) When c is unknown: the maximum likelihood estimates of σ, θ and c can be obtained from (3.9) by solving the following three equations as

$$\frac{n}{c}-\left(\frac{\sigma}{y_{L(n)}-\theta}\right)^{c}\log\left(\frac{\sigma}{y_{L(n)}-\theta}\right)-\sum_{i=1}^{n}\log\left(\frac{y_{L(i)}-\theta}{\sigma}\right)=0, \tag{3.12}$$

$$\sigma = n^{1/c}\left(y_{L(n)}-\theta\right), \tag{3.13}$$

$$(c+1)\sum_{i=1}^{n}\left(y_{L(i)}-\theta\right)^{-1}-\frac{nc}{y_{L(n)}-\theta}=0. \tag{3.14}$$

4 Relative Efficiency

To compare between the BLUEs and the MLEs obtained in Sections 2 and 3, we calculate the relative efficiency in some cases as follows:

1) For the scale case we have $RE(\sigma_1^*,\tilde{\sigma}_1)=1$,

2) For the location-scale case we have

$$RE(\theta_2^*,\hat{\theta}_2)=\frac{Var(\theta_2^*)}{S^2_{\hat{\theta}_2}},\ and\ RE(\sigma_2^*,\hat{\sigma}_2)=\frac{Var(\sigma_2^*)}{S^2_{\hat{\sigma}_2}}.$$

Table 5 given below show the relative efficiency be-

Table 4. The bias and estimated variance of the MLEs.

c	n	$Bias(\hat{\theta}_2)$	$Bias(\hat{\sigma}_2)$	$S^2_{\hat{\theta}_2}$	$S^2_{\hat{\sigma}_2}$
3	4	0.05838	0.04222	0.35204	0.96583
3	5	0.04556	0.01134	0.30296	0.96390
3	6	0.06768	-0.11985	0.21086	0.45111
3	7	0.18492	-0.41586	0.14638	0.32753
4	4	0.17717	-0.13855	0.39000	0.58523
4	5	0.06416	-0.00930	0.35768	0.48147
4	6	0.10480	-0.13350	0.21182	0.39013
4	7	0.02483	-0.01781	0.15638	0.34454
5	4	0.05524	0.01944	0.51265	0.92268
5	5	0.15418	-0.14972	0.26795	0.42049
5	6	0.14928	-0.16165	0.21410	0.38703
5	7	0.12132	-0.14616	0.18654	0.28381

Table 5. The relative efficiency.

	$c = 3$		$c = 4$		$c = 5$	
n	$RE(\theta_2^*,\hat{\theta}_2)$	$RE(\sigma_2^*,\hat{\sigma}_2)$	$RE(\theta_2^*,\hat{\theta}_2)$	$RE(\sigma_2^*,\hat{\sigma}_2)$	$RE(\theta_2^*,\hat{\theta}_2)$	$RE(\sigma_2^*,\hat{\sigma}_2)$
4	0.89535	0.70758	0.80205	0.96475	0.61153	0.54786
5	0.61889	0.50846	0.55888	0.85945	0.77701	0.88920
6	0.59850	0.84303	0.66802	0.83434	0.70808	0.76480
7	0.62577	0.94739	0.68359	0.77785	0.62721	0.86220

-tween the different estimates of the location and scale parameters. From **Table 5**, we see that, the BLUEs give more efficiency than the MLEs.

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