Distributed $H_\infty$ Consensus of High-Order Multi-Agents with Nonlinear Dynamics*

Jianzhen Li

School of Automation, Nanjing University of Science and Technology, Nanjing, China
E-mail: jianzhenli1983@yahoo.com.cn
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Abstract

This paper deals with the distributed consensus problem of high-order multi-agent systems with nonlinear dynamics subject to external disturbances. The network topology is assumed to be a fixed undirected graph. Some sufficient conditions are derived, under which the consensus can be achieved with a prescribed $H_\infty$ norm bound. It is shown that the parameter matrix in the consensus algorithm can be designed by solving two linear matrix inequalities (LMIs). In particular, if the nonzero eigenvalues of the laplacian matrix according to the network topology are identical, the parameter matrix in the consensus algorithm can be designed by solving one LMI. A numerical example is given to illustrate the proposed results.

Keywords: Consensus, Multi-Agent Systems, Nonlinear Dynamics, External Disturbances

1. Introduction

The consensus problem of multi-agent systems has been researched extensively in recent years. This is because of its widely application in much areas such as flocking [1-2], synchronization of coupled oscillators [3], formation control of mobile robots [4-5], distributed computation [6] and information fusion in wireless sensor networks [7]. The object of consensus control is to design consensus protocol such that the group of agents can asymptotically agree upon certain quantities of interest based on information received from their neighbors.

Most of the work on the consensus problem focuses on the multi-agent systems with first-order dynamics. In particular, [8] deals with the first-order multi-agent systems with switching topologies and time delays in a continuous setting. The fist-order multi-agent systems with switching topologies is investigated in [9] in a discrete-time setting. The consensus problem has also been investigated from many other aspects such as reference signals [10], asynchronous sampling time [11], and so on. Recently, the consensus problem of second-order multi-agent systems has been investigated extensively [12-14]. In particular, the consensus problem of second-order multi-agent systems with nonlinear dynamics was investigated in [15]. The nonlinear dynamics can be taken as the potential functions or the desired final dynamics of the agents. There is also some work on the consensus problems of high-order multi-agent systems [16-17].

Generally speaking, the consensus cannot be achieved accurately if there are external disturbances. To deal with this problem, the $H_\infty$ consensus problem is considered [18-21]. It is shown that for undirected network topologies, the desired parameter matrix in the consensus algorithm can be designed by solving two LMIs, which relate to the system matrix of the agents and the smallest eigenvalues of the laplacian matrix corresponding to the network topology. The $H_2$ consensus problem was investigated in [22].

In the aforementioned work on the $H_\infty$ or $H_2$ consensus problem, the nonlinear dynamics was not considered. As is mentioned in [14-15] much multi-agent systems have nonlinear dynamics. Motivated by this, this paper considers the $H_\infty$ consensus problem of high-order multi-agent systems with nonlinear dynamics. To the best of the author’s knowledge, this problem has not been considered in the literature. Some sufficient conditions will be derived, under which the consensus can be achieved with a prescribed $H_\infty$ norm bound. It will be shown that the parameter matrix in the consensus algorithm can be designed by solving two LMIs, which relate to the system matrix of the agents and the smallest and

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the biggest nonzero eigenvalue of the laplacian matrix corresponding to the network topology. In particular, if the nonzero eigenvalues of the laplacian matrix according to the network topology are identical, the parameter matrix in the consensus algorithm can be designed by solving one LMI.

2. Preliminary Notations and Problem Formulation

Let \( G = (\mathcal{V}, E, A) \) be a weighted undirected graph of order \( N \), where \( \mathcal{V} = \{1, \ldots, N\} \) is the node set. \( E \subseteq \mathcal{V} \times \mathcal{V} \) is a set of unordered pairs of nodes, and \( A \) is the adjacency matrix. An undirected path is a sequence of edges in a undirected graph of the form \((v_i, v_j), (v_j, v_k), \ldots, (v_l, v_i)\), where \( v_i, v_j, \ldots \in \mathcal{V} \). An undirected graph is called connected if for any two nodes of the graph, there exists a path that follows the edges of the graph. The adjacency matrix is a nonnegative matrix \( A_{ij} = [a_{ij}] \in \mathbb{R}^{N \times N} \) satisfying \( a_{ii} = 0 \) for any \( i \in \mathcal{V} \), \( a_{ij} = a_{ji} > 0 \) if \( (j,i) \in E \), and \( a_{ij} = 0 \) if agents \( j \) and \( i \) are not adjacent. The Laplacian matrix of the graph is defined as \( L = [l_{ij}] \in \mathbb{R}^{N \times N} \) with \( l_{ii} = \sum a_{ij} \) and \( l_{ij} = -a_{ij}, \forall i \neq j \). We can see that \( L \) satisfies \( L1 = 0 \) and \( 1^T L = 0 \) where \( 1 = [1, \ldots, 1]^T \). For matrices \( M \) and \( N \), \( M \otimes N \) denotes their Kronecker product.

It is well known that if the undirected network topology \( G \) is connected, the Laplacian matrix corresponding to \( G \) has \( N-1 \) positive eigenvalues and a simple zero eigenvalue.

Consider a group of \( N \) agents with the following dynamics:

\[
\dot{x}_i = Ax_i + Bu_i + B_1\omega(t) + B_2 f(x_i),
\]

where \( x_i(t) \in \mathbb{R}^m \), \( u_i(t) \in \mathbb{R}^p \) are, respectively, the state, the control input of agent \( i \), \( \omega(t) \in \mathbb{R}^m \) is the external disturbance which belongs to \( L^2_2 [0, \infty) \), and \( f(x_i) \in \mathbb{R}^m \) is a nonlinear function.

**Assumption 1:** There exists a positive scalar \( \alpha \) such that

\[
\left[ f(x_i) - f(x_j) \right]^T \left[ f(x_i) - f(x_j) \right] \leq \alpha (x_i - x_j)^T (x_i - x_j),
\]

\( \forall x_i, x_j \in \mathbb{R}^m \).

**Remark 1:** Assumption 1 is similar to the Assumption 1 in [14]. It is a Lipschitz-type condition satisfied by many systems.

**Definition 1:** We say algorithm \( u_i \) solves the consensus problem if

\[
x_i - \sum_{j=1}^N \frac{x_j}{N} \to 0, \quad t \to \infty, \quad \forall i \in \mathcal{V}.
\]

**Definition 2:** We say algorithm \( u_i \) solves the consensus problem with \( H_{\infty} \) norm bound \( \gamma \) if the following two conditions are satisfied:

1. Algorithm \( u_i \) solves the consensus problem if \( \omega = 0 \);
2. If \( z_0 = 0 \), the following inequality is satisfied:

\[
\int_0^t \| \dot{z} \|^2 dt < \gamma^2 \int_0^t |\omega|^2 dt,
\]

where

\[
z_i = x_i - \sum_{j=1}^N \frac{x_j}{N},
\]

\[
z = \left[ z_1^T \cdots z_N^T \right]^T,
\]

\[
\omega = \left[ \omega_1^T \cdots \omega_N^T \right]^T,
\]

and \( z_0 \) is the initial value of \( z \).

The object of the \( H_{\infty} \) consensus control is to design consensus algorithms such that the consensus problem is solved for a prescribed \( H_{\infty} \) norm bound.

**Lemma 1:** (Schur complement [23]) Let \( S \) be a symmetric matrix of partitioned form \( S = [S_{ij}] \) with \( S_{11} \in \mathbb{R}^{r \times r}, S_{12} \in \mathbb{R}^{r \times (n-r)} \) and \( S_{21} \in \mathbb{R}^{(n-r) \times r} \). Then, \( S < 0 \) if and only if \( S_{11} < 0, S_{22} - S_{12}S_{11}^{-1}S_{21} < 0 \) or equivalently \( S_{22} - S_{12}S_{21}^{-1}S_{11} < 0 \).

**Lemma 2:** For matrices \( A, B, C, D \) with appropriate dimensions, one has

\[
(A \otimes B)^T = A^T \otimes B^T, \quad (CD) = (A \otimes C)(B \otimes D)
\]

and \( (A+B) \otimes C = A \otimes C + B \otimes C \).

3. Results

In this section, the \( H_{\infty} \) consensus problem of multi-agent systems with nonlinear dynamics will be investigated. Considers the following state feedback consensus algorithm:

\[
u_i(t) = K \sum_{j=1}^N a_{ij} (x_i - x_j)
\]

With (2), system (1) becomes

\[
\dot{x}_i = Ax_i + BK \sum_{j=1}^N a_{ij} (x_i - x_j) + B_1\omega(t) + B_2 f(x_i),
\]

which can be written in a compact form as

\[
\dot{x} = (I_N \otimes A + L \otimes BK) x + (I_N \otimes B_1) \omega + (I_N \otimes B_2) f,
\]

where \( x = \left[ x_1^T \cdots x_N^T \right]^T \) and \( f = \left[ f^T(x_1) \cdots f^T(x_N) \right]^T \).

By the definition of \( z \) we have the consensus is achieved if and only if \( z \to 0 \) as \( t \to \infty \). It is easy to see that
\[ z = (H \otimes I_m)x, \quad (5) \]

where \( H \in \mathbb{R}^{N \times N} \) with \( H_{ij} = \begin{cases} N-1 & \text{if } i = j \\ -1/N & \text{if } i \neq j. \end{cases} \)

It can be seen that \( H = I_N - 11^T/N, \) \( H^2 = H, \) \( 1^T_i H = 0^T_i, \) \( H1_N = 0_N \) and \( H\mathcal{L} = \mathcal{L} = \mathcal{L}. \)

**Lemma 3:** There exists an orthogonal matrix \( U \in \mathbb{R}^{N \times N} \) with last column \( 1_N/\sqrt{N} H \) such that
\[
U^T HU = \begin{bmatrix} I_{N-1} & 0_{N-1} \\ * & 0 \end{bmatrix}, \quad U^T LU = \begin{bmatrix} \Delta & 0_{N-1} \\ * & 0 \end{bmatrix}.
\]

From (4) and (5) we have
\[
\dot{x} = (H \otimes I_m)x \\
= (H \otimes A + L \otimes BK)x + (H \otimes B_i)\omega + (H \otimes B_j)f \\
= (H \otimes A + L \otimes BK)\dot{x} + (H \otimes B_i)\omega + (H \otimes B_j)f.
\]

**Proof:** Assume that the undirected graph \( \mathcal{G} \) is connected, we have \( \lambda_2 > 0. \) Suppose there exists a symmetric positive definite matrix \( X \) and a matrix \( W \) such that the LMIs
\[
\begin{bmatrix}
\Xi & B_1 & X B_1^T & X \\
B_1^T & -\gamma^2 I_m & 0 & 0 \\
X & 0 & -1 & 0 \\
0 & 0 & 0 & I_m
\end{bmatrix} < 0,
\]
\( i = 2, N, \) hold, where
\[
\Xi = AX + X A^T + \lambda_i(BW + W^T B_i).
\]

In this case, the parameter matrix in (2) can be chosen as \( K = WX^{-1}. \)

Note that \( z^T z = \delta^T \delta = \delta^T \delta = \xi^T \xi, \) we conclude that algorithm (2) solves the consensus problem with \( H_\infty \) norm bound \( \gamma \) if and only if system (9) is asymptotically stable with \( \|T_{\infty}\| < \gamma, \) where \( \|T_{\infty}\| \) denotes the \( H_\infty \) norm of the transfer function matrix from \( \omega \) to \( \xi. \)

**Theorem 1:** Suppose the undirected graph \( \mathcal{G} \) is connected and the nonzero eigenvalues of \( \mathcal{L} \) are \( \lambda_2, \ldots, \lambda_N. \) Using algorithm (2), the consensus is achieved with \( H_\infty \) norm bound \( \gamma \) if there exists a symmetric positive definite matrix \( X \) and a matrix \( W \) such that the LMIs
\[
\begin{bmatrix}
\Psi & PB_1 & B_1^T & I_m \\
B_1^T P & -\gamma^2 I_m & 0 & 0 \\
I_m & 0 & -1 & 0 \\
0 & 0 & 0 & I_m
\end{bmatrix} < 0,
\]
\( i = 2, N, \) hold, where
\[
\Psi = PA + A^T P + \lambda_i(BW + W^T B_i).
\]

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\[ \Omega \triangleq \begin{bmatrix} \Gamma & \mathbf{I}_{N-1} \otimes (PB_i) \\ \mathbf{I}_{N-1} \otimes (PB_i)^T & -\gamma I_{(N-1)m} \end{bmatrix}, \]

where

\[ \Gamma = \mathbf{I}_{(N-1)m} + \text{diag} [\lambda_2, \ldots, \lambda_N] \otimes \left( \mathbf{PB} + \mathbf{K}^T \mathbf{B}^T \mathbf{P} \right) + \mathbf{I}_{N-1} \otimes (\mathbf{B}_2^T \mathbf{B}_2) + \mathbf{I}_{N-1} \otimes \left( \mathbf{PA} + \mathbf{A}^T \mathbf{P} + \alpha \mathbf{I}_m \right). \]

From (12) we know that \( \Omega < \mathbf{0} \).

Next prove that the consensus is achieved if \( \omega = 0 \).

If \( \omega = 0 \), (9) becomes

\[
\dot{\xi} = \left( \mathbf{I}_{N-1} \otimes \mathbf{A} + \text{diag} [\lambda_2, \ldots, \lambda_N] \otimes \mathbf{BK} \right) \xi + \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes \mathbf{B}_2 \right) f.
\]

(13)

Consider the Lyapunov function

\[ V(\xi) = g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} \right) \xi. \]

Because \( \mathbf{P} \) symmetric positive definite, we have \( V(\xi) \) is symmetric positive definite with respect to \( \xi \).

Taking derivative of \( V(\xi) \) along (13), we have

\[
\dot{V}(\xi) = 2g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} \right) (I_{N-1} \otimes \mathbf{A}) \xi + 2g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} \right) \left( \text{diag} [\lambda_2, \ldots, \lambda_N] \otimes \mathbf{BK} \right) \xi + 2g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} \right) \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes \mathbf{B}_2 \right) f.
\]

(14)

Because \( \mathbf{U} \) is an orthogonal matrix, one has

\[ \mathbf{I}_N = \mathbf{U}^T \mathbf{U}: \quad \mathbf{U}^T \mathbf{U} = \mathbf{I}_{N-1}. \]

Then we have

\[
2g^T \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes \mathbf{B}_2 \right) f \leq g^T \left[ \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{U}_i \right) \otimes (\mathbf{B}_2^T \mathbf{B}_2) \right] \xi + g^T \left[ (\mathbf{H}^T) \otimes \mathbf{I}_m \right] f.
\]

(15)

Notice that

\[
f^T \left[ (\mathbf{H}^T) \otimes \mathbf{I}_m \right] f = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{N} \left[ f(x_i) - f(x_j) \right]^T \left[ f(x_i) - f(x_j) \right] \\
\leq \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{N} (x_i - x_j)^T (x_i - x_j)
\]

\[= \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{N} (x_i - x_j)^T (x_i - x_j) = \alpha \mathbf{I}_m, \]

(16)

From (14)-(16) we have

\[
\dot{V}(\xi) \leq g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} + \text{diag} [\lambda_2, \ldots, \lambda_N] \otimes \left( \mathbf{PB} + \mathbf{K}^T \mathbf{B}^T \mathbf{P} \right) \right) \xi + g^T \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes \mathbf{B}_2 \right) f.
\]

(17)

It follows from (12) that \( \Psi + \mathbf{B}_2^T \mathbf{B}_2 + (\alpha + 1) \mathbf{I}_m < \mathbf{0} \), which, together with (17) implies that \( \dot{V}(\xi) \) is negative definite with respect to \( \xi \). Then it follows that \( \xi \rightarrow 0 \) asymptotically. From the analysis above we know the consensus can be achieved.

Assume that \( \omega \neq 0 \). Taking derivative of \( V(\xi) \) along (9), we have

\[
\dot{V}(\xi) = g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} \right) (I_{N-1} \otimes \mathbf{A}) \xi + g^T \left( \mathbf{I}_{N-1} \otimes \mathbf{P} \right) \left( \text{diag} [\lambda_2, \ldots, \lambda_N] \otimes \mathbf{BK} \right) \xi + g^T \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes \mathbf{B}_2 \right) f
\]

(18)

Assume that \( z_0 = 0 \), which implies that \( \xi_0 = 0 \), where \( \xi_0 \) is the initial state of \( \xi \). It follows that

\[
\int_0^\infty \| \xi \|^2 dt - \gamma \int_0^\infty \| \omega \|^2 dt
\]

\[= \int_0^\infty \left[ \| \xi \|^2 - \gamma \| \omega \|^2 \right] dt - V(\xi) + V_0 \]

\[\leq \int_0^\infty \| \xi \|^2 dt - \gamma \int_0^\infty \| \omega \|^2 dt \]

\[+ g^T \left( \mathbf{I}_{N-1} \otimes \left( \mathbf{PA} + \mathbf{A}^T \mathbf{P} + \mathbf{B}_2^T \mathbf{B}_2 + \alpha \mathbf{I}_m \right) \right) \xi \]

\[+ g^T \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes \left( \mathbf{PB} + \mathbf{K}^T \mathbf{B}^T \mathbf{P} \right) \right) \xi
\]

\[+ 2g^T \left( \mathbf{F}^T \mathbf{U}_i^T \mathbf{H} \otimes (\mathbf{P} \mathbf{B}) \right) \omega \]

(19)

\[= \int_0^\infty \eta^T \tilde{\Pi} \eta dt, \]

where

\[\eta = \left[ g^T \omega^T \right]^T, \quad V_0 = 0 \] is the initial value of \( V(\xi) \), and

\[\tilde{\Pi} \triangleq \begin{bmatrix} \Gamma & (\mathbf{F}^T \mathbf{U}_i^T \mathbf{H}) \otimes (\mathbf{P} \mathbf{B}) \\ (\mathbf{H} \mathbf{U}_i \mathbf{F}) \otimes (\mathbf{P} \mathbf{B}) & -\gamma^2 \mathbf{I}_{Nm} \end{bmatrix}.\]
By Lemma 1 we know that \( \Pi < 0 \) if and only if
\[
\Gamma + \text{diag} \left[ \lambda_2, \ldots, \lambda_N \right] \otimes \left( PBK + K^T B^T P \right)
\]
\[
+ \frac{1}{\gamma^2} \left[ \left( F^T U^T H \right) \otimes (PB_i) \right] \left[ \left( HU_i F \right) \otimes (PB_i)^T \right]
\]
\[
= \Gamma + \text{diag} \left[ \lambda_2, \ldots, \lambda_N \right] \otimes \left( PBK + K^T B^T P \right)
\]
\[
+ \frac{1}{\gamma^2} \left[ I_{N-1} \otimes \left( PB_i B_i^T P \right) \right] < 0.
\]

Also by Lemma 1 we know that (20) is equivalent to \( \Xi \leq \gamma \), which has been proved in the above analysis. So we have that \( \Pi \) is symmetric negative definite. It follows from (19) that

\[
\int_0^\infty \| x \|_2^2 dt - \gamma^2 \int_0^\infty \| x \|_2^2 dt \leq 0.
\]

Therefore, the consensus is achieved with \( H_\infty \) norm bound \( \gamma \). The proof is completed.

Sometimes, the laplacian matrix has \( N-1 \) identical nonzero eigenvalues, i.e. \( 0 < \lambda_2 = \cdots = \lambda_N \). Take the complete graph for example. Consider the complete graph with \( N \) nodes. The laplacian matrix is chosen as

\[
\mathcal{L} = \begin{bmatrix}
\frac{N-1}{N} & -1 & \cdots & -1 \\
-1 & \frac{N-1}{N} & \cdots & -1 \\
\vdots & \vdots & \ddots & \vdots \\
-1 & -1 & \cdots & \frac{N-1}{N}
\end{bmatrix}
\]

By some calculations we have the eigenvalues of \( \mathcal{L} \) are \( 0, \frac{1}{N-1}, \ldots, \frac{1}{N-1} \). In this case, we have the following corollary.

**Corollary 1:** Suppose the undirected graph \( G \) is connected and the nonzero eigenvalues of \( \mathcal{L} \) satisfy \( \lambda_2 = \cdots = \lambda_N \). Using algorithm (2), the consensus is achieved with \( H_\infty \) norm bound \( \gamma \) if there exists a symmetric positive definite matrix \( X \) and a matrix \( W \) such that the LMI

\[
\begin{bmatrix}
\Xi & B_i X B_i^T & X \\
B_i^T & -\gamma^2 I_m & 0 & 0 \\
B_2 & 0 & -I_m & 0 \\
X & 0 & 0 & -\frac{1}{\alpha+1} I_m
\end{bmatrix} < 0,
\]

holds, where \( \Xi = AX + XA^T + BM + M^T B^T \). In this case, the parameter matrix in (2) can be chosen as \( K = WX^{-1} \).

**Proof:** Assume that there exists a symmetric positive definite matrix \( X \) and a matrix \( W \) such that (21) holds.

Define \( XM \triangleq \frac{1}{\lambda_2} W \). It follows that

\[
\begin{bmatrix}
\Xi & B_i X B_i^T & X \\
B_i^T & -\gamma^2 I_m & 0 & 0 \\
B_2 & 0 & -I_m & 0 \\
X & 0 & 0 & -\frac{1}{\alpha+1} I_m
\end{bmatrix} < 0,
\]

holds for \( i = 2 \). From Theorem 1 we have the consensus is achieved with \( H_\infty \) norm bound \( \gamma \), and the parameter matrix can be chosen as \( K = WX^{-1} \).

**Remark 2:** From Corollary 1 one has that if the nonzero eigenvalues of the laplacian matrix are identical, the \( H_\infty \) performance is determined by \( \lambda_2 \) and the system matrices of the agents. It has no relationship with the number of the agents.

### 4. A Numerical Example

Consider a multi-agent systems consisted of \( N \) nodes with the following second-order

\[
\ddot{x}_i = v_i + \omega_i, \quad \dot{v}_i = u_i + \omega_i + \sin(0.5x_i),
\]

where \( \omega_i = \sin \left( \frac{1}{t+1} \right) \) is the external disturbance. This multi-agent system can be written in the form of (1) with

\[
A = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \end{bmatrix}.
\]

The communication topology is given in Figure 1. The laplacian matrix is chosen as

\[
\mathcal{L} = \begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 2 & -1 & 0 \\
0 & -1 & 2 & -1 \\
0 & 0 & 1 & -1
\end{bmatrix}.
\]

The eigenvalues of \( \mathcal{L} \) are \( 0, 0.5858, 2, 3.4142 \). Solving the LMIs in (10) with \( \lambda_2 = 0.5858 \), \( \lambda_4 = 3.4142 \), \( \alpha = 0.5 \) and \( \gamma = 1.29 \), we can get

\[
X = \begin{bmatrix}
0.0455 & -0.6688 \\
-0.6688 & 23.2996
\end{bmatrix}, \quad W = \begin{bmatrix} 0.1 & -2605.7 \end{bmatrix}.
\]

From Theorem 1 we know that \( K \) can be chosen as \( K = WX^{-1} = \begin{bmatrix} -2844.2 & -193.5 \end{bmatrix} \).

**Figure 2** shows the trajectory of the external disturbance. **Figures 3** and **4** show, respectively, the position and velocity responses of nodes \( 1 \rightarrow 4 \).
The $H_\infty$ consensus problem has been investigated in this paper, for the high-order multi-agent systems with nonlinear dynamics. Sufficient conditions have been given in the forms of LMIs, under which the $H_\infty$ consensus problem can be solved. The parameter matrix in the consensus algorithm can be designed by solving two LMIs. If the nonzero eigenvalues of the laplacian matrix according to the network topology are identical, the parameter matrix in the consensus algorithm can be designed by solving one LMI. The numerical simulation confirmed the proposed results.

6. References


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