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Economic Growth and Optimal Taxation: A Case of Mongolian Economy

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Abstract

We examined three cases of Schully model [1] [2] for Mongolian economy. In the first case, we consider the production function with constant returns to scale. In the second case, we employ an econometric model for the production function with none constant return to scale without constraints on parameters of elasticities. Finally, the constrained regression model has been implemented by solving a convex minimization problem over a convex set. Also, we have proved that Schully production function namely, "U shape function" in the literature [1] [2] [3] in fact is concave function under some assumptions.

Keywords

Tax Burden Rate, Growth, Scully Model, Log-Log Model

1. Introduction

Economic growth is the main macroeconomic indicator. There are many works [1]-[8] devoted to economic growth. In papers [1] [9] [10] [11] economic growth and taxation have been studied. Growth can be reached by the accumulation of capital and from innovations which lead to technical progress. Accumulation and innovation increase the productivity of inputs into production and provides the potential level of output.

The rate of growth can be affected by policy choices through the taxation. An increase in taxation reduces the returns to investment. Lower returns mean less accumulation and innovation, and hence a lower rate of growth. This is the negative aspect of taxation. Taxation also has a positive aspect. Some public expenditure can enhance productivity, such as the provision of infrastructure, public education, and health care. Taxation provides the means to finance these expenditures and, indirectly, can contribute to an increase in the growth rate. In most developed countries the level of taxes rose steadily over the course of the twentieth century: an increase from about 5% - 10% of gross domestic product (GDP) at the turn of the century to 30% - 40% at the end is typical [3]. Much of the literature on economic growth focused on the long-run equilibrium where output per head was constant or modelled growth through exogenous technical progress. By definition, when technical progress is exogenous it cannot be affected by policy. The development of endogenous growth theory has overcome these limitations by explicitly modelling the process through which growth is generated.

But the major question is what size of tax revenue is optimally suitable for a country to maximize growth. Barro (1990) developed a model showing growth-maximizing condition of lump sum taxation under the assumption of a balanced budget. His model indicates that maximum growth is achieved when the size of government equals to the share it would hold when public services were delivered competitively as an input of private sector.

Although his model indicates inverse U-shaped relationship between taxes and growth, the model has not been popular to quantify growth-maximizing tax level. But Scully (1995, 1996) developed a model to estimate optimal tax burden rate showing that tax-growth relationship is inverse U-shaped. While some recent studies use quadratic method to find the optimal size of tax revenue. They clearly show that relationship is positive up to a certain level and starts to become negative after crossing that level.

Unlike many traditional approaches which depict linear relationship between taxes and growth, nonlinear approach showing inverse U-shape is highly recognized as it admits both positive and negative behavior of taxation. However, a majority of the conclusions in this regard are drawn from developed countries and linear regression results. This research tries to estimate the growth-maximizing tax burden rate using the data from Mongolian economic statistics.

Scully himself conducted a series of studies to find the optimal tax size and found that around 20 percent of GDP is suitable for the higher growth. Scully (1991) used quadratic method by taking the data from 103 countries and observed that 19.3 percent of GDP is the growth-maximizing tax rate. In 1995, Scully surveyed the data of USA (1949-1989), and concluded that growth-maximizing tax rate was between 21.5 and 22.9 percent of GNP. Similarly, Scully (2003) used two models—Barro and Scully. He found that growth-maximizing tax rate for the USA is 25.1 and 19.3 percent, respectively. Scully (2006) again surveyed the data of USA (1929-2004), by using his own method developed in 1996, and found that the optimal tax rate for the highest growth is 23 percent of GDP.

Following the Scully method, some other studies have tried to calculate the optimal tax rate, however, the optimal size observed across the studies is not uniform. Chao and Grubel (1998) applied Scully method for Canadian data and explored that the optimal size of taxes for growth maximization in Canada is 34 percent of national income. Keho (2010) used the data of Cote d'Ivorie (1960 to 2006) and observed that optimal tax rate is between 22.1 and 22.3 percent of GDP.

Similarly, Abdullaev and Konya (2014) used the same method for Uzbekistan (1996-2011) and found that the optimal tax rate is 22 percent of GDP for prior to 2001 and 31.25 percent for post 2001. Davidson (2012) slightly modified the Scully (1996) method and applied in randomly selected 12 countries for the period of 1982-2012 and found a low tax rate of 11.1 percent of GDP. Similarly, a study conducted by Saibu (2015), using the data of South Africa and Nigeria, found that the growth-maximizing tax rate for South Africa is 15 percent and Nigeria is 30 percent of GDP. Likewise, Husnain, Haider and Salman (2015) used Scully model, with the inclusion of deficit term, for 4 South Asian countries and found that 13.78 % is the growth-maximizing tax rate. A brief review of related literature has been summarized in [3].

As it is noticed in [3] researches which cover low-income countries in Asia are very rare. Hence, in this paper, we did an attempt to find the optimal level of tax rate of Mongolian economy using Scully's model.

2. Methodology and Concavity Property of the Production Function

Based on Schully model (1996), we examine a relationship between economic growth and tax. We assume that there are two sectors in the economy which consists of government sector and private sector. Using labor and capital, government provides the public goods which are due to taxation. Goods of the private sector are results of the untaxed part of the national income. Government and private sectors are used in production of the final goods. Hence, total output is composition of the output of two sectors. The production function we use is a type of the Gobb-Douglas form [1] [2]:

$$Y_t = \alpha_0 \left(G_{t-1}\right)^{\alpha_1} \left[\left(1-T\right) Y_{t-1}\right]^{\alpha_2} \tag{1}$$

where, Y is the output, G is the government spending on public goods, T lamp sum tax rate for the time period t, α_1 and α_2 are elasticity coefficients. The government budget requires that tax revenue equals the cost of public goods provided which means that:

$$G_t = TY_t \tag{2}$$

where, T is the tax rate or a proportion of tax revenue in GDP.

Then combining (1) and (2), we can write

$$Y_t = \alpha_0 \left(TY_{t-1}\right)^{\alpha_1} \left[\left(1-T\right) Y_{t-1}\right]^{\alpha_2} \tag{3}$$

By definition of growth rate, we have

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \theta \text{ or } \frac{Y_t}{Y_{t-1}} = \theta + 1$$

Dividing both sides of the expression (3) by Y_{t-1}, we get

$$\frac{Y_t}{Y_{t-1}} = \alpha_0 T^{\alpha_1} \left(1-T\right)^{\alpha_2} \left(Y_{t-1}^{\alpha_1+\alpha_2-1}\right) \tag{4}$$

or

$$\theta = \alpha_0 T^{\alpha_2} \left(1-T\right)^{\alpha_2} \left(Y_{t-1}^{\alpha_1+\alpha_2-1}\right) - 1 \tag{5}$$

If we fix parameters α_1, α_2 and α_0 then θ is a function of *T*. Introducing a constant *A* as

$$A = \alpha_0 Y_{t-1}^{\alpha_1+\alpha_2-1}$$

we can write the function θ as

$$\theta = AT^{\alpha_1} \left(1-T\right)^{\alpha_2} - 1 \tag{6}$$

For the further purpose, we assume that:

$$\alpha_1 > 0, \alpha_2 > 0, \alpha_1 + \alpha_2 \le 1 \tag{7}$$

Now we prove the following assertion.

Lemma 1: Under the assumption (7) the function θ is concave.

Proof: The function θ can be presented a function of two variables x and y in the following:

$$\theta = f\left(x, y\right)$$

where $f\left(x, y\right) = Ax^{\alpha_1} y^{\alpha_2} - 1$, $x = T$ and $y = 1 - T$.

In order to prove concavity of the function θ, we need to check a sign of its second order derivative.

Compute first order derivative of the function θ as composite function.

$$\frac{\partial \theta}{\partial T} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial T} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial T} = \alpha_1 A x^{\alpha_1 - 1} y^{\alpha_2} - A\alpha_2 x^{\alpha_1} y^{\alpha_2 - 1}$$

It can be easily checked that the second order derivative is:

$$\begin{aligned} \frac{\partial^2 \theta}{\partial T^2} &= \frac{\partial^2 f}{\partial x^2} - 2\frac{\partial x}{\partial T} + \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 y}{\partial y^2} \\ &= \alpha_1\left(\alpha_1 - 1\right) x^{\alpha_1 - 2} y^{\alpha_2} \\ &= 2\alpha_1 \alpha_2 x^{\alpha_1 - 1} y^{\alpha_2 - 1} + \alpha_2\left(\alpha_2 - 1\right) x^{\beta_1} y^{\alpha_2 - 2} \\ &= x^{\alpha_1 - 2} y^{\alpha_2 - 2}\left[\alpha_1\left(\alpha_1 - 1\right) y^2 - 2\alpha_1\alpha_2 xy + \beta\alpha_2\left(\alpha_2 - 1\right) x^2\right] \end{aligned} \tag{8}$$

Now introduce the function $\varphi\left(x, y\right)$ as

$$\varphi\left(x, y\right) = \alpha_1\left(\alpha_1 - 1\right) y^2 - 2\alpha_1\alpha_2 xy + \alpha_2\left(\alpha_2 - 1\right) x^2 \tag{9}$$

and show that $\varphi\left(x, y\right) < 0$ for arbitrary $x > 0, y > 0$.

Indeed, we have

$$\varphi\left(x, y\right) = \frac{1}{x^2}\left[\alpha_1\left(\alpha_1 - 1\right) z^2 - 2\alpha_1\alpha_2 z + \alpha_2\left(\alpha_2 - 1\right)\right],$$

where, $z = \dfrac{y}{x}$.

$$\varphi\left(x, y\right) = \frac{1}{x^2} q\left(z\right),$$

where, $q\left(z\right) = \alpha_1\left(\alpha_1 - 1\right) z^2 - 2\alpha_1\alpha_2 z + \alpha_2\left(\alpha_2 - 1\right)$.

Define the discriminant of the quadratic term as

$$\begin{aligned} D &= \left[4\alpha_1^2\alpha_2^2 - 4\alpha_1\alpha_2(\alpha_1 - 1)(\alpha_2 - 1)\right] \\ &= 4\alpha_1\alpha_2\left[\alpha_1\alpha_2 - \alpha_1\alpha_2 + \alpha_1 + \alpha_2 - 1\right] \\ &= 4\alpha_1\alpha_2(\alpha_1 + \alpha_2 - 1) \end{aligned}$$

Hence, we have $q(z) \le 0$, and consequently, $\frac{\partial^2\theta}{\partial T^2} \le 0$ which proves the concavity of θ.

Now we consider the problem of maximizing economic growth with respect to tax rate. This problem is:

$$\max_T \theta \tag{10}$$

From (6) we can get

$$\frac{\partial\theta}{\partial T} = A\left[\alpha_1 T^{\alpha_1 - 1}(1-T)^{\alpha_2} - \alpha_2(1-T)^{\alpha_2 - 1}T^{\alpha_1}\right] = 0$$

$$\alpha_1 T^{\alpha_1 - 1}(1-T)^{\alpha_2} - \alpha_2(1-T)^{\alpha_2 - 1}T^{\alpha_1} = 0$$

$$\alpha_1(1-T) = \alpha_2 T$$

Hence, we have

$$T^* = \frac{\alpha_1}{\alpha_1 + \alpha_2} \tag{11}$$

which provides the maximum value to function θ.

Note that if assumption (7) violates, in other words, if $\alpha_1 + \alpha_2 > 1$ then function θ may not be concave on [0, 1], but T^* is still its maximum point.

It is easy to check that if $\alpha_1 + \alpha_2 = 1$ then the function $(\theta + 1)$ is constant returns to scale. Indeed, for any $\lambda > 0$, we can write

$$\begin{aligned} (\theta+1)\left(\lambda T, (1-\lambda)^T\right) &= A\left(\lambda T^{\alpha_1}\right)\left[\lambda(1-T)^{\alpha_2}\right] \\ &= \lambda^{\alpha_1+\alpha_2} A T^{\alpha_1}(1-T)^{\alpha_2} \\ &= \lambda\left[\theta(T, 1-T) + 1\right]. \end{aligned}$$

Then

$$T^* = \alpha_1 \tag{12}$$

For constant returns to scale case, the production function (5) has the form:

$$(\theta+1) = \alpha_0 T^{\alpha_1}(1-T)^{\alpha_2}, \alpha_1 + \alpha_2 = 1$$

Taking log from both sides, we obtain

$$\begin{aligned} \ln(1+\theta) &= \ln\alpha_0 + \alpha_1 \ln T + \alpha_2 \ln(1-T) \\ &= \ln\alpha_0 + \alpha_1 \ln T + \ln(1-T) - \alpha_1 \ln(1-T) \end{aligned}$$

or

$$\ln\left(\frac{1+\theta}{1-T}\right) = \ln\alpha_0 + \alpha_1 \ln\frac{T}{1-T} \tag{13}$$

3. Data Description

In 2018, the government budget revenue reached a historical record for the first time, exceeding over 10 trillion MNT.

Figure 1 shows government revenue and spending since 1991, the Mongolian transition to market economy period.

The government budget loss reached over 100 billion MNT for periods of 1997-2001 due to transition economy. Next budget difficulties occurred in the period of 2012-2017, the loss was over1 trillion MNT. The maximum loss reached in 2016 and it was 3.7 trillion MNT or 15.3 percent of GDP [12].

As it is shown in Figure 2, the government budget revenue to GDP ratio reached a maximum of 37.9 percent in 2007 and a minimum of 22.1 percent in 1996. An average, tax rate is 28.6 percent.

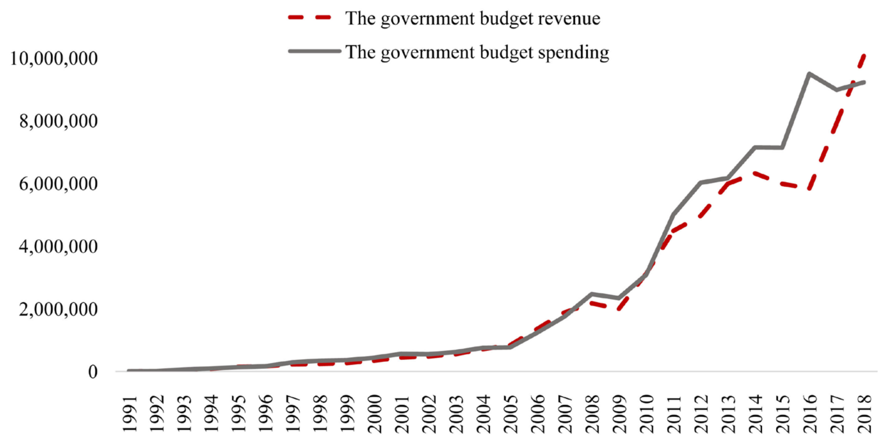

Figure 1. The government budget revenue and spending. Source: National Statistical Office. http://www.1212.mn/

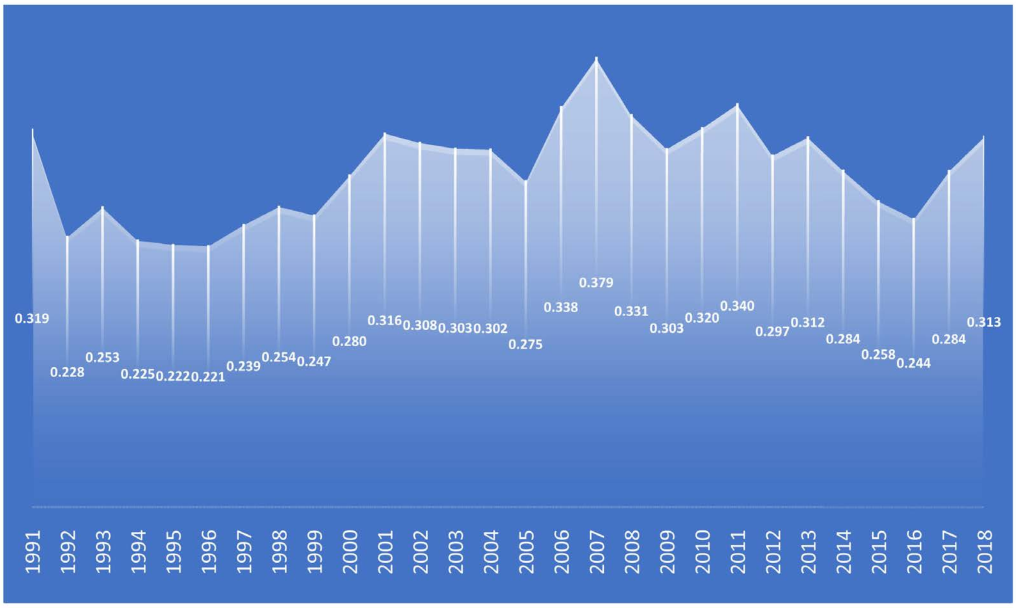

Figure 2. Tax rate. Source: National Statistical Office. http://www.1212.mn/

4. Numerical Results

For econometric analysis we use the following data which shows relationship between economic growth and tax burden of Mongolia for period 1991-2018 (**Table 1**).

Econometric analysis of Schully model [1] which employs the production function with constant returns to scale gives the following result:

$$\ln\left(\frac{1+G_t}{1-T_t}\right)=0.764+0.413\cdot\ln\left(\frac{T_t}{1-T_t}\right)$$
$$(16.5)\qquad(8.5)$$
$$R^2=0.73, DW=0.61$$

Tax rate $\alpha_1=0.413$ is too high for Mongolian economy so this estimation is not acceptable in practice. But estimation of parameters of the model by econometric analysis without constraints on parameters leads to the equation:

$$\ln\left(Y_t\right)=0.5+0.356\cdot\ln\left(T_{t-1}Y_{t-1}\right)+0.668\cdot\ln\left(\left(1-T_{t-1}\right)Y_{t-1}\right)$$
$$(2.5)\qquad(7.5)\qquad\qquad(11.7)$$
$$R^2=0.99, DW=0.97$$

If we compare the value of R-squared with the previous one, this was increased by 0.26. However, Durbin-Watson's test is low so there is supposed to be a long-term equilibrium between independent and dependent variables.

Johansen Cointegration test allows determining the number of cointegrated equations between the integrated terms with the same order. This test uses two statistics to determine the number of cointegrating vectors:

Table 1. Economic growth and tax burden.

Year	GDP, mln. MNT	Economic growth	Tax burden	Year	GDP, mln. MNT	Economic growth	Tax burden
1991	4,774,283.9	−8.7	0.319	2005	7,128,340.3	7.3	0.275
1992	4,330,275.5	−9.3	0.228	2006	7,738,256.8	8.6	0.338
1993	4,191,706.6	−3.2	0.253	2007	8,531,274.8	10.2	0.379
1994	4,279,732.5	2.1	0.225	2008	9,290,589.8	8.9	0.331
1995	4,553,635.4	6.4	0.222	2009	9,172,729.6	−1.3	0.303
1996	4,653,815.3	2.2	0.221	2010	9,756,588.6	6.4	0.320
1997	4,835,314.1	3.9	0.239	2011	11,443,578.3	17.5	0.340
1998	4,994,879.5	3.3	0.254	2012	12,853,406.8	12.5	0.297
1999	5,149,720.8	3.1	0.247	2013	14,350,689.1	11.6	0.312
2000	5,206,367.7	1.1	0.280	2014	15,482,273.4	8.1	0.284
2001	5,362,558.7	3	0.316	2015	15,847,217.2	2.5	0.258
2002	5,614,599.0	4.7	0.308	2016	16,047,782.7	1.4	0.244
2003	6,007,881.9	7	0.303	2017	16,873,380.6	5.2	0.284
2004	6,646,243.1	10.6	0.302	2018	18,059,484.1	7.0	0.313

- Trace test with the hypothesis corresponding to the existence of at most r cointegrating vectors;
- The maximum Eigen-value test with hypothesis corresponding to the existence of exactly r cointegrating vectors.

The results of the test based on Table 2 are given below.

The empirical results show that the null hypothesis ($r=0$ or $r\le 1$) for Trace test and null hypothesis ($r=0$ or $r=1$) for Maximum Eigen-value test which was not rejected at 5 percent. Consequently, these two cointegration tests cannot be confirmed that variables are cointegrated. Then, the optimal tax rate computed by Formula (11) is

$$T^* = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{0.356}{0.356+0.668} = 0.348$$

This result is still high so it cannot be accepted. That is why we need to implement the constrained regression model. For this case, in order to estimate parameters of nonconstant return production function by econometric model, we solve the following constrained convex minimization problem.

$$\min F\left(\alpha_0,\alpha_1,\alpha_2\right)$$
$$=\sum_{i=1}^{m}\left[\alpha_1\left(\ln T_i+\ln Y_{i-1}\right)+\alpha_2\left(\ln\left(1-T_i\right)+\ln Y_{i-1}\right)+\ln\alpha_0-\ln Y_{i-1}-\ln\left(1+\hbar_i\right)\right]^2$$

subject to constraint

$$\begin{cases}\alpha_1+\alpha_2\le 1\\ \alpha_1\ge 0,\alpha_2\ge 0\end{cases}$$

The problem was solved on Mongolian economic data from 2009 to 2018 by Matlab. The solution was $\alpha_1 = 0.29, \alpha_2 = 0.70$.

Then, the optimal tax rate computed by Formula (11) is

$$T^* = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{0.29}{0.29+0.70} = 0.296$$

5. Conclusions

In this paper, we define the optimal tax rate to provide economic growth of Mongolian economy based on Scully's model. We improve and modify the model by proving the concavity of the production function used in it and also reducing the parameter estimation problem to constrained optimization problem.

Table 2. Johansen cointegration test.

Trace тест				Maximum Eigen-Value тест			
H_0	H_1	Trace	5%	H_0	H_1	Max	5%
$r=0$	$r\ge 1$	22.6	29.79	$r=0$	$r=1$	15.89	21.13
$r\le 1$	$r\ge 2$	6.71	15.49	$r=1$	$r=2$	6.67	14.26
$r\le 2$	$r\ge 3$	0.03	3.84	$r=2$	$r=3$	0.032	3.84

We find an optimal tax rate as 29.6 percent of the GDP of Mongolia which is greater than average tax rate 28.6 by one percent. It means that at this point of tax rate economic growth will reach the maximum and policymakers should take this value into account in their decision making.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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