

Evaluation of Social Risk Using Structural Equation Model

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ABSTRACT

A quantitative method to evaluate social risk using structural equation model (SEM) is developed. Evaluation of social risk is the essential step in early warning of social risk. On the basis of the society stability, a scientific and applicable index system of social risk is put forward, which includes 6 first-level indexes and 40 second-level indexes. Based on these indexes and relationships among them, a structural equation model is introduced, and an improved partial least square (PLS) algorithm by finding the best iterative initial value is proposed.

Keywords: Social Risk Evaluation, Structural Equation Models, Unit Vector Constraint

1. Introduction

Nowadays, social conflict of different interest has become more and more intense, which consequently result in potential social risks. In 1986, Ulrich Beck the German prominent sociologist put forward the theory of risk society in his best works, Risk Society. Social risk refers to the uncertainty that endangers the social stabilities, equilibrium and development. Generally speaking, our social risk management gets along with economic mechanism very well, but because of the relationship and sequence between the reformation and social risk management, it's inevitable that social risk management policy often lags behind the reform. This kind of lagging makes social risk management unable to resolve the rough social problems in time. Therefore, it's necessary to appraise and supervise the social risk, to the effect that we can detect the incompatibility between the policies and the development of the society, and make necessary adjustment in time. Therefore, an evaluation system for social risk should be established in order to accurately predict and rapidly react in very early stage, with the purpose of minimizing social and economic losing.

Social risk evaluation is a broader concept for the ideas of the so-called social indicators movement which goes back to a classical work of Raymond A. Bauer, Social Indicators, from 1966 [1]. The Club of Rome raised considerable public attention in 1972 with its report The Limits to Growth, a brilliant publication which is influential still today. It predicted that economic growth could not continue indefinitely because of the limited

availability of natural resources [2]. Prof. Linfei Song has begun the research on social risk early warning using the methods of social indicators since 1990s. And in 1995 he put forward the social risk synthesized index system (SRSS), which including 50 indexes. But the summarizing coefficients in this index system are designed beforehand. In 2004, Prof. Yaojun Yan built up the social stability early warning system, which includes 55 indexes containing the domain of political, economic, social natural and international environment. There were greatly improved and development between the latter index system and the former in both the scientificness and rationality [3,4].

At present, main systematic evaluation methods of social risk, which are in common use, includes Analytic Hierarchy Process (AHP), Systematic Grading Method and Fuzzy Systematic Grading Method and so on. There are always index systems in these evaluation methods, and these indexes need to be summarized. Traditionally the summarizing coefficients are designed beforehand, usually in the form of expert grading or questionnaire investigating. However, in this paper, we introduce a method in which the summarizing coefficients are calculated by samples, so it is more objective and convincing, and could offer more deep analysis for the index systems.

2. Measure the Social Stability by SEM

SEM is a rapid-developing embranchment of Application Statistics, which has a wide application in the area of Psychology and Sociology, especially in Customer Satisfaction Index (CSI) model which is required by a series

of ISO9000 criterions. This model not only studies the interior relationship among various factors, but also the relative and causal relations among latent variables.

There are always two systems of equations in a SEM. One is a structure system of equations among structural variables, and the other one is a measurement system of equations between structural variables and observed variables. It has been thought and said that establishing a scientific and rational evaluation index system can not

only provide a credible gist for decision-making analysis of social risk, but also maintain our social stability and harmony. Therefore, it's important and urgent to quantitative analysis and build effective social risk index immediately. Now we build a SEM for evaluation of social risk. The model includes 6 structural variables and 40 observed variables. The variables are listed in **Table 1** as follows: Notice that the numbers of observed variables corresponding each structure variable are 6, 6, 8, 8, 5, 7.

Table 1. Index of variables

Structural variables		Observed variables	
<u>Situation</u> of external environment ξ_1	The influence degree by the world economic recession x_{11}	The influence degree by economic sanctions and economic frictions x_{12}	The effect <u>degree</u> by armed interventions and terrorist attacks x_{13}
	The quantity of people died in disasters x_{14}	the proportion of damage area caused by disasters x_{15}	Proportion of economic loss caused by disasters x_{16}
Economic power index η_1	Per capita GDP growth rate y_{11}	Per capita state revenue growth rate y_{12}	Growth rate of agriculture added value y_{13}
	Contributions rate of scientific and technological to the economic growth y_{14}	Urban residents per capita disposable income growth rate y_{15}	The consumer price index(CPI) y_{16}
	Household annual savings ratio y_{21}	Society-wide general retail price index y_{22}	Engel's coefficient y_{23}
<u>Social security</u> level η_2	Medical insurance coverage rate y_{24}	Urban unemployment rate y_{25}	Endowment insurance coverage rate y_{26}
	Unemployed insurance coverage rate y_{27}	Population proportion below security line for <u>minimum</u> subsistence y_{28}	
	the number of police officers for <u>every</u> 10,000 people y_{31}	Incidence rate of crime by taking advantage of duty of national public servants y_{32}	The incidence rate of major criminal cases y_{33}
Capability of Social controlling η_3	Activity level of Religious activities y_{34}	The incidence rate of major accidents y_{35}	The quantity of letters and visits from the people y_{36}
	Divorce rate y_{37}	The incidence rate of major economic cases y_{38}	
	per capita income ratio of the highest-income industries and the lowest-incomes industries y_{41}	the income ratio of the 10% of the highest-income earners with 10% of the lowest-income earners y_{42}	the gini coefficient ratio of rural residents and urban residents y_{43}
Distribution of social wealth η_4	gini coefficient y_{44}	Residents' disposable income ratio of urban and rural y_{45}	
	the <u>confidence</u> level of the social development prospect y_{51}	Satisfaction rate with the relations between cadres and the masses y_{52}	Satisfaction rate with the economic income y_{53}
Social psychology η_5	Gross National tolerance on the corruptions y_{54}	Tolerance on the inequity of the administration of justice y_{55}	National Tolerance on the income differentials y_{56}
	Gross National Happiness index (life satisfaction and subjective well-being <u>index</u>) y_{57}		

There exists 13 relationships among the 6 structural variables (latent variables), which are expressed in **Figure 1** below (The relationships among variables are $\gamma_1 \sim \gamma_5$, expressed with dashed arrowheads; The relationships among independent variables are β_{ij} , expressed with real-line arrowheads). The structural relationship among the latent variables (structural model) can be put as follows:

Among the structural variables there are some path relationships or causalities. These causalities among the structural variables can be expressed as equations as below.

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \beta_{21} & 0 & 0 & 0 & 0 \\ 0 & \beta_{32} & 0 & 0 & 0 \\ \beta_{41} & 0 & \beta_{43} & 0 & 0 \\ \beta_{51} & \beta_{52} & \beta_{53} & \beta_{54} & 0 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} + \begin{pmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{14} \\ \gamma_{15} \end{pmatrix} \xi_1 + \begin{pmatrix} \varepsilon_{\eta 1} \\ \varepsilon_{\eta 2} \\ \varepsilon_{\eta 3} \\ \varepsilon_{\eta 4} \\ \varepsilon_{\eta 5} \end{pmatrix} \quad (1)$$

In general, suppose that $\eta_1 \sim \eta_m$ are m dependent variables, arranging them as a vector η by column as (1); and $\xi_1 \sim \xi_k$ are k independent variables, arranging them as a vector ξ by column also. Then B square matrix B is the coefficient matrix of η , then

$m \times k$ matrix Γ is the coefficient matrix of ξ , ε_η is the residual vector, then SEM (1) may be extended as:

$$\eta = B\eta + \Gamma\xi + \varepsilon_\eta \quad (2)$$

The structural variables are implicit and cannot be observed directly. Each structural variable is corresponding with many observed variables.

Suppose that there are M observed variables and each one has N observed values, then we will get a $N \times M$ matrix. The relationships between the structural variables and the observed variables can also be expressed as follows:

$$\begin{pmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{111} \end{pmatrix} = \begin{pmatrix} \nu_{11} \\ \nu_{12} \\ \vdots \\ \nu_{111} \end{pmatrix} \xi_1 + \begin{pmatrix} \varepsilon_{x11} \\ \varepsilon_{x12} \\ \vdots \\ \varepsilon_{x111} \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} y_{i1} \\ \vdots \\ y_{iL(i)} \end{pmatrix} = \begin{pmatrix} \lambda_{i1} \\ \vdots \\ \lambda_{iL(i)} \end{pmatrix} \eta_i + \begin{pmatrix} \varepsilon_{yi1} \\ \vdots \\ \varepsilon_{yiL(i)} \end{pmatrix} \quad i = 1, \dots, 4 \quad (4)$$

where x_{ij} , $j = 1, \dots, 5$ (There are 5 observed variables in **Table 1**) are the observed variables corresponding to ξ_t , $t = 1, \dots, L(i)$ ($L(i)$ are respectively 6, 6, 8, 8, 5, 7, in **Table 1**) are the observed variables corresponding to η_i , $i = 1, \dots, m$, ν_{ij} , λ_{ij} are load items. We call (2) (3) (4) a SEM for evaluating the social risk.

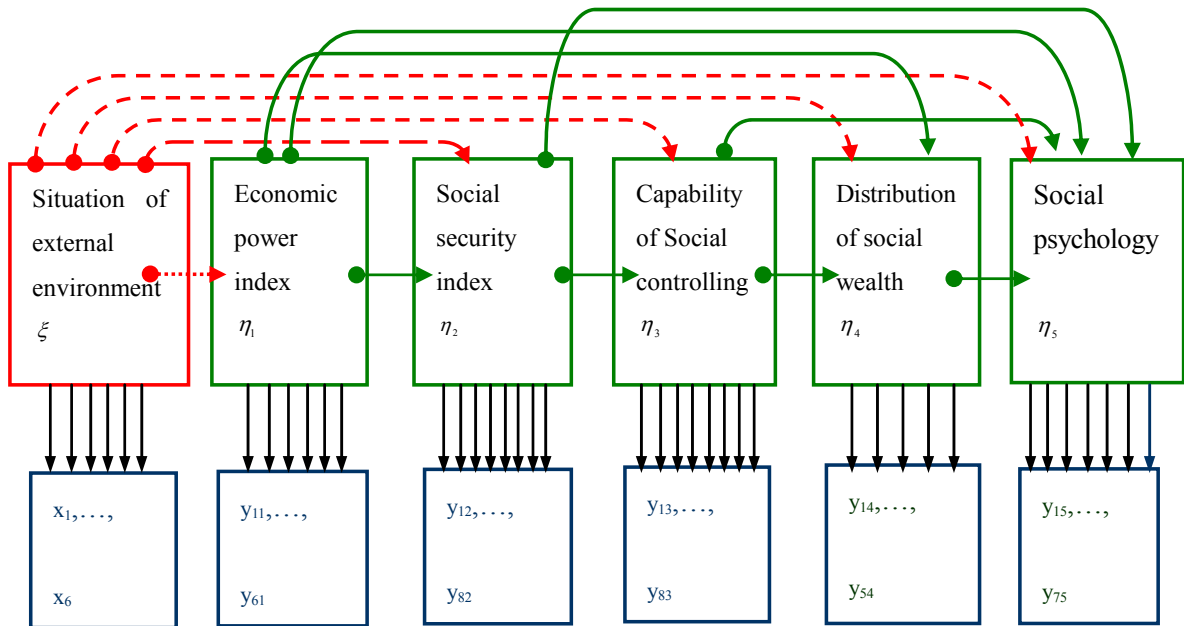


Figure 1. Social risk index model

3. The Best Initial Value in PLS Algorithm of SEM

At present, there exist two algorithms to solve SEM, one is Linear Structure Relationship (LISREL) which has a abundant theory system but lacks of practicability, the other one is PLS which is widely applied in many areas but its convergence can not be assured or its convergence rate is too slow, since its iterative initial value is given arbitrarily. However, we find that arbitrary initial value is not necessary and PLS can be calculated by a suitable iterative initial value based on the least square estimation in the observation equations.

Equation (4) can be written as:

$$Y_i = \eta_i \Lambda_i + \delta \tag{5}$$

And $Y_i' Y_i \approx \Lambda_i' \eta_i' \eta_i \Lambda_i = \eta_i' \eta_i \Lambda_i' \Lambda_i$, it we set structural variables as unit vectors, namely $\eta_i' \eta_i = 1$, then:

$$Y_i' Y_i \approx \Lambda_i' \Lambda_i \tag{6}$$

Which is an approximate equality between two $L(i) \times L(i)$ matrixes under the meaning of least square and its detail form is:

$$\begin{pmatrix} y_{i1} y'_{i1} & y_{i1} y'_{i2} & \cdots & y_{i1} y'_{iL(i)} \\ y_{i2} y'_{i1} & y_{i2} y'_{i2} & \cdots & y_{i2} y'_{iL(i)} \\ \cdots & \cdots & \cdots & \cdots \\ y_{iL(i)} y'_{i1} & y_{iL(i)} y'_{i2} & \cdots & y_{iL(i)} y'_{iL(i)} \end{pmatrix} \approx \begin{pmatrix} \lambda_{i1}^2 & \lambda_{i1} \lambda_{i2} & \cdots & \lambda_{i1} \lambda_{iL(i)} \\ \lambda_{i2} \lambda_{i1} & \lambda_{i2}^2 & \cdots & \lambda_{i2} \lambda_{iL(i)} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{iL(i)} \lambda_{i1} & \lambda_{iL(i)} \lambda_{i2} & \cdots & \lambda_{iL(i)}^2 \end{pmatrix} \tag{7}$$

Be attention each element in the left is the product of two vectors, while element in the right is the product of two numbers. Next set diagonal elements in the two sides equal, then:

$$\lambda_{ki}^2 = y'_{ki} y_{ki}, \quad k = 1, \dots, j \tag{8}$$

We can do it for variable ξ with the same method. In this way we get the initial value of coefficients between observed variables and structural variables under the meaning of least square, namely the estimation value $\hat{\lambda}_i = (\hat{\lambda}_{i1}, \dots, \hat{\lambda}_{iL(i)})'$ of matrix Λ_i .

Next we will estimate structural variable η_i . Suppose $\eta_i = (\eta_{i1}, \eta_{i2}, \dots, \eta_{iN})'$, then the vector form of (13) is

$$\begin{pmatrix} y_{i1s} \\ \vdots \\ y_{iL(i)s} \end{pmatrix} \approx \begin{pmatrix} \lambda_{i1} \\ \vdots \\ \lambda_{iL(i)} \end{pmatrix} \eta_{is}, \quad s = 1, \dots, N \tag{9}$$

where vector $Y_s = (y_{i1s}, \dots, y_{iL(i)s})'$ is the transversal

vector of matrix y_i . Let vector $\hat{\lambda}_i$ multiple two sides of (9), we get the least square estimation of η_{is}

$$\begin{aligned} \lambda_i' \lambda_i \eta_{is} &= \sum_{k=1}^{L(i)} \lambda_{ik}^2 \eta_{is} \\ &= (\lambda_{i1}, \dots, \lambda_{iL(i)}) \begin{pmatrix} y_{i1s} \\ \vdots \\ y_{iL(i)s} \end{pmatrix} = \lambda_i' Y_s \end{aligned} \tag{10}$$

$$\hat{\eta}_{is} = \frac{\hat{\lambda}_i' Y_s}{\hat{\lambda}_i' \hat{\lambda}_i'}, \quad s = 1, \dots, N \tag{11}$$

where $\hat{\lambda}_i$ has been estimated before. We can also get estimate v_{ij} and ξ_i in the same way. Then we get all estimation values of structural variables, they satisfy

$$\| \eta_i - \sum_{j=1}^{L(i)} \omega_{ij} y_{ij} \| \rightarrow \min \tag{12}$$

Its geometrical meaning is to seek the distance between a unit sphere and a hyper-plane and its solution is unique under the condition that it is not linearly dependent among vectors y_{ij} .

After getting the least square solution $\hat{\xi}_i, \hat{\eta}_i$ of the structural system of equations, we can easily gain the solution of (2) with two-phase least square method. In other words, it dose not need to iterate since we get the least square solution $\hat{\xi}_i, \hat{\eta}_i$ of the observed equation system based on unit vector constraint and the solution satisfy the two systems of equations of SEM.

4. Final Remarks

In this paper, we propose structural equation model to measure social risk. It is more objective and scientific to use SEM in the evaluation of social risk compared with traditional methods, such as AHP, Fuzzy Systematic Grading Method and so on, because the summarizing coefficients of this evaluation system are calculated by samples rather than designed beforehand. Therefore, we can have a better understanding the relationships among the indexes, which will do a great favor to decision-making analysis of the social stability.

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