Using Piezocone Dissipation Test to Estimate the Undrained Shear Strength \( (s_u) \), \( N_{kt} \) and \( N_{\Delta u} \) Factors in Cohesive Soils

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Abstract

The current practice of geotechnical engineering commonly uses a combination of theoretical and empirical correlations to estimate the soil undrained shear strength in clays from the piezocone test. In order to complement the use of such correlations, the application of a method to estimate the soil undrained shear strength, using measures of the excess pore pressure in dissipation tests of piezocone is presented. In cohesive soils, excess pore pressure and undrained shear strength are dependent on the same variables (stress state, stress history, soil stiffness), which allows them to be related by the theoretical cavity expansion-critical state framework. This paper mentions the mathematical formulation that supports the theoretical framework used, its relationship with the \( N_{kt} \) and \( N_{\Delta u} \) factors and their estimation in a case studied. The results obtained are consistent within the dispersion found in the international literature and encourage the use of the method in engineering practice.

Keywords

CPTu Piezocone, Overconsolidation Ratio, Undrained Shear Strength, Dissipation Test

1. Introduction

Prediction of soil properties from piezocone test (CPTu) data in cohesive soils is routinely carried out in geotechnical design. This is possible because there is a general recognition that analytical and numerical analysis techniques and constitutive models of soil behaviour are now sufficiently developed to produce
good agreement between cone parameters and independently measured soil properties (e.g. Lunne et al., 1997; Yu, 2004; Schnaid, 2009). A theoretical frame is essential, because the CPTu cannot measure the undrained shear strength directly and therefore CPTu assessment of $s_u$ relies on a combination of theory and empirical correlations.

Penetration tests in clay are generally undrained, and therefore the cone tip resistance $q_c$ can be related to $s_u$ as follows:

$$q_c = N_{kt} \cdot s_u + \sigma_0$$

(1)

where $N_{kt}$ is a theoretical cone factor and $\sigma_0$ is the in situ total stress. The cone factor may be determined using simple bearing capacity formulations, cavity expansion or strain path method (e.g. Terzaghi, 1943; Meyerhof, 1956; Caquot & Kerisel, 1953; Baligh, 1985; Teh & Houlbsy, 1991; Yu & Whittle, 1999). Yu (2004) pointed out that while each theory may be used alone for cone penetration analysis, better predictions of cone penetration mechanisms may be achieved if some of the methods are used in combination. A combination of strain path analysis and finite element calculations was used by Teh & Houlbsy (1991) to model cone penetration in a Von Mises soil. Yu & Whittle (1999) proposed a cone factor estimated from both strain path analysis and cavity expansion methods. In this approach, the strain path solution developed by Baligh (1985) was used to estimate the size of the plastic zone produced by penetration. Once the plastic zone is established, spherical cavity expansion was used to determine the stress distribution and therefore cone resistance. Burns & Mayne (1998) use cavity expansion-critical state framework to model monotonic and dilatory response with regard to time.

Whereas theoretical solutions have been contributing in the understanding of the fundamental mechanics of cone penetration, empirical correlations are still widely used in practice to estimate $s_u$ from cone resistance. Values of cone factor ($N_{kt}$) often fall in the range from 10 to 20 and are influenced by soil plasticity, overconsolidation ratio, sample disturbance, strain rate and scale effects, as well as the reference test from which $s_u$ has been established (e.g. Aas et al., 1986; Mesri, 1975; Lunne et al., 1997). However in overconsolidated clays the values of $N_{kt}$ often fall outside the predicted range and there are no constitutive modes to support empirical evidences.

A potential alternative to overcome the existing uncertainty related to $N_{kt}$ is to use the excess pore pressure to estimate $s_u$. Several relationships have been proposed based on theoretical or semi-theoretical approaches using cavity expansion theory (Battaglio et al., 1986; Campanella et al., 1985; Randolph & Wroth, 1979; Vesic, 1972):

$$s_u = \frac{\mu_s - \mu_u}{N_{3u}}$$

(2)

where $N_{3u}$ is shown from cavity expansion to vary in the range of 2 to 20. Lunne et al. (1997) recommend using a value of $N_{3u}$ of between 7 and 10. It has been advocated that these methods have the advantage of increased accuracy in the
measurement of $\Delta u$, mainly in soft clays where $\Delta u$ can be very large (e.g. Campanella et al., 1985).

The rationality in using Equation (2) is that cone resistance and excess pore pressures generated during cone penetration into fine grained soils will be dependent on the same parameters—stress state, stress history, soil stiffness—and can therefore be associated in CPTu predictions. Proposed semi-empirical solutions (Massarch & Broms, 1981; Campanella et al., 1985) attempt to capture the reduction in excess pore pressures with increasing overconsolidation ratio. In addition, there has been some research work suggesting that $N_{cu}$ and $N_{\Delta u}$ is related to the $B_p$ parameter but while this appears to hold on a site specific basis, no global correlation has been identified.

A new method proposed by Mántaras et al. (2014) is applied here. This analysis advocates a different approach using dissipation tests and linking the measured piezocone maximum dissipation excess pore-water pressure ($\Delta u_{\text{max}}$) to the undrained shear strength ($s_u$). The mathematical solution proposed by Burns & Mayne (1998) is used as reference given the fact that excess pore water pressures are computed through a combination of the octahedral and the shear-induced components, allowing both normally and overconsolidated clays to be modeled from pore pressures measured immediately behind the cone shoulder ($u_2$).

The verification of the aforementioned method requires well-documented cases, which are not always available in engineering practice. This work aims to expand the database on which the method is based, and to verify the consistency of results within the empirical framework that relates cone factors and normalized excess pore pressure.

2. Mathematical Formulation (Mántaras et al., 2014)

The mathematical solution proposed by Burns & Mayne (1998) is based on the cavity expansion-critical state framework for the monotonic and dilatory response with regard to time. The excess pore water pressures, $\Delta u$, at any time can be compared with the initial values during penetration, $\Delta u_i = u_{ij} - u_0$, represented as:

$$\Delta u_i = (\Delta u_{\infty})_{ij} + (\Delta u_i),$$

where:

$$(\Delta u_{\infty})_{ij} = (2/3)M \sigma_{ij}^\prime (OCR/2) \ln I_i$$

is the octahedral component during penetration and

$$(\Delta u_i) = \sigma_{ij} [1 -(OCR/2)]^\Lambda$$

is the shear-induced component during penetration, with OCR (the overconsolidation ratio), $\phi'$ the effective friction angle, $M = (6 \sin \phi')/(3 - \sin \phi')$ and $\Lambda$ the compressibility ratio $(1 - C_s/C_b)$. Departing from this concept, Burns & Mayne (1998) derived the following equation for the normalized excess pore pressure:

$$\frac{\Delta u}{\sigma_c'} = \frac{2}{3} M \cdot \ln (I_i) \left[ \frac{OCR}{2} \right]^\Lambda \frac{1 - \left[ \frac{OCR}{2} \right]^\Lambda}{1 + 5000 \cdot T}$$

(3)

where: $I_i$ is the rigidity index and $T$ a dimensionless time factor.

Being $T_{\text{max}}$ the value of $T$ where the excess pore pressures reaches its maxi-
mum value, it is possible to found it analytically from the equation:
\[ \frac{\partial}{\partial T} (\Delta u/\sigma') = 0 \]
resulting the expression:

The maximum value of \( T \) could be obtained

\[
T_{\text{max}} = \frac{-1}{50} \left[ \alpha \left( \frac{OCR}{2} \right)^{\alpha} + 1 - \left( \frac{OCR}{2} \right)^{\alpha} - 9.9 \sqrt{\alpha \left( \frac{OCR}{2} \right)^{\alpha} \left( \frac{OCR}{2} \right)^{\alpha} - 1} \right] 
\]

\[
100 \cdot \alpha \left( \frac{OCR}{2} \right)^{\alpha} + 1 - \left( \frac{OCR}{2} \right)^{\alpha} 
\]

Note that \( T_{\text{max}} \) is the value of \( T \) where the excess pore pressure reaches its maximum value taking the first derivative of Equation (3) and setting the gradient of the function equal to zero, it is possible to determine the maximum value of the normalized excess pore pressure in relation to time factor \( T \).

\( (T_{\text{max}}) \) combined with \( \Delta u_{\text{max}} \) and related to the normalized undrained shear strength from direct shear undrained test (Wroth, 1984) yields the undrained pore pressure ratio \( N_{u_{\text{u,max}}} \left( = \Delta u_{\text{max}}/s_u \right) \) defined as the ratio of the maximum excess pore pressure and the undrained shear strength:

\[
N_{u_{\text{u,max}}} = \frac{\left( \Delta u_{\text{max}} \right)}{s_u} = \frac{2}{3} M \cdot \ln (I_r) \left[ \frac{OCR}{2} \right]^{\alpha} + \frac{1 - \left( \frac{OCR}{2} \right)^{\alpha}}{1 + 5000 \cdot T_{\text{max}}/T_{\text{max}} \cdot \sin (\phi) \cdot OCR^{\alpha} \cdot \log (I_r)} 
\]

Based on Equation (5), the derived formulation shows little sensitivity to variations on \( OCR \) and \( \phi' \), and for typical soil parameters can be reduced to a relatively simple expression of a constant times the logarithm of the rigidity index, \( I_r \) (as illustrated on Figure 1 for \( OCR \)).

**Figure 1.** Undrained pore pressure ratio \( N_{u_{\text{u,max}}} \) (Mántaras et al., 2014).
Advantages of interpreting pore pressures measurements compared to the more conventional piezocone penetration data are: (a) the cone factor $N_{u\text{max}}$ is less affected by soil rigidity and stress history because measurements result from pore pressure flow around the probe rather than of complete soil displacement caused by the cone penetration test and (b) well-defined failure mechanisms of flow around the probe allow for sound theoretical analysis of pore pressure dissipation and shear strength.

From the mathematical viewpoint the above expression is rigorously consistent; expressing the ratio between values of pore pressure measured during piezocone dissipation and the undrained shear strength in Direct Simple Shear conditions.

3. Case Studied

To illustrate the possibilities offered by the approach proposed by Mántaras et al. (2014), it will be applied to data from a well-documented site research program of a soft clay deposit in the metropolitan area of Porto Alegre, Rio Grande do Sul, Brazil. The complete research program consists of eleven study points using different types of tests in each one.

In order to visualize the results of the different tests and the application of the method, two points of very different characteristics were chosen to be presented in this work. In both cases, undrained shear strength values predicted from classical approaches are consistent between them ($s_u$ obtained from Vane test, $N_{u}$ and $N_{\Delta u}$).

The first boring is illustrated in Figure 2, showing a homogeneous clay profile deposit. Undrained strength, $s_u$, obtained from the dissipation test method proposed by Mántaras et al. (2014) using $N_{u\text{max}} = 8$ ($N_{u\text{max}} = 4 + \log I_r$, with $I_r$ equal to 100 = 8) are of the same order of magnitude as those calculated from other methods.

![Figure 2](Image)

*Figure 2.* Metropolitan Region Porto Alegre, Brazil. (a) Homogeneous geotechnical profile. (b) Dissipation tests.
The second boring is illustrated in Figure 3, showing a layered deposit. The different grain size of layers could also be estimated from assessment of horizontal coefficient of consolidation, \( C_h \), from dissipation tests. Once again, dissipation test results yielded \( s_u \) values of the same order of magnitude as those calculated from other methods.

Table 1 summarized the results for the whole campaign. Aiming to verify the consistency of assessed undrained strength with this novel approach, the cones factor \( N_{ku} \) and \( N_{\Delta u} \) values were back calculated in Table 1 and presented in Figure 4 and Figure 5 in the same framework proposed by Lunne et al. (1997).

Despite the dispersion of the results, it is clear the trends of \( N_{ku} \) vs. \( B_q \) and \( N_{\Delta u} \) vs. \( B_q \) values and are consistent with those found in international literature.

Figure 3. Metropolitan Region Porto Alegre, Brazil. (a) Erratic geotechnical profile. (b) Dissipation tests.

Figure 4. Back calculated cone factors \( N_{\Delta u} \) vs. \( B_q \) using \( s_u \) (\( \Delta u_{\text{opt}} \) method).
Table 1. All the results obtained in the complete study.

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<th>Boring</th>
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<th>u eq kPa</th>
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Figure 5. Back calculated cone factors $N_{st}$ vs. $B_q$ using $s_r$ ($\Delta u_{\text{max}}$ method).
4. Closing Remarks

The paper applies a new method proposed by Mántaras et al. (2014) to estimate the undrained shear strength from piezocone dissipation tests. Stress history, shear strength and compressibility are the critical factors affecting the accuracy of predictions and are properly taken into account. The method yields an undrained pore pressure ratio \( N_{\Delta u}\max \) of about 8 which is within the range of early recommended values.

Also the \( N_u \) and \( N_{\Delta u} \) factors calculated from the results of undrained resistance obtained from dissipation tests applying this new mathematical formulation are consistent with those referenced in the international literature and encourage its use in the engineering practice.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References


