Robust $H_\infty$ Observer-Based Tracking Control for the Photovoltaic Pumping System

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Abstract

In this paper, we propose a $H_\infty$ robust observer-based control DC motor based on a photovoltaic pumping system. Maximum power point tracking is achieved via an algorithm using Perturb and Observe method, with array voltage and current being used to generate the reference voltage which should be the PV panel's operating voltage to get maximum available power. A Takagi-Sugeno (T-S) observer has been proposed and designed with non-measurable premise variables and the conditions of stability are given in terms of Linear Matrix Inequality (LMI). The simulation results show the effectiveness and robustness of the proposed method.

Keywords

Photovoltaic, Pumping System, Fuzzy Controller, $H_\infty$, Takagi-Sugeno (T-S) Fuzzy Model, Observer, Stability, Linear Matrix Inequalities (LMIs), Maximum Power Point Tracking (MPPT), Unmeasurable Premise Variables

1. Introduction

The standalone photovoltaic pumping systems have become a favorable solution for water supply. It earns a

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more market share, particularly in rural areas that have a substantial amount of insolation and have no access to an electric grid. The maximization of these systems via maximum power point tracking (MPPT) has been sufficiently exploited in the literature [1]. As a result, most commercial photovoltaic pumping systems either use conventional MPPT control (P & dO, Incremental) or not use the MPPT control [2]. For a better optimization of the energy, a pump controller is necessary. For photovoltaic pumping systems, the pump controller is essentially a DC-DC, the duty ratio is controlled by a tracking system of the maximum power point tracking (MPPT) [3] [4]. This is used to adjust the motor armature voltage and in turn the motor speed and the hydraulic power of the pump according to the irradiation level. Different types of dc-dc converters have been employed including the buck converter [5] [6] and the boost converter [7] depending on the voltage rating of the motor and the PV array.

Based on these works, we propose in this paper a $H_\infty$ observer-based on tracking controller based on TS representation of the PV pumping system, where the weighting functions depend on non-measurable premise variables [8]-[10]. Sufficient conditions, based on Lyapunov approach and the convex optimization techniques, are formulated as an efficient one step LMI [11] to avoid the complexity of separate design steps and can be solved using MATLAB software. The block diagram of a DC pumping system is shown in Figure 1 [12].

The structure of this paper is organized as follows. In Section II we describe the process of getting a T-S fuzzy model of PV system, with the DC motor pumping [13]. Section III presents the proposed control design. The stability conditions of the closed loop system are proposed in this section in terms of LMIs. In Section IV, we present the proposed strategy of the observer and the controller. In Section V, the simulation results are presented and discussed. Finally, conclusions are made in Section VI.

2. Model of Pumping System

The water pumping system is considered in this work as a standalone system, without batteries. This is a complex and nonlinear system. The complete model is difficult to use in control applications. We need an easy model to use for the synthesis of observers and controllers. It also allows estimating or adjusting the values of dynamic parameters in real time. The system consists of a single PV module, an MPPT System, and a DC water pump. In the literature, different models of PV and of the pump were used. Figure 2 shows the equivalent circuit of the boost of the PV system with water pump.

The system [3] average model is given in (1):

$$\begin{align*}
\frac{dI_L}{dt} &= \frac{V_{pv}}{L} - \frac{(R_s + R_L)}{L} I_L - \frac{k}{L} W - \frac{V_{fw}}{L} + \left(\frac{(R_s - R_m)}{L} I_L + V_{fw} + k\omega\right)u \\
\frac{dV_{pv}}{dt} &= \frac{I_{pv}}{C} - \frac{I_L}{C} \\
\frac{d\omega}{dt} &= \left(\frac{k}{j} - \frac{k_1}{j}\right) I_L - \frac{k_2}{j} W - \frac{k_1}{j} 
\end{align*}$$

(1)

where $V_{pv}$ and $i_{pv}$ refer to the PV panel voltage and current respectively. $L$, $R_s$ and $I_L$ are the self-inductance, resistance and current. $R_m$ is a resistance characterizing IGBT lost.

$C$, $V_{fw}$ and $\omega$ are the input capacitance, the diode forward voltage and speed. $u$ is the control input. Considering the PV current as an exogenous input, we get the state representation (2).

$$\begin{align*}
\frac{d}{dt}\begin{pmatrix} I_L \\ V_{pv} \\ \omega \end{pmatrix} &= \begin{pmatrix} \frac{R_s + R_m}{L} & \frac{1}{L} & \frac{k}{L} \\
-\frac{1}{C} & 0 & 0 \\
k & 0 & -\frac{k_2}{j} \end{pmatrix} \begin{pmatrix} I_L \\ V_{pv} \\ \omega \end{pmatrix} + \begin{pmatrix} \frac{(R_s - R_m)}{L} I_L + V_{fw} + k\omega \\
0 \\
-k_1 \end{pmatrix} \frac{u}{L} + \begin{pmatrix} \frac{V_{fw}}{L} \\
-\frac{I_{pv}}{C} \\
-\frac{k_1}{j} \end{pmatrix}.
\end{align*}$$

(2)

In order to guarantee zero steady state regulation error, we develop an integral T-S fuzzy control. Let $V_{ref}$ be a constant reference, the objective is to achieve that $V_{pv} \rightarrow V_{ref}$ when $t \rightarrow \infty$. To this end, we introduce an added state variable to account for the integral of output regulation error. Let us define the new state variable as (3):
The new augmented state vector then becomes:

\[
\begin{bmatrix}
  i_x \\
  V_{pv} \\
  \omega \\
  x_e
\end{bmatrix}
\]

(4)

So this second integrator included previous to the input would make it smoother, benefiting the implementation, and therobustness [14].

The augmented system can be written as (5):

\[
\dot{x} = Ax + B(x)u + B_x d
\]

(5)

where

\[
A = \begin{bmatrix}
    -\frac{(R_u + R_i)}{L} & \frac{1}{L} & -\frac{k}{L} & 0 \\
    \frac{1}{C} & 0 & 0 & 0 \\
    \frac{k}{j} & 0 & 0 & 0 \\
    0 & -1 & 0 & 0
\end{bmatrix}
\]

\[
B(x) = \begin{bmatrix}
    \frac{(R_u - R_w)}{L}i_x + V_{pv} + k\omega \\
    0 \\
    -\frac{k}{j}i_x \\
    0
\end{bmatrix}
\]

\[
d = \begin{bmatrix}
    \frac{1}{i_{pv}} \\
    \frac{1}{V_{ref}} \\
\end{bmatrix}
\]

\[
B_x = \begin{bmatrix}
    -\frac{v_{fu}}{L} & 0 & 0 & 0 \\
    0 & \frac{1}{C_i} & 0 & 0 \\
    0 & 0 & -\frac{k}{j} & 0 \\
    0 & 0 & 0 & -1
\end{bmatrix}
\]
3. Control Strategy

In this work two control strategies are studied.

3.1. Perturb and Observe Algorithm

The P&P algorithm acts periodically by giving a perturbation to operating voltage $V$ and observing the power variation $P = V I_a$ order to deduct the direction of evolution to give to the voltage reference $V_{ref}$. Taking into account power-voltage characteristic curve $P - V$ obtained under given conditions, the goal is to track the operating point at the MPP as shown in Figure 3. This algorithm measures at each $z$ instant variable $(z)$ and $v(z)$ and calculates $p(z)$, then compares with the power calculated at $(z-1)$ instant $p(z-1)$.

For all the operating points where the power and current variations are positive, the algorithm that continued to perturb the system in the same direction of perturbation is reversed. The increasing of reference voltage $V_{ref}$, otherwise, if these variations are negative, the direction of perturbation is reversed. The increasing or decreasing of reference $V_{ref}$ is done by tracking step $\Delta V$. The flow chart of the P&O algorithm is presented in Figure 4. Theoretically, the algorithm is simple to implement in its basic form. However, it was noticed some oscillations around the MPP in steady state operating and this causes power loss [15]. Its functioning depends on the tracking step size applied to voltage reference $V_{ref}$. For the same sample time of the system, the oscillations and consequently the power loss could be minimized if the tracking step would continuously get smaller [16]. Nevertheless, the response of the algorithm becomes slower.

![Figure 3. Power-Voltage characteristic of PV panel.](image)

![Figure 4. Flow chart of the P&O algorithm.](image)
3.2. Takagi-Sugino Fuzzy Model

In this section, we present T-S fuzzy control approach that ensures robust regulation under disturbance. Consider a general nonlinear system as follows:

\[
\begin{align*}
\dot{x} &= Ax + B(x)u + B_d d \\
z &= C_1x \\
y &= C_2x
\end{align*}
\]

where \( z \) is the controlled output variable and \( y \) is the output vector.

By observing the functions of \( A(x), B(x) \), the fuzzy premise variables are chosen as \( z_1 = i_L \) and \( z_2 = \omega \). Then, the system (2)-(5) can be represented by the following T-S fuzzy rules: IF \( z_1(t) \) is \( F_{i_1} \) and \( z_2 \) is \( F_{i_2} \), then [17].

\[
\dot{x} = Ax + B(x)u + B_d d \\
z(t) = C_1x \quad i = 1,2,\ldots,r. \\
y(t) = C_2x
\]

where \( F_j (j=1,2) \) are the fuzzy sets, \( r \) is the number of fuzzy rules, and \( A, B_j, C_1, \) and \( C_2 \) are appropriate subsystem matrices.

The global T-S model is then inferred as follows:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(t)(Ax + B(x)u + B_d d) \\
z(t) &= C_1x \\
y(t) &= C_2x
\end{align*}
\]

where \( h(t) = [z_1(t) \quad z_2(t)]^T \), \( \mu_i(h(t)) = \frac{w_i(h(t))}{\sum_{i=1}^{r} w_i(h(t))} \geq 0 \), with \( w_i(h(t)) = \prod_{j=1}^{2} F_{i_j}(z_j(t)) \) so that \( \sum_{i=1}^{r} \mu_i(h(t)) = 1 \) for all \( t \).

To obtain an exact fuzzy representation of dynamic (2), the membership functions of \( z_1 \) and \( z_2 \) should be chosen so that \( A = \sum_{i=1}^{r} \mu_i(h(t))A_i \) and \( B(x) = \sum_{i=1}^{r} \mu_i(h(t))B_i \). For simplification, let us write the membership function in the general form [18].

\[
S_{ij} = -\frac{d_j}{D_j} - d_j + \left( \frac{1}{D_j - d_j} \right) z_j(t) \quad S_{ij} = 1 - S_{ij}
\]

where \( D_j = \max_{\tau \Omega_j} z_j(t) \) and \( d_j = \min_{\tau \Omega_j} z_j(t) \), for \( j = 1,2 \) and the discussion set:

\( \Omega_j = \{ \tau = (i_L, \omega) | \tau \in [\ell_{\min}, \ell_{\max}], \quad i = 1,2 \} \). (Note that \( \ell_{\min} \) and \( \ell_{\max} \) are the lower and upper bounds of variable \( \ell \) respectively).

Hence, functions \( \mu_i(h(t)) \) are the weighting functions depending on variable \( h(t) \) which can be measurable (as the input or the output of the system) or unmeasurable variables [19] [20] (as the state of the system) and verify the following properties:

\[
\begin{align*}
\sum_{i=1}^{4} \mu_i(h(t)) &= 1 \\
0 &\leq \mu_i(h(t)) \leq 1 \quad \forall i \in \{1,\ldots,4\}
\end{align*}
\]

In the following, we assume that the weighting functions depend on the system state. Then, TS fuzzy system
(7) becomes:

\[ \dot{x}(t) = \sum_{i=1}^{4} \mu_i(\hat{x}(t))(A_i x + B_i u + B_u d) + w(t) \]

\[ y(t) = C_i x(t) \quad (9) \]

where \( w(t) = \sum_{i=1}^{4} \left( \mu_i(x(t)) - \mu_i(\hat{x}(t)) \right) (A_i x + B_i u + B_u d) \).

4. Fuzzy Observer Based Tracking Controller

The aim of this section is the design of the observer-based tracking control of the photovoltaic Pumping System. As the motor speed model states are not fully measurable, the designed state feedback control is based on the estimated states. We define the fuzzy controller as follows:

\[ u(t) = \sum_{i=1}^{4} \mu_i(\hat{x}(t)) \left( K_i \hat{x}(t) \right) \quad (10) \]

where \( \hat{x}(t) \in \mathbb{R}^4 \) is the estimated state and \( K_i \in \mathbb{R}^{4 \times 4} \) \( (i = 1, \ldots, 4) \) are the controller gains to be determined.

Based on T-S fuzzy model of solar pump (1), the structure of the observer is defined as:

\[ \dot{\hat{x}}(t) = \sum_{i=1}^{4} \mu_i(\hat{x}(t))(A_i \hat{x}(t) + B_i u + L_i (y - \hat{y})) \quad \hat{y}(t) = C_2 \hat{x}(t) \quad (11) \]

where \( \hat{y}(t) \in \mathbb{R}^3 \) is the estimated output, \( L_i \in \mathbb{R}^{4 \times 4} \) \( (i = 1, \ldots, 4) \) are the observer gains to be determined.

Let’s define error of state estimation as \( e_0(t) = x(t) - \hat{x}(t) \); then, we can find the estimation error dynamics as follows:

\[ \dot{e}_0(t) = \sum_{i=1}^{4} \mu_i(\hat{x}(t)) \left( (A_i - L_i C_2) e_0(t) \right) + B_u d + w(t) \quad (12) \]

Augmented system \( \tilde{x} = \begin{bmatrix} x(t) \\ e_0(t) \end{bmatrix} \); can be written as

\[ \dot{\tilde{x}} = \sum_{i=1}^{4} \sum_{j=1}^{4} \mu_i(\hat{x}(t)) \mu_j(\hat{x}(t)) \left( \tilde{A}_{ij} \hat{x}(t) + \tilde{B}_j \tilde{w}(t) \right) \]

\[ y(t) = \tilde{C}_2 \tilde{x}(t) \quad (13) \]

with \( \tilde{A}_{ij} = \begin{bmatrix} A_i - B_i K_j & B_i K_j \\ 0 & A_i - L_i C_2 \end{bmatrix} \); \( \tilde{B}_j = \begin{bmatrix} B_j & I_4 \\ B_w & I_4 \end{bmatrix} \}; \( \tilde{w} = \begin{bmatrix} w(t) \\ d \end{bmatrix} \); \( \tilde{C}_2 = \begin{bmatrix} C_2 \ & 0 \end{bmatrix} \).

The estimation error asymptotically converges around zero and satisfies the following \( H_\infty \) performance under zero initial conditions:

\[ \| e_0(t) \|_F \leq \gamma, \quad \text{for} \quad w(t) \neq 0. \quad (14) \]

where \( \gamma \) is the desired disturbance attenuation parameter.

**Theorem 1**: if there exists symmetric matrices \( X_i > 0 \), \( P_i > 0 \), matrices \( Y_j, J_j \) and a prescribed \( \gamma^2 > 0 \), such that the following LMI holds [21]:

\[
\begin{bmatrix}
\Lambda_{ij} & B_j Y_j & B_w & I & 0 & 0 & 0 \\
* & -2\mu I & 0 & 0 & \mu I & 0 & 0 \\
* & * & -2\mu I & 0 & 0 & \mu I & 0 \\
* & * & * & -2\mu I & 0 & 0 & \mu I \\
* & * & * & * & \Gamma_{2j} & P_2 B_w & P_2 \\
* & * & * & * & * & -\gamma^2 I & 0 \\
* & * & * & * & * & * & -\gamma^2 I
\end{bmatrix} \leq 0
\quad (15)
\]
where
\[ A_y = A_x + X_A \Gamma_y \Gamma_y^T - B_y Y_y - Y_y^T B_y^T \] (16)

Controller gains \( K_y \) and observer gains \( L_y \) are given by:
\[ K_y = Y_y X_y^{-1} \] (17)
\[ L_y = P_y^{-1} J_y \] (18)

**Proof**

Consider the following Lyapunov function candidate:
\[ V(x(x)) = x^T(t) P x(t), \quad P = P^T > 0. \] (19)

The time derivative of \( V(t) \) (12) is given by
\[ \dot{V}(x(t)) = \sum_{i=1}^{4} \sum_{j=1}^{4} \mu_i(x(t)) \mu_j(x(t)) \left[ \tilde{x}^T(t) \left( P \tilde{x}_y + \tilde{x}_y^T P \right) \tilde{x}(t) + \tilde{x}(t) P \tilde{H} \tilde{w}(t) + \tilde{w}^T(t) \tilde{H}^T P \tilde{x}(t) \right]. \]

The closed loop system with controller-based observer is stable and has \( H_{\infty} \) norm limited by \( \gamma \) if and only if:
\[ \dot{V}(\tilde{x}(t)) + \gamma \tilde{w}^T(t) \tilde{w}(t) < 0 \] (20)

Therefore, we have
\[ \sum_{i=1}^{4} \sum_{j=1}^{4} \mu_i(x(t)) \mu_j(x(t)) \left[ \tilde{x}^T(t) \left( P \tilde{x}_y + \tilde{x}_y^T P \right) \tilde{x}(t) + \tilde{x}(t) P \tilde{H} \tilde{w}(t) + \tilde{w}^T(t) \tilde{H}^T P \tilde{x}(t) \right] \leq 0 \] (21)

where \( \Xi_y = \left[ \tilde{A}_y^T P + P \tilde{A}_y + \tilde{H}^T \tilde{H} - \gamma^2 I_4 \right]. \)

Inequality (19) is satisfied if condition 20 holds:
\[ \Xi_y \leq 0 \] (22)

Let us consider the following particular form of \( P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \), \( P_1 = P_1^T > 0 \), \( P_2 = P_2^T > 0 \) Then after simple manipulation, inequality (17) can be reformulated as:
\[ \Xi_y = \begin{bmatrix} \Gamma_{1y} & P B_y K_y & P B_y \\ * & \Gamma_{2y} & P B_y \\ * & * & -\gamma^2 I_4 \end{bmatrix} \leq 0 \] (23)

with
\[ \Gamma_{1y} = P_1 \left( A - B K_y \right)^T P_1 \] and \( \Gamma_{2y} = P_2 \left( A - L C_y \right)^T P_2 + I_4 \).

It should be noted that condition (18) is nonlinear with respect variables \( P_1 \) and \( K_y \). Then, the objective is to formulate (18) in LMI constraints.

Hence, after partitioning the matrix inequality shown in (18), \( \Xi_y \) becomes:
\[ \Xi_y = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} \] (24)

\( \Xi_{12} \) is the upper right block of (22) and \( \Xi_{22} \) is the lower right block of (22).

**Lemma 1**: (Congruence) Let two matrices \( P \) and \( Q \), if \( P \) is positive definite and if \( Q \) is a full column rank matrix, then matrix \( PQ^T \) is positive definite.
\[ Q = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & X \end{bmatrix} \text{ and } X = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & I \end{bmatrix} \]

Post and pre-multiplying inequality (21) by \( Q \), it follows that (23) can be rewritten as:

\[ \begin{bmatrix} P_1^{-1} \Xi_{11} P_1^{-1} & P_1^{-1} \Xi_{12} X \\ \ast & X \Xi_{22} X \end{bmatrix} \leq 0 \]

(25)

**Lemma 2:** Considering \( \Xi_{22} \leq 0 \) a matrix \( X \) and a scalar \( \mu \), the following holds [21][22]:

\[
(\begin{bmatrix} X + \mu \Xi_{22} \end{bmatrix})^T \Xi_{22} (\begin{bmatrix} X + \mu \Xi_{22} \end{bmatrix}) \leq 0 \iff X \Xi_{22} X \leq -2 \mu X - \mu \Xi_{22}^{-1}
\]

(26)

By substituting (22) into (21) and using the Schur complement, then inequality (21) holds if (23), displayed below, is satisfied:

\[ \begin{bmatrix} P_1^{-1} \Xi_{11} P_1^{-1} & P_1^{-1} \Xi_{12} X & 0 \\ \ast & -2 \mu X & \mu I \\ \ast & \ast & \Xi_{22} \end{bmatrix} \leq 0 \]

(27)

By changing matrices \( \Xi_{11}, \Xi_{12} \) and \( \Xi_{22} \) by their expressions from (19) and considering \( X_1 = P_1^{-1}, Y_j = K_j X_1 \) we obtain an LMI in (14).

## 5. Simulation Results

To illustrate the proposed method, the Observer based robust controller law is tested by considering the T-S Model of the photovoltaic pumping. The controller is tested by simulation. This section shows the efficiency of the designed control of system through computer simulations.

**Figure 5** and **Figure 6** present the climatic conditions on one day (temperature and solar irradiance).

To validate our approach, we compare these results with those given by the classical P&O method.

In **Figure 7**, the simulation result shows that the power obtained in our case is better than using the P&O method.

**Figure 8** shows the comparison of motor speed using the two methods. We can see the difference and the importance of our approach.

Finally, the water flow obtained by the two methods is presented in **Figure 9**.

The errors of state estimation are given in **Figure 10**, where all errors asymptotically converge to zero.

The Simulation results showing the confusion of the P&O algorithm with reference voltage perturbation by a step increase in solar irradiance (**Figure 11**).
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Figure 6. The solar irradiance for a day.

Figure 7. Maximum power obtained by two methods.

Figure 8. Simulated motor speed.
Figure 9. Simulated flow rate.

Figure 10. Estimation error of (a) inductance current; (b) PV array Voltage; and (c) Motor speed.

Figure 11. Voltage Vpv and your reference.
6. Conclusion

In this paper, a robust $H_{\infty}$ observer-based tracking control of the photovoltaic Pumping System has been proposed. The designed controller ensures the optimum of power and guarantees a better flow of water. The stability conditions are given in terms of LMIs, which can be solved in one single step-procedure to determine the observer and the controller gains. From simulation results, the performances of the designed robust observer-based controller are satisfactory and the capability of this controller is shown under critical situations. To put that in perspective, we will test this control law on a test branch that we are developing in our laboratory.

References


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