Modeling and Simulation of Laparoscopic Tools for Autonomously Positioning Laparoscope in Laparoscopic Surgery

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ABSTRACT

In laparoscopic surgery, the surgeons are equipped with the suitable tools for the surgery, while the laparoscope is used to capture the operation environment and displays it on a monitor. This paper presents the mathematical kinematic position modeling of the laparoscopic tools used for autonomous positioning of a laparoscope in such operations. These models are obtained using Denavit-Hartenberg (D-H) Notations and Homogenous Transformation Matrix (HTM). The laparoscopic tools are considered as six degrees of freedom (DOF) mechanisms while the laparoscope has four DOF. The 3D loop closure equation is used to obtain the laparoscope kinematic position models in terms of those of the laparoscopic tools. These models are used to simulate and align the laparoscope camera with the surgeon’s laparoscopic Tools Center Points (TCP). The obtained results show the smooth positioning of the laparoscope camera for better visualization of laparoscopic surgery environments.

Keywords: Laparoscopic Surgery; Homogenous Transformation Matrix; Denavit-Hartenberg Notations

1. Introduction

In laparoscopic surgery, the surgeons are equipped with the suitable tools for the surgery, while the laparoscope is used to capture the operation environment and displays it on a monitor. This paper presents the mathematical kinematic position modeling of the laparoscopic tools used for autonomous positioning of a laparoscope in such operations. These models are obtained using Denavit-Hartenberg (D-H) Notations and Homogenous Transformation Matrix (HTM). The laparoscopic tools are considered as six degrees of freedom (DOF) mechanisms while the laparoscope has four DOF. The 3D loop closure equation is used to obtain the laparoscope kinematic position models in terms of those of the laparoscopic tools. These models are used to simulate and align the laparoscope camera with the surgeon’s laparoscopic Tools Center Points (TCP). The obtained results show the smooth positioning of the laparoscope camera for better visualization of laparoscopic surgery environments.

Keywords: Laparoscopic Surgery; Homogenous Transformation Matrix; Denavit-Hartenberg Notations

2. Kinematic Modeling of 6 DOF Laparoscopic Tools

In laparoscopic surgery, Surgeons use two 6 DOF laparoscopic tools (right & left) to carry out complex operations. The four Denavit-Hartenberg (D-H) parameters ($\theta_i$, $r_i$, $a_i$, and $\alpha_i$ for $i = 1, 2 \cdots 6$) are used for generating the elementary HTM’s $(T_{i,i+1})$ of the laparoscopic tools (see Figure 1, and Table 1) [6].

By substituting the D-H parameters in Table 1 into (1) the 6 elementary HTM’s $(T_{i,i+1})$ for $i = 1, 2 \cdots 6$ are determined which are used in (2) to obtain the generalized HTM $(T_{0,7})$.
Figure 1. Laparoscopic tool kinematic diagram.

Table 1. D-H parameters.

<table>
<thead>
<tr>
<th>Joint i</th>
<th>Parameter s</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>$\pi/2$</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>0</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td>$q_i$</td>
</tr>
<tr>
<td>$r_i$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>$a_i$</td>
<td>0</td>
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<td>0</td>
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<td>$a_i$</td>
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<tr>
<td>$a_i$</td>
<td>$-\pi/2$</td>
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<td>$\pi/2$</td>
<td>$\pi/2$</td>
<td>$-\pi/2$</td>
<td>0</td>
</tr>
</tbody>
</table>

This generalized HTM is used with (3) representing the desired laparoscopic tool generalized HTM ($T_{0,7}$) to obtain the expressions of the laparoscopic tool direct kinematic position model (DKPM) and the position and orientation of its TCP are obtained. The inverse kinematic position model (IKPM) is also derived using the same generalized HTM. In the next equations $S = \sin$ and $C = \cos$.

Elementary HTM:

$$T _ {i,i+1} = \begin{bmatrix} CC\theta_i & -S\theta_i CC\alpha_i & SS\theta_i SS\alpha_i & a_i CC\theta_i \\ S\theta_i CC\theta_i CC\alpha_i & -C - C\theta_i SS\alpha_i & a_i SS\theta_i \\ 0 & SS\alpha_i & CC\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(1)

Generalized HTM:

$$T _ {0,7} = \prod _ {i=1} ^ 7 T _ {i,i+1}$$

(2)

$$T _ {0,7} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & n \\ r_{21} & r_{22} & r_{23} & m \\ r_{31} & r_{32} & r_{33} & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3)

DKPM & IKPM: An algebraic approach is used to calculate the laparoscopic tool DKPM and IKPM as follows:

First (2) is pre-multiplied by the inverse of the HTM ($T _ {0,3}$) as presented in (4) resulting 6 Equations (5, 6, 7, 8, 9, and 10). Second (2) is pre-multiplied by the inverse of the HTM ($T _ {0,2}$) and post-multiplied by the inverse of the HTM ($T _ {6,7}$) as presented in (11) resulting 2 Equations (12, 13). The obtained eight equations are used to calculate the expressions of the laparoscopic tool DKPM which are solved to calculate its IKPM.

$$\left(T _ {0,3} \cdot T _ {1,2} \cdot T _ {2,3}\right) ^ {-1} \cdot T _ {0,7} = T _ {3,4} \cdot T _ {4,5} \cdot T _ {5,6} \cdot T _ {6,7}$$

(4)

$$r_{21} \cdot S q_1 - r_{31} \cdot C q_1 = S q_1 \cdot C q_2 \cdot C q_6 - C q_3 \cdot S q_6$$

(5)

$$r_{22} \cdot S q_1 - r_{32} \cdot C q_1 = S q_1 \cdot C q_5$$

(6)

$$m \cdot S q_1 - n \cdot C q_1 = a_k \cdot S q_3 \cdot C q_3 \cdot C q_6 - C q_3 \cdot S q_6 + a_k \cdot S q_3 \cdot C q_5$$

(7)

$$r_{11} \cdot C q_2 - S q_2 \cdot (r_{21} \cdot C q_1 + r_{31} \cdot S q_1) = S q_5 \cdot C q_6$$

(8)

$$r_{13} \cdot C q_2 + S q_2 \cdot (r_{23} \cdot C q_1 + r_{33} \cdot S q_1) = S q_5 \cdot S q_6$$

(9)

$$l \cdot C q_2 - S q_2 \cdot \left(m \cdot C q_1 + n \cdot S q_1\right) = a_k \cdot S q_3 \cdot C q_6 + a_k \cdot S q_5 - q_4$$

(10)

$$\left(T _ {0,3} \cdot T _ {1,2}\right) ^ {-1} \cdot T _ {0,7} \cdot T _ {4,5} \cdot T _ {5,6} = T _ {2,3} \cdot T _ {3,4} \cdot T _ {4,5} \cdot T _ {5,6}$$

(11)

$$C q_6 \cdot \left(r_{21} \cdot S q_1 - r_{31} \cdot S q_1\right) + S q_6 \cdot \left(r_{23} \cdot S q_1 - r_{33} \cdot C q_1\right) = S q_3 \cdot C q_5$$

(12)

$$S q_6 \cdot \left(r_{21} \cdot S q_1 - r_{31} \cdot S q_1\right) + C q_6 \cdot \left(r_{23} \cdot S q_1 - r_{33} \cdot C q_1\right) = -S q_5 \cdot S q_5$$

(13)

The following is the IKPM of the 6 DOF laparoscopic tools

$$a_1 = \frac{m^2}{a_5^2} + \left(\frac{a_5^2}{a_5^2} - 1\right) \cdot r_{21} \cdot 2a_5 \cdot m \cdot r_{21} + r_{22} - r_{23}^2$$

(14)

$$b_1 = \frac{2m}{a_5} - 2 \left(\frac{a_5^2}{a_5^2} - 1\right) r_{21} \cdot r_{31} + \frac{2a_5}{a_5} \left(m \cdot r_{21} + n \cdot r_{21}\right)$$

(15)
\[ c_1 = \frac{n^2}{a_5^2} + \left( \frac{a_5^2}{a_5^2} - 1 \right) r_1^2 - \frac{2a_5 \cdot n}{a_5^2} r_{31} + r_{31}^2 - r_{33}^2 \]  
\[ q_1 = \text{atan2} \left( \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \right) \]  
\[ d_{61} = r_{23} \cdot S_{q_1} - r_{32} \cdot C_{q_1} \]  
\[ d_{62} = r_{21} \cdot S_{q_1} - r_{31} \cdot C_{q_1} \]  
\[ c_{61} = r_{32} \cdot C_{q_1} - r_{22} \cdot S_{q_1} \]  
\[ c_{62} = \frac{m \cdot S_q \cdot q_n \cdot C_{q_1} - a_6 \cdot d_{12}}{a_5} \]  
\[ \begin{bmatrix} C_{q_6} \\ S_{q_6} \end{bmatrix} = \begin{bmatrix} d_{61} \\ d_{62} \end{bmatrix} \begin{bmatrix} c_{61} \\ c_{62} \end{bmatrix} \]  
\[ q_n = \text{atan2} \left( S_{q_6}, C_{q_6} \right) \]  
\[ q_5 = \text{atan} \left( \frac{-d_{61}}{d_{62}} \right) \]  
\[ \begin{bmatrix} S_{q_3} \\ C_{q_3} \end{bmatrix} = \begin{bmatrix} C_{q_5} & 0 \\ C_{q_5} \cdot C_{q_6} & -S_{q_6} \end{bmatrix} \begin{bmatrix} c_{62} \\ \end{bmatrix} \]  
\[ q_3 = \text{atan2} \left( S_{q_3}, C_{q_3} \right) \]  
\[ d_{41} = m \cdot C_{q_1} + n \cdot S_{q_1} \]  
\[ q_4 = -l \cdot C_{q_3} + S_{q_2} \cdot d_{41} + c_{41} \]  
\[ c_{41} = a_6 \cdot S_{q_5} \cdot C_{q_6} + a_5 \cdot S_{q_6} \]  
\[ c_{42} = a_6 \cdot \left( C_{q_5} \cdot C_{q_3} \cdot C_{q_6} + S_{q_3} \cdot S_{q_6} \right) + a_5 \cdot C_{q_3} \cdot C_{q_5} \]  
\[ \begin{bmatrix} S_{q_2} \\ C_{q_2} \end{bmatrix} = \begin{bmatrix} d_{41} & -I \\ I & d_{41} \end{bmatrix} \begin{bmatrix} q_4 - c_{41} \\ c_{42} \end{bmatrix} \]  
\[ q_3 = \text{atan2} \left( S_{q_3}, C_{q_3} \right) \]  

3. Positioning of a Robot Assisted Surgery Laparoscope

It’s supposed that the laparoscope is always tracking the midpoint between the TCPs of the right and left laparoscopic tools. The 3D loop closure theory is used to get the HTM of the laparoscope in terms of the left and right laparoscopic tools joint angles [7]. Figures 2 and 3 present the closed loops formed by the HTM of the right, left laparoscopic tool, and laparoscope. The product of these HTMs is equal to a unity matrix as in (33) and (39).

\[ T_{AD} = T_{AB} T_{BC} T_{CD} \]  

\[ T_{BA} = \begin{bmatrix} R_{BA}(3 \times 3) & P_{BA}(3 \times 1) \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  

\[ T_{AB} = \begin{bmatrix} R_{AB}(3 \times 3) & -R_{BA}(3 \times 3)^T P_{BA}(3 \times 1) \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  

\[ T_{CD} = \begin{bmatrix} R_{CD}(3 \times 3) & P_{CD}(3 \times 1) \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  

\[ T_{BC} = \begin{bmatrix} I_P & 0_{bc} \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  

\[ T_{AD} = \begin{bmatrix} R_{AD}(3 \times 3) R_{BA}(3 \times 3) P_{BA}(3 \times 1) \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  

\[ T_{EF} = T_{EB} T_{BA} T_{AB} \]  

\[ T_{EB} = \begin{bmatrix} 1 & P_{EB}(3 \times 1) \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]
\[ T_{AF} = \begin{bmatrix} R_{Ba} R_{cd} & r \left[ R_{Ba}^t \left( P_{Ba} + P_{cd} \right) - R_{Ba}^t P_{Ba} \right] \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  \tag{41}

\[ T_{EF} = \begin{bmatrix} R_{cd} & r \left[ R_{Ba}^t \left( P_{Ba} + P_{cd} \right) - R_{Ba}^t P_{Ba} \right] \\ 0_{(1 \times 3)} & 1 \end{bmatrix} \]  \tag{42}

where \( T_{BA} \) is the HTM of the right laparoscopic tool, \( T_{AB} \) is the inverse of the HTM of the right laparoscopic tool, \( T_{CD} \) is the HTM of the left laparoscopic tool, \( T_{BC} \) is HTM between the right laparoscopic tool fixed coordinates at point \( B \) and the left laparoscopic tool fixed coordinates at \( C \). \( T_{AD} \) is the homogenous transformation matrix between the movable coordinates of the right laparoscopic tool at point \( A \) and the movable coordinates the left laparoscopic tool at \( D \).

\( T_{EF} \) is the HTM of the laparoscope, \( T_{EB} \) is the HTM between the laparoscope fixed coordinates at \( E \) and the right laparoscopic tool fixed coordinates at \( B \). It’s assumed that the coordinates in the same direction so the rotation matrix is unity matrix and there are only translation in \( x, y, z \) directions. \( T_{de} \) is the HTM between the movable right laparoscopic tool coordinates at \( A \) and the coordinates at the target point (\( F \)) but the translation vector is a ratio (\( r \)) of the translation vector of \( T_{AF} \). The orientation and position (DKPM) of the laparoscope Camera are calculated directly from \( T_{EF} \). The laparoscope parameters are determined in terms of those of the laparoscopic tools [6].

4. Simulation

During surgery operation, the laparoscopic tool is moved from one point to another on a certain trajectory. Free and guided trajectories may be selected to avoid obstacles inside the human body. The laparoscopic tools’ models are executed using any desired time function for the path of its TCP to perform a certain task. The obtained mathematical models are simulated to show the best positioning of the laparoscope in a surgery environment. The spline trajectory is used to simulate these models [6]. The simulation results show that the laparoscope movements are smooth so the monitor’s vision will be stable as shown in Figure 4.

5. Conclusion

In this paper, D-H parameters and HTM technique are used to obtain the expressions of the DKPM and IKPM models of two laparoscopic tools and one laparoscope used for its autonomous positioning in laparoscopic surgery. The laparoscopic tools and laparoscope models are used to simulate and align the laparoscope camera with the surgeon’s laparoscopic tools’ TCPs. The obtained results show a smooth positioning of the laparoscope camera for better visualization in laparoscopic surgery environments.

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