Research on the Application of AHP and Fuzzy Comprehensive Evaluation of Teaching Quality in Basic Mathematics Classroom

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Abstract

With the development and innovation of basic mathematics education, the classroom teaching quality has become a hot spot of basic mathematics education curriculum and teaching reform. The high classroom teaching quality can comprehensively and deeply reflect the teaching level of teachers, improve the level of students’ mathematical key competencies, and realize the fast improvement of mathematics learning process. Therefore, the evaluation of it has become not only important but necessary. As a result, we argue that evaluation indicator system of teaching quality should be well researched. Based on the multi-layered and fuzzy-paste nature of the evaluation problem of classroom teaching quality, the evaluation model of classroom teaching quality is tried to construct. By using analytic hierarchy process, the weight of each indicator is determined, and the teaching quality is determined by using the model of fuzzy comprehensive evaluation. The evaluation method improves the reliability and objectivity of the evaluation results, promoting students’ efficient learning of mathematics and teaching level.

Keywords
Classroom Teaching Quality, Analytic Hierarchy Process, Fuzzy Comprehensive Evaluation Model, Evaluation Indicator System

1. Introduction
1.1. Background

In mathematics classroom teaching, it is divided into lecture-type and open-type. The lecture-type means that teachers use certain teaching methods to directly teach students new teaching content, so that students can obtain knowledge and
class-type skills. The open-type means that teaching is to correctly handle the relationship between “process” and “conclusion” in teaching according to the law of cognition, restoring the proper status of “process”, and giving students the power to study independently. The teachers allow students to enjoy the vast space and time of independent discovery, handing-on experience and independent exploration.

In the work of basic mathematics education, how to evaluate the quality of efficient mathematics classroom teaching and which method should be chosen have always been different from each other. It is generally accepted that “it is not difficult to finish a lesson, but it is not easy to finish a perfect lesson” (Fu & Jia, 2009). In mathematics classroom teaching, it is divided into lecture-type and open-type. And in the case of mathematics teaching in basic education stage, the traditional lecture-type teaching mode still occupies a decisive position, and most of the leading actor in class is still teacher, ignoring the students’ behavior who are the main body of the classroom. Mathematical knowledge cannot be well interpreted. Learning interest is not completely inspired. Teaching effect cannot be perfectly displayed. So the traditional teaching quality evaluation method cannot improve the level of mathematics classroom teaching. In the new era, how should we reform the evaluation mode of traditional mathematics classroom teaching quality, turn the boring knowledge into vivid, and create a dynamic and efficient mathematics class?

1.2. Significance of the Study

The construction of the evaluation system of mathematics education quality with novel ideas can not only improve students’ learning methods and styles, promote students’ all-round development and improve students’ core qualities, but also promote the transformation of teachers’ roles and improve teaching methods. This is the orientation and guidance of the development direction of basic mathematics education. It is the self-improvement of the reform system of basic mathematics education and the regression of basic mathematics education standard functions (Gao & Fu, 2017).

The teacher should design the teaching process carefully around the learning community in the dynamic process of spiritual communication, diverting students’ thinking and developing students’ perspective, moderately conveying the subject to students. Exploration problems at different levels should be carefully designs. Appropriate methods and skills for knowledge in different fields should be appropriately selected. Effective mathematical learning activities need students’ independent inquiry, cooperative communication and hands-on practice, which should be a lively and active process. The learning contents should be realistic, meaningful and challenging. Teachers need to accurately locate the recent development area of students’ thinking, carry out questioning, and establish the correct education quality view in mathematics class. To fully understand the quality evaluation of efficient mathematics classroom teaching, we should not
only pay attention to teachers’ performance and students’ acceptability, but also discover and develop students’ potential in all aspects, guide students’ self-confidence, self-reliance and self-improvement, and promote students’ all-round development.

Based on the multi-layered and fuzzy-paste nature of the evaluation problem of classroom teaching quality, the academic significance is that the evaluation model of classroom teaching quality is constructed. It is creative in teaching reform. By using analytic hierarchy process and fuzzy comprehensive evaluation, the teaching quality is determined. The evaluation method improves the reliability and objectivity of the evaluation results, promoting students’ efficient learning of mathematics and teaching level (Li, 2011).

2. Process of the Study

2.1. The Meaning of Mathematics Teaching Quality Evaluation

The premise of mathematics teaching evaluation is to have clear and definite teaching objectives. The teaching goal is not single, but is composed by many objective factors. Therefore, in the implementation of mathematics education evaluation, it is necessary to comprehensively investigate diversified mathematics objective factors, guide students to develop comprehensively and individualized, and make the results of mathematics teaching quality evaluation more objective and accurate.

2.2. The Concept of Quality Evaluation System in Mathematics Classroom

The effective evaluation of mathematics classroom teaching quality should focus on whether students can acquire comprehensive quality, reasonable education and true quality, and whether students can achieve comprehensive development. Its basic spirit lies in paying attention to the benefit of mathematics teaching class, that is, how to consume the least time and energy of teachers and students to maximize the benefit of efficient mathematics class, and finally achieve “win-win” situation between teachers and students. The idea of mathematics classroom teaching quality evaluation is to establish an evaluation system to promote students’ all-round development, and improve teaching practice. The concept of evaluation is not merely pay attention to a particular element, but focus on students to seek the process and strategy by using knowledge to solve practical problems, to provide students a variety of opportunities. Such diversity evaluation indicator system should be constructed by using appropriate method. It makes the evaluation results more valuable and more effectively (Zhang & Ma, 2013).

3. Construction of Evaluation Model

3.1. Evaluation System

In order to make the evaluation results of teaching quality scientific and reached the target, it is crucial to determine scientific and rational evaluation criteria and
to make them concrete and operable. In accordance with the original principle of guiding, science, hierarchy, practicability and development, this paper adopts the research methods of visiting and investigating, literature review, expert discussion, in-class lecture, empirical analysis and other research methods to analyze and deal with the problems existing in mathematics classroom teaching content. We attempt to construct a set of more scientific and reasonable evaluation indicator system for classroom teaching quality which makes the evaluation results more effective and math class returned to students so that the class full of passion and vitality. They can guide teachers to adopt more appropriate teaching methods to improve students’ learning effect (Zhang & Ma, 2013).

Students are the image of teaching, the main body of learning, the direct experience of teachers’ teaching results and the beneficiaries. As a result, students’ evaluation results play a key role in the evaluation of classroom teaching quality. There are 6 first-level indicators and 22 second-level indicators in evaluation system, as shown in Table 1.

### 3.2. Evaluation Model

The weight of each indicator is determined by the method of analytic hierarchy process. AHP is a method to calculate the weight of each indicator which is put forward by American professor T.L. Saaty et al. in 1970s. Comparing the indicators in pairs, we get judgment matrix, in addition, through the results of consistency check, it is possible to overcome the disadvantages created by determining the weights of indicators by subjective view analysis, improving the objectivity and scientific nature of the analysis (McDermott & Shaffer, 1992; Wosilait, Heron, Shaffer, & McDermott, 1998).

Using the software of yaahp, hierarchical structure model between levels is constructed. The weight matrix of primary and secondary indicator is established, and the weight of each indicator is calculated separately. The model is shown in Figure 1. The specific results are shown in Figures 2-6.

### 4. Fuzzy Comprehensive Evaluation Method

Using the method of analysis analytic hierarchy process to calculate the weight of each indicator of classroom teaching quality, however, there is no quantitative description of the evaluation. For this reason, the application of the fuzzy comprehensive evaluation method provides a new path and a new method for such problems (Armstrong & Baron, 1998).

First, each first-level indicators are evaluated by this method, and then to the classroom teaching quality is given a fuzzy comprehensive evaluation.

### 4.1. The Evaluation Factors of Mathematics Classroom Teaching Quality Are Determined

We set up the evaluation factor set according to the establishment of quality evaluation system of mathematics classroom teaching...
The second-level factor set of subfactor set $U_i (i = 1, 2, \cdots, n)$ is

$$U_i = \{U_{i1}, U_{i2}, \cdots, U_{in}\}$$ (2)
Figure 2. The weight of each first-level indicator.

Figure 3. The weight of each second-level indicator.

Figure 4. The weight of each indicator.
4.2. The Indicator Comment Set of Mathematics Classroom Teaching Quality Are Determined

$V$ is divided into 5 evaluation levels, generally are excellent, good, general, poor, very poor.

$$V = \{V_1, V_2, \cdots, V_5\}$$  \hspace{1cm} (3)

We suppose its comment vector is

$$V = \{\mu_{11}, \mu_{12}, \cdots, \mu_{15}\}$$  \hspace{1cm} (4)
\[ \mu_{i_k} \in [0,1] (k = 1,2,\cdots, p) \] It’s the comment of level \( k \) relative to the affiliation degree of \( V \).

### 4.3. The Weight of Evaluation Factors of Mathematics Classroom Teaching Quality Is Determined

Through analysis the role of \( U_i \) in evaluation process, we get weight vector \( A \) of \( U \)

\[ A = (a_1, a_2, \cdots, a_n) \]  

\[ a_i \geq 0 , \text{ and } \sum_{j=1}^{n} a_i = 1 (i = 1,2,\cdots,n) . \]

The weight vector \( A_i \) of \( U_i \)

\[ A_i = (a_{i1}, a_{i2}, \cdots, a_{im}) \]  

\[ a_{ij} \geq 0, (j = 1,2,\cdots,m) , \text{ and } \sum_{j=1}^{m} a_{ij} = 1 (i = 1,2,\cdots,n) . \]

### 4.4. The Method of Single Factor Evaluation of Mathematics Classroom Teaching Quality

We evaluate single factor \( U_j \), the vector value of the \( j \) subfactor of \( U \) is

\[ r_{ij} = (r_{ij1}, r_{ij2}, \cdots, r_{ijp}) \]  

\[ \sum_{k=1}^{p} r_{ijk} = 1 , \text{ } r_{ijk} \text{ is the affiliation degree about level } k \text{ relative to the } j \text{ subfactor of } U_i . \]

The single factor evaluation matrix \( R_j \) of the sub-factor set \( U_i \) which have \( j \) sub-single factors is Saaty, 1980

\[ R_j = \begin{bmatrix} r_{i11} & r_{i12} & \cdots & r_{i1p} \\ r_{i21} & r_{i22} & \cdots & r_{i2p} \\ \vdots & \vdots & \ddots & \vdots \\ r_{im1} & r_{im2} & \cdots & r_{imp} \end{bmatrix} \]  

### 4.5. The Method of the First-Level Fuzzy Comprehensive Evaluation of Mathematics Classroom Teaching Quality

Each \( U_i \) is got fuzzy comprehensive evaluation, we get fuzzy vector of the first-level fuzzy comprehensive evaluation

\[ B_{i} = (b_{i1}, b_{i2}, \cdots, b_{ip}) = A_i \circ R_i \]  

\[ b_{ik} = \max \{a_{ik}, r_{ijk}\} , \cdots , \min (a_{im}, r_{imk}) \]  

\[ i = 1,2,\cdots,n , j = 1,2,\cdots,m , k = 1,2,\cdots,p . \]

### 4.6. The Method of the Second-Level Fuzzy Comprehensive Evaluation of Mathematics Classroom Teaching Quality

The \( n \) sub factors of \( U \) are considered as \( n \) single factor, and then get the second-level fuzzy comprehensive evaluation. The fuzzy evaluation matrix \( R \)
consisting of $B_n$ which are the results of the first-level comprehensive evaluation of $U_j$ is

$$
R = \begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= \begin{bmatrix}
b_{11} & b_{12} & \cdots & b_{1p} \\
b_{21} & b_{22} & \cdots & b_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
b_{n1} & b_{n2} & \cdots & b_{np}
\end{bmatrix}
$$

The fuzzy vector $B$ of the second-level comprehensive evaluation is

$$
B = (b_1, b_2, \cdots, b_p) = A \cdot R
$$

The final evaluation level is determined according to the principle of enlargement.

5. Using Fuzzy Comprehensive Evaluation Method to Evaluate the Quality of Classroom Teaching

We take the lead in selecting a certain section of the middle school mathematics classroom as an example to make attempt to explore and obtain a more objective, fair and comprehensive evaluation.

5.1. Factor Set and Evaluation Set Are Determined

$$
U = \{U_1, U_2, U_3, U_4, U_5, U_6\},
$$

$$
U_1 = \{U_{11}, U_{12}\},
$$

$$
U_2 = \{U_{21}, U_{22}, \cdots, U_{28}\},
$$

$$
U_3 = \{U_{31}, U_{32}, U_{33}, U_{34}\},
$$

$$
U_4 = \{U_{41}, U_{42}, U_{43}, U_{44}\},
$$

$$
U_5 = \{U_{51}, U_{52}\},
$$

$$
U_6 = \{U_{61}, U_{62}\}
$$

$$
V = \{V_1, V_2, V_3, V_4, V_5\} \text{ are excellent, good, general, poor, very poor.}
$$

Determination of weights is a very important part of the evaluation. It actually determines the position of each specific evaluation in the entire indicator system. We determine the vector of the factors in a by consulting the experts and issuing the questionnaire.

The weight vector $A_i$ of each factor $U_i$ are that

$$
A_1 = \{0.65, 0.35\}
$$

$$
A_2 = \{0.19, 0.05, 0.06, 0.15, 0.12, 0.16, 0.14, 0.13\}
$$

$$
A_3 = \{0.35, 0.35, 0.10, 0.20\}
$$

$$
A_4 = \{0.30, 0.20, 0.30, 0.20\}
$$

$$
A_5 = \{0.40, 0.60\}
$$

$$
A_6 = \{0.60, 0.40\}
$$
5.2. Fuzzy Evaluation Matrix of Single-Factor Evaluation Is Determined

The expert judges were invited to evaluate each factor in a certain section of mathematics classroom teaching, and 100 questionnaires were issued. When 26 people rated “excellent”, the statistical result was 0.26. To determine the affiliation degree of to the evaluation level of each factor in $V$.

$$R_1 = \begin{bmatrix} 0.26 & 0.33 & 0.31 & 0.10 & 0.00 \\ 0.21 & 0.29 & 0.35 & 0.15 & 0.00 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.17 & 0.49 & 0.18 & 0.16 & 0.00 \\ 0.14 & 0.21 & 0.43 & 0.22 & 0.00 \\ 0.15 & 0.32 & 0.36 & 0.17 & 0.00 \\ 0.18 & 0.32 & 0.40 & 0.10 & 0.00 \\ 0.16 & 0.33 & 0.37 & 0.14 & 0.00 \\ 0.13 & 0.34 & 0.32 & 0.21 & 0.00 \\ 0.12 & 0.36 & 0.34 & 0.18 & 0.00 \\ 0.15 & 0.33 & 0.35 & 0.17 & 0.00 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.36 & 0.41 & 0.23 & 0.00 & 0.00 \\ 0.16 & 0.38 & 0.27 & 0.19 & 0.00 \\ 0.00 & 0.20 & 0.31 & 0.49 & 0.00 \\ 0.10 & 0.34 & 0.29 & 0.27 & 0.00 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} 0.44 & 0.31 & 0.25 & 0.00 & 0.00 \\ 0.19 & 0.44 & 0.29 & 0.08 & 0.00 \\ 0.21 & 0.38 & 0.29 & 0.12 & 0.00 \\ 0.32 & 0.31 & 0.22 & 0.15 & 0.00 \end{bmatrix}$$

$$R_5 = \begin{bmatrix} 0.19 & 0.24 & 0.37 & 0.20 & 0.00 \\ 0.17 & 0.29 & 0.29 & 0.25 & 0.00 \end{bmatrix}$$

$$R_6 = \begin{bmatrix} 0.31 & 0.28 & 0.22 & 0.19 & 0.00 \\ 0.27 & 0.33 & 0.25 & 0.15 & 0.00 \end{bmatrix}$$

By calculating, we get the fuzzy vector of comprehensive evaluation.

$$B_1 = (0.26 \ 0.33 \ 0.31 \ 0.15 \ 0.00)$$

$$B_2 = (0.17 \ 0.19 \ 0.18 \ 0.16 \ 0.00)$$

$$B_3 = (0.35 \ 0.35 \ 0.20 \ 0.20 \ 0.00)$$

$$B_4 = (0.30 \ 0.30 \ 0.29 \ 0.12 \ 0.00)$$

$$B_5 = (0.19 \ 0.29 \ 0.29 \ 0.25 \ 0.00)$$

$$B_6 = (0.31 \ 0.33 \ 0.25 \ 0.19 \ 0.00)$$

Based on experience and expert group advice, the relative weights of each sub-factor set $U_i$ of evaluation factor set $U$ are given.

$$A = \{0.16, 0.34, 0.12, 0.14, 0.13, 0.11\}$$
The fuzzy evaluation matrix for the second-level comprehensive evaluation $U$ is that

$$ R = \begin{pmatrix} B_1 & B_2 & B_3 & B_4 & B_5 \end{pmatrix}^T $$

The fuzzy evaluation vector for the second-level comprehensive evaluation is that

$$ B = \begin{pmatrix} 0.17 & 0.19 & 0.18 & 0.16 & 0.00 \end{pmatrix} $$

As $0.17 + 0.19 + 0.18 + 0.16 \neq 1$ then normalize it to get that

$$ \hat{B} = \begin{pmatrix} 0.243 & 0.272 & 0.257 & 0.228 & 0.00 \end{pmatrix} $$

According to the principle of taking large, the fuzzy comprehensive evaluation level of mathematics teaching quality in the middle school is good.

**6. Concluding Remarks**

We use the method of analytic hierarchy process to determine the weight. According to the fuzzy comprehensive evaluation model of classroom teaching quality evaluation, it has the following characteristics:

Firstly, on basis of the knowledge of experts and subjective experiences, the strict mathematical method can be used to remove the subjective experiences as many as possible. Secondly, it can reasonably determine the weight of evaluation indicators and have good combination of qualitative and quantitative analysis. And according to the root, it is judged whether the matrix has a satisfactory consistency, so that the weight is more consistent with the objective reality. The evaluation results are reliable and useful. At last, the evaluation implementation process is operable. It is easier to express the advantage of this method. It is good at handling vague and uncertain information. The analysis of determinism and the description of determinism are better integrated. It can overcome the ideality of subjects in evaluation work (Yu, 2008; Dong & Qi, 2017).

All in all, the methods of analytic hierarchy process and fuzzy comprehensive evaluation are used comprehensively in the quality evaluation of teachers’ teaching and educating, and the reform of the quality evaluation model of traditional mathematics teaching can weaken subjective factors. It is innovative and practical. It provides ideas for the construction of high quality and efficient classroom, and improves the rationality, science, and effectiveness of mathematics classroom teaching. More innovative improvements and applications are the direction of our ongoing research in the future.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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