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Study of Thermally Induced Vibration of Non-Homogeneous Trapezoidal Plate with Parabolically Thickness Variation in Both Directions

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Abstract

The present analysis demonstrates the thermal effect on vibrations of a symmetric, non-homogeneous trapezoidal plate with parabolically varying thickness in both directions. The variation in Young's modulus and mass density is the main cause for the occurrence of non-homogeneity in plate's material. In this consideration, density varies linearly in one direction. The governing differential equations have been derived by Rayleigh-Ritz method in order to attain fundamental frequencies. With C-S-C-S boundary condition, a two term deflection function has been considered. The effect of structural parameters such as taper constants, thermal gradient, aspect ratio and non-homogeneity constant has been investigated for first two modes of vibration. The obtained numerical results have been presented in tabular and graphical form.

Keywords

Vibration, Trapezoidal Plate, Taper Constants, Thermal Gradient, Aspect Ratio, Non-Homogeneity, Parabolically Thickness, Linearly Density

1. Introduction

People became interested in vibration when the first musical instruments, probably whistles or drums were discovered. Since then people have applied ingenuity and critical investigation to study the phenomenon of vibra-

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tion. Many studies in existing period have been aggravated by the engineering applications of vibration, such as design of machines, foundations, structures, engines and turbine systems. Most major movers have vibrational problems because of the inbuilt unbalance in the engines. In spite of its detrimental effects, vibration can be utilized profitably in several industrial and consumer applications.

In many engineering applications different types of plates such as rectangular, parallelogram, circular etc. act as an integral part of the system. In contrast of uniform thickness of plate, the suitable variation in thickness in plate has a significant effect on its vibration. Thus, the choice of material depends on suitable properties of materials. On the whole, non-homogeneity is a significant constituent of any design which occurs as a result of variation in density. Literature shows that the vibration analysis has inspired many researchers to do work in this direction. Out of them few are given under. Kumar and Lal [1] worked on the vibrations of non-homogeneous orthotropic rectangular plates with bilinear thickness variation resting on Winkler foundation. Kumar and Tomar [2] had studied the free transverse vibrations of monoclinic rectangular plates with continuously varying thickness and density. Johri and Johri [3] had worked on the exponential thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional linear variation in thickness. Gupta *et al*. [4] did the vibration analysis of non-homogeneous circular plate of non-linear thickness variation by differential quadrature method. Li and Zhou [5] discussed the shooting method for non-linear vibration and thermal buckling of heated orthotropic circular plates. Chakraverty *et al*. [6] studied the effect of non-homogeneity on natural frequencies of vibration of elliptic plates. Gupta *et al*. [7] discussed the vibration of visco-elastic orthotropic parallelogram plate with linear thickness variation in both directions. Chen *et al*. [8] worked on the free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plates. Gurses *et al*. [9] analyzed the shear deformable laminated composite trapezoidal plates. Kitipornchai *et al*. [10] presented a global approach for vibration of thick trapezoidal plates. Sayad and Ghazy [11] studied the rayleigh-ritz method for free vibration of midline trapezoidal plates. Leung *et al*. [12] had studied the free vibration of laminated composite plates subjected to in-plane stresses using trapezoidal p-element. McGee and Butalia [13] presented the natural vibrations of shear deformable cantilevered skewed trapezoidal and triangular thick plates. Qatu [14] studied the vibrations of laminated composite completely free triangular and trapezoidal plates. Grigorenko *et al*. [15] used spline functions to solve boundary-value problems for laminated orthotropic trapezoidal plates of variable thickness. Feng and Min [16] worked on the vibrations of axially moving visco-elastic plate with parabolically varying thickness. Gupta and Sharma [17] evaluated the forced axisymmetric response of an annular plate of parabolically varying thickness. Liew and Lim [18] studied the transverse vibration of trapezoidal plates of variable thickness: symmetric trapezoids. Maruyama *et al*. [19] presented an experimental study of the free vibration of clamped trapezoidal plates. Karami *et al*. [20] used a differential quadrature method for skewed and trapezoidal laminated plates. Huang *et al*. [21] carried out experimental and numerical investigations for the free vibration of cantilever trapezoidal plates. Gupta and Sharma [22] studied the effect of thermal gradient on transverse vibration of non-homogeneous orthotropic trapezoidal plate of parabolically varying thickness. Gupta and Sharma [23] observed the effect of linear thermal gradient on vibrations of trapezoidal plates whose thickness varies parabolically. Gupta and Sharma [24] study the thermally induced vibration of non-homogeneous trapezoidal plate with varying thickness and density.

The existing work is an attempt to investigate the thermal effect on vibration of non-homogeneous trapezoidal plate of bi-parabolically varying thickness with linear density variation. To attain the natural frequencies for the first two modes of vibration Rayleigh-Ritz's method has been applied. The deflection function has been taken to satisfy the C-S-C-S boundary condition. All the obtained results have been presented in tabular and graphical form.

2. Mathematical Formulation

2.1. Geometry of the Plate

For the study of transverse vibration a thin, symmetric, non-homogeneous trapezoidal plate with varying thickness and density has been taken. The geometry of the plate is shown in **Figure 1**.

2.2. Thickness and Density

The thickness of the plate which varies parabolically in both directions can be expressed as

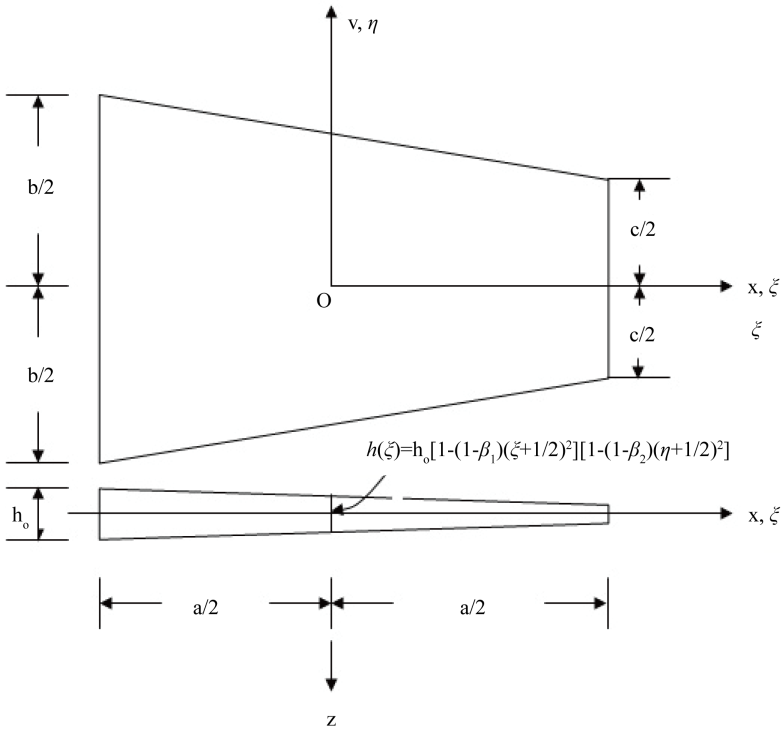


Figure 1. Geometry of the trapezoidal plate.

$$h(\xi) = h_0\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right] \tag{1}$$

The non-homogeneity occurs in the bodies because of imperfection of materials and it is assumed to arise due to the linear variation in density along the length of the plate. So, it can be stated as

$$\rho = \rho_0\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right] \tag{2}$$

It is assumed that the temperature of the non-homogeneous trapezoidal plate varies linearly along x-axis and is of the form

$$\tau = \tau_0\left(\frac{1}{2}-\xi\right) \tag{3}$$

where τ represent the excess above the reference temperature at a distance $\xi = \frac{x}{a}$ and τ_0 denotes the temperature excess above the reference temperature at the end $\xi = -\frac{1}{2}$.

The temperature dependence of the modulus of elasticity for most of the engineering materials is specified as [25]

$$E = E_0(1-\gamma\tau) \tag{4}$$

where E_0 denotes the value of Young's modulus at reference temperature $\tau = 0$ and γ is the slope of variation of E with τ.

Using Equation (3) into Equation (4), one obtain

$$E = E_0\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right) \tag{5}$$

where $\alpha = \gamma\tau_0\ (0 \le \alpha \le 1)$ known as thermal gradient.

3. Equation of Motion

The governing differential equation for kinetic energy T and strain energy V for a non-homogeneous trapezoidal plate with bi-parabolically varying thickness can be expressed as

$$T=\frac{ab}{2}\omega^2\int_A h(\xi)\rho w^2\,\mathrm{d}A \tag{6}$$

$$V=\frac{ab}{2}\int_A D(\xi)\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}A \tag{7}$$

where ν is the Poisson ratio, ω is the angular frequency of vibration and A is the area of the plate.

Flexural rigidity $D(\xi)$ of the plate is given by

$$D(\xi)=D_0\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3 \tag{8}$$

where $\xi=\frac{x}{a}$, $\eta=\frac{y}{b}$ are non-dimensional variables. Here,

$$D_0=\frac{Eh_0^3}{12\left(1-\nu^2\right)} \tag{9}$$

On using Equation (9) and (5), Equation (8) gives the value of flexural rigidity as follows

$$D(\xi)=\frac{E_0h_0^3}{12\left(1-\nu^2\right)}\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right) \tag{10}$$

Now after putting Equations (1), (2) into Equation (6) and (10) into Equation (7), kinetic energy and strain energy become

$$T=\frac{ab}{2}\rho_0h_0\omega^2\int_A\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\times\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right]w^2\,\mathrm{d}A \tag{11}$$

And

$$\begin{aligned}V=&\frac{ab}{2}\frac{E_0h_0^3}{12\left(1-\nu^2\right)}\int_A\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)\\&\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}A\end{aligned} \tag{12}$$

Two terms deflection function for a C-S-C-S trapezoidal plate can be defined as,

$$\begin{aligned}w=&A_1\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^2\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\left(\frac{b+c}{4}\right)\right\}\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\left(\frac{b+c}{4}\right)\right\}\\&+A_2\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^3\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\left(\frac{b+c}{4}\right)\right\}^2\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\left(\frac{b+c}{4}\right)\right\}^2,\end{aligned} \tag{13}$$

where A_1 and A_2 are unknowns to be calculated.

In this manner, for vibrational analysis a trapezoidal plate whose two sides are clamped and two are simply-supported has been considered. The deflection function which is already discussed by Equation (13) satisfies the boundary conditions and presents an excellent evaluation to the frequency. Thus, the boundaries are given by four straight lines as follows:

$$\eta = \frac{c}{4b} - \frac{\xi}{2} + \frac{1}{4} + \frac{c\xi}{2b};$$

$$\eta = -\frac{c}{4b} + \frac{\xi}{2} - \frac{1}{4} - \frac{c\xi}{2b};$$

$$\xi = -\frac{1}{2};$$

and

$$\xi = \frac{1}{2}. \tag{14}$$

4. Method of Solution

In addition the frequency is calculated through Rayleigh-Ritz technique which is based on the principle of conservation of energy i.e. the maximum strain energy must be equal to the maximum kinetic energy. Therefore, the resulting equation can be described by

$$\delta(V - T) = 0. \tag{15}$$

Using boundary condition (14) into Equation (11) and (12), one gets

$$T = \frac{ab}{2}\rho_0 h_0 \omega^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}} \left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right] \times\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right] w^2 \,\mathrm{d}\eta\,\mathrm{d}\xi \tag{16}$$

And

$$V = \frac{ab}{2}\frac{E_0 h_0^3}{12(1-\nu^2)} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}} \left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3 \left(1-\alpha\left(\frac{1}{2}-\xi\right)\right) \times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2 - 2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2} - \left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}\eta\,\mathrm{d}\xi. \tag{17}$$

Now Equations (16) and (17) consists the values of T and V so, put these values into Equation (15), we obtain

$$\delta\left(V_1 - \lambda^2 T_1\right) = 0, \tag{18}$$

where

$$T_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}} \left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\times\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right] w^2 \,\mathrm{d}\eta\,\mathrm{d}\xi, \tag{19}$$

$$V_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}} \left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)^2\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)^2\right]\right]^3 \left(1-\alpha\left(\frac{1}{2}-\xi\right)\right) \times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2 - 2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2} - \left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}\eta\,\mathrm{d}\xi, \tag{20}$$

And

$$\lambda^2 = \frac{12\omega^2\rho_0 a^4\left(1-\nu^2\right)}{E_0 h_0^2}. \tag{21}$$

is a frequency parameter.

Equation (18) includes two unknowns A_1 and A_2 which occurs as a result of using the deflection function. These two unknown can be evaluated from Equation (18) as follow:

$$\frac{\partial}{\partial A_m}\left(V_1-\lambda^2 T_1\right)=0, \quad m=1,2. \tag{22}$$

On simplifying (22), we get

$$b_{m1}A_1+b_{m2}A_2=0, \quad m=1,2. \tag{23}$$

where b_{m1}, b_{m2} (m = 1, 2) involves parametric constants and the frequency parameter. For a non-zero solution, the determinant of co-efficient of Equation (23) must vanish. Therefore, for a (C-S-C-S) trapezoidal plate the frequency equation can be obtained as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix}=0. \tag{24}$$

The quadratic equation in λ^2 is obtained through the Equation (24) which presents the two values of λ^2 known as first and second modes of vibration respectively.

5. Results and Discussion

The existing work deals with the vibration behavior of a non-homogeneous trapezoidal plate whose thickness varies bi-parabolically and density varies linearly in one direction. All results acquired by Equation (24) provide the values of frequency parameter for different values of taper constants, thermal gradient, aspect ratios and non-homogeneity constant. The natural frequencies are estimated for first two modes of vibration. The value of Poisson's ratio is considered as 0.33. With the help of tables and graphs all the results have been displayed.

Table1 includes the values of frequency parameter λ for a non-homogeneous trapezoidal plate where taper constant β_1 varies from 0.0 to 1.0, taper constant $\beta_2=0.6$, thermal gradient $\alpha=0.0,0.4$, non-homogeneity constant $\beta=0.4,1.0$ and aspect ratios $a/b=1.0, c/b=0.5$. It is evident from the table that as taper constant β_1 increases the values of frequency parameter also increases for both the modes of vibration. In addition when the value of non-homogeneity constant β increases the frequency parameter decreases.

Table 2 contains the values of frequency parameter λ in which taper constant β_2 varies from 0.0 to 1.0, taper constant $\beta_1=0.6$, thermal gradient $\alpha=0.0,0.4$, non-homogeneity constant $\beta=0.4,1.0$ and aspect ratios $a/b=1.0$, $c/b=0.5$. It is clear from the table that as taper constant β_2 increases the values of frequency parameter also increases for both the modes of vibration. Moreover when the value of non-homogeneity constant β increases the frequency parameter decreases.

Table 3 depicts the values of frequency parameter λ for different values of thermal gradient α from0.0 to 1.0, taper constants $\beta_1=0.0,0.2$ & $\beta_2=0.0,0.6$, non-homogeneity constant $\beta=0.4,1.0$ and aspect ratios

Table 1. Values of frequency parameter (λ) for different values of taper constant (β_1) and constant aspect ratios (a/b = 1.0, c/b = 0.5).

β_1	$\beta=0.4,\beta_2=0.6$				$\beta=1.0,\beta_2=0.6$			
	$\alpha=0.0$		$\alpha=0.4$		$\alpha=0.0$		$\alpha=0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	35.8473	186.784	34.1974	174.059	31.2208	160.827	29.7859	149.860
0.2	36.2405	191.572	34.4581	177.844	31.4975	164.217	29.9506	152.438
0.4	36.7986	197.234	34.8498	182.389	31.9218	168.423	30.2334	155.736
0.6	37.5366	203.722	35.3829	187.645	32.5055	173.384	30.6424	159.691
0.8	38.4626	210.980	36.0631	193.562	33.2545	179.034	31.1815	164.246
1.0	39.5780	218.950	36.8920	200.088	34.1693	185.312	31.8514	169.342

Table 2. Values of frequency parameter (λ) for different values of taper constant (β_2) and constant aspect ratios (a/b = 1.0, c/b = 0.5).

β_2	$\beta = 0.4, \beta_1 = 0.6$				$\beta = 1.0, \beta_1 = 0.6$			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	32.7939	188.474	30.9171	173.561	28.4001	160.136	26.7765	147.455
0.2	34.0657	189.852	32.1129	174.859	29.4996	161.419	27.8104	148.661
0.4	35.6479	194.902	33.6028	179.522	30.8691	165.806	29.0999	152.712
0.6	37.5366	203.722	35.3829	187.645	32.5055	173.384	30.6424	159.691
0.8	39.7106	216.110	37.4334	199.048	34.3906	183.988	32.4205	169.452
1.0	42.1373	231.683	39.7236	213.381	36.4958	197.295	34.4074	181.699

Table 3. Values of frequency parameter (λ) for different values of thermal gradient (α) and constant aspect ratios (a/b = 1.0, c/b = 0.5).

α	$\beta = 0.4$				$\beta = 1.0$			
	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = 0.2, \beta_2 = 0.6$		$\beta_1 = \beta_2 = 0.0$		$\beta_1 = 0.2, \beta_2 = 0.6$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	31.3526	173.173	36.2405	191.572	27.3064	148.899	31.4975	164.217
0.2	30.6376	167.329	35.3613	184.836	26.6845	143.869	30.7345	158.437
0.4	29.9036	161.273	34.4581	177.844	26.0462	138.657	29.9506	152.438
0.6	29.1488	154.982	33.5283	170.567	25.3899	133.242	29.1438	146.194
0.8	28.3709	148.424	32.5691	162.966	24.7135	127.598	28.3115	139.671
1.0	27.5668	141.564	31.5767	154.994	24.0146	121.693	27.4508	132.829

$a/b = 1.0$, $c/b = 0.5$. It is obvious from the table that as thermal gradient α increases, the values of frequency parameter decreases for both the modes of vibration. It is also noted that on increasing the value of non-homogeneity constant β, the frequency parameter decreases.

Table 4 and **Table 5** consist the values of frequency parameter λ for a trapezoidal plate for different combinations of thermal gradient α and taper constants β_1 & β_2 as

a. $\beta_1 = \beta_2 = 0.0$, $\alpha = 0.0$.
b. $\beta_1 = \beta_2 = 0.0$, $\alpha = 0.4$.
c. $\beta_1 = \beta_2 = 0.6$, $\alpha = 0.0$.
d. $\beta_1 = \beta_2 = 0.6$, $\alpha = 0.4$.

The value of non-homogeneity constant $\beta = 0.4$, the values of aspect ratio a/b are 0.75 and 1.0 and the values of aspect ratio c/b are (0.25, 0.50, 0.75, 1.0).

It is obvious from the above discussed **Table 4** and **Table 5** that the values of frequency parameter λ decrease by increasing the aspect ratio c/b for both the modes of vibration. Moreover as taper constant increases frequency parameter also increases. It has been observed from the comparison of **Table 4** and **Table 5** that as one increases the aspect ratio (a/b) from 0.75 to 1.0 the frequency parameter also increases.

Table 6 contains the values of frequency parameter λ for a non-homogeneous trapezoidal plate for which non-homogeneity constant β varies from 0.0 to 1.0, the values of taper constants $\beta_1 = 0.0, 0.2$ & $\beta_2 = 0.0, 0.6$, thermal gradient $\alpha = 0.0, 0.4$ and aspect ratios $a/b = 1.0$, $c/b = 0.5$. From this table one can observe that as non-homogeneity constant β increases, the frequency parameter decreases for both the modes of vibration.

First mode and second mode of vibrations are presented in **Figure 2(a)** and **Figure 2(b)** respectively.

Table 4. Values of frequency parameter (λ) for different combinations of thermal gradient (α), taper constants (β_1 & β_2) fixed value of non-homogeneity constant ($\beta = 0.4$) and aspect ratio ($a/b = 0.75$).

c/b	$\beta = 0.4$							
	$\beta_1 = \beta_2 = 0.0$, $\alpha = 0.0$		$\beta_1 = \beta_2 = 0.0$, $\alpha = 0.4$		$\beta_1 = \beta_2 = 0.6$, $\alpha = 0.0$		$\beta_1 = \beta_2 = 0.6$, $\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.25	35.5810	155.784	34.3084	147.199	41.5586	181.123	39.8231	169.602
0.50	27.7864	123.040	26.6608	115.999	32.7334	142.816	31.0989	133.005
0.75	22.4677	97.7182	21.4242	91.9546	27.0622	114.537	25.3862	105.948
1.0	19.1007	79.4839	18.0837	74.6004	23.9156	95.5363	22.0626	87.5099

Table 5. Values of frequency parameter (λ) for different combinations of thermal gradient (α), taper constants (β_1 & β_2) fixed value of non-homogeneity constant ($\beta = 0.4$) and aspect ratio ($a/b = 1.0$).

c/b	$\beta = 0.4$							
	$\beta_1 = \beta_2 = 0.0$, $\alpha = 0.0$		$\beta_1 = \beta_2 = 0.0$, $\alpha = 0.4$		$\beta_1 = \beta_2 = 0.6$, $\alpha = 0.0$		$\beta_1 = \beta_2 = 0.6$, $\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.25	39.8350	214.445	38.1884	200.589	47.2984	252.697	44.9746	234.315
0.50	31.3526	173.173	29.9036	161.273	37.5366	203.722	35.3829	187.645
0.75	25.3738	137.923	24.0616	127.998	30.9373	162.881	28.8299	149.064
1.0	21.4087	109.929	20.1717	101.728	26.9381	131.323	24.7445	119.341

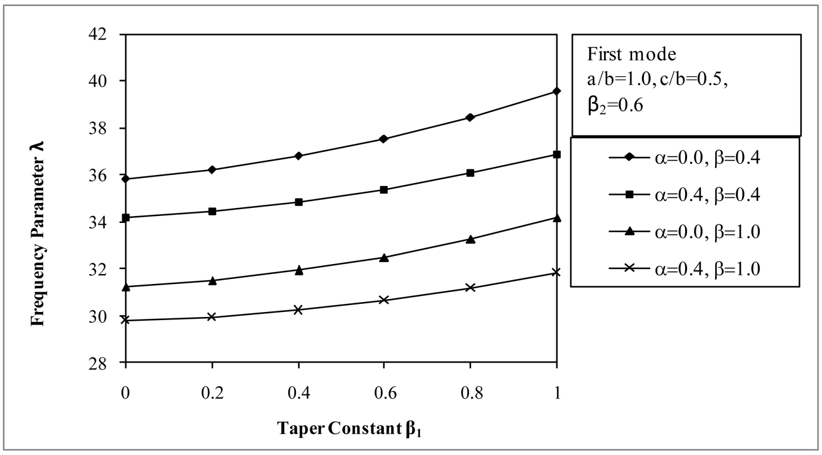
Table 6. Values of frequency parameter (λ) for different values of non-homogeneity constant (β) and constant aspect ratios ($a/b = 1.0$, $c/b = 0.5$).

β	$\beta_1 = \beta_2 = 0.0$				$\beta_1 = 0.2, \beta_2 = 0.6$			
	$\alpha = 0.0$		$\alpha = 0.4$		$\alpha = 0.0$		$\alpha = 0.4$	
	First mode	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**	**First mode**	**Second mode**
0.0	35.3229	198.154	33.6878	184.552	40.9252	220.053	38.9092	204.301
0.2	33.1609	184.388	31.6273	171.724	38.3702	204.313	36.4817	189.679
0.4	31.3526	173.173	29.9036	161.273	36.2405	191.572	34.4581	177.844
0.6	29.8110	163.801	28.4341	152.541	34.4299	180.977	32.7375	168.004
0.8	28.4764	155.814	27.1618	145.099	32.8660	171.981	31.2513	159.649
1.0	27.3064	148.899	26.0462	138.657	31.4975	164.217	29.9506	152.438

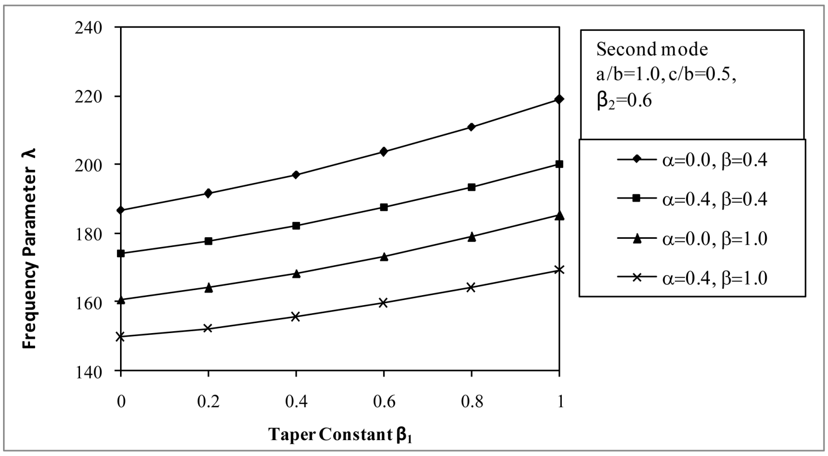
Figure 2 depicts the behaviour of frequency parameter λ with taper constant β_1. For first two modes of vibration the values of various plate parameters are taken as follows:

- $\beta_1 = 0.0$ to 1.0.
- $\beta_2 = 0.6$.
- $\alpha = 0.0, 0.4$.
- $\beta = 0.4, 1.0$.
- $a/b = 1.0, c/b = 0.5$.

It is evident from the **Figure 2** that frequency for both the modes of vibration increases as taper constant β_1 increases.



(a)



(b)

Figure 2. (a) & (b) Frequency parameter λ vs. taper constant β_1.

Figure 3 represents the variation of frequency parameter λ with taper constant β_2. For first two modes of vibration the values of various plate parameters are taken as follows:

- $\beta_2 = 0.0$ to 1.0.
- $\beta_1 = 0.6$.
- $\alpha = 0.0, 0.4$.
- $\beta = 0.4, 1.0$.
- $a/b = 1.0, c/b = 0.5$.

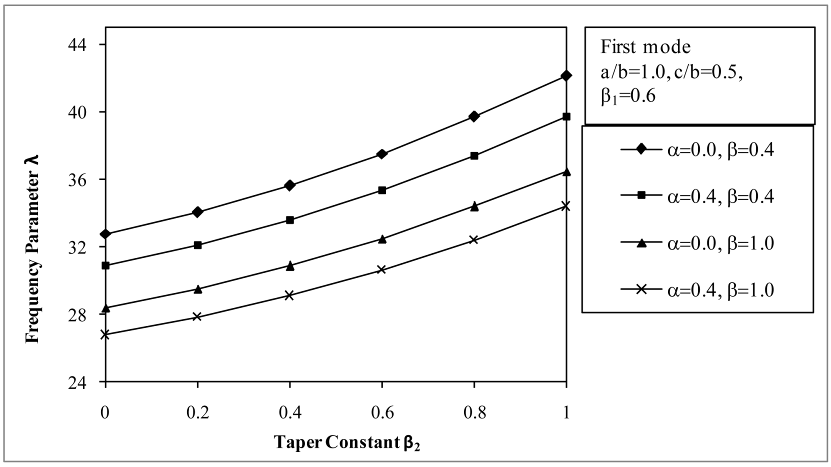
This figure explicates that as taper constant β_2 increases, the frequency parameter also increases for both the modes of vibration.

Figure 4 shows the variation of frequency parameter λ with thermal gradient α. For first two modes of vibration the values of various plate parameters are taken as follows:

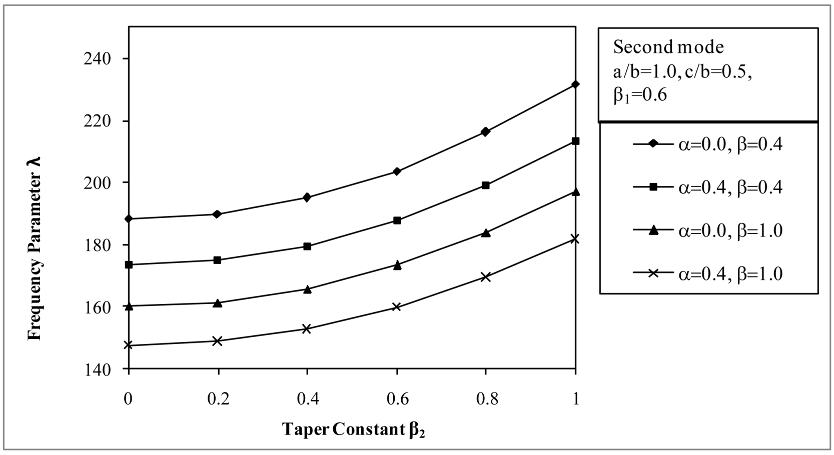
- $\alpha = 0.0$ to 1.0.
- $\beta_1 = \beta_2 = 0.0$.
- $\beta_1 = 0.2, \beta_2 = 0.6$.
- $\beta = 0.4, 1.0$.
- $a/b = 1.0, c/b = 0.5$.

From the discussed **Figure 4** the behaviour of the frequency parameter can be examined. It is found that as thermal gradient increases the values of frequency parameter decreases for both the modes of vibration.

Figure 5 displays the effect of aspect ratio *c*/*b* varies from 0.25 to 1.0, on the frequency parameter λ for different combinations of taper constants β_1 & β_2 and thermal gradient α as follows:



(a)



(b)

Figure 3. (a) & (b) Frequency parameter λ vs. taper constant β_2.

- $\beta_1 = \beta_2 = 0.0$, $\alpha = 0.0$.
- $\beta_1 = \beta_2 = 0.0$, $\alpha = 0.4$.
- $\beta_1 = \beta_2 = 0.6$, $\alpha = 0.0$.
- $\beta_1 = \beta_2 = 0.6$, $\alpha = 0.4$.

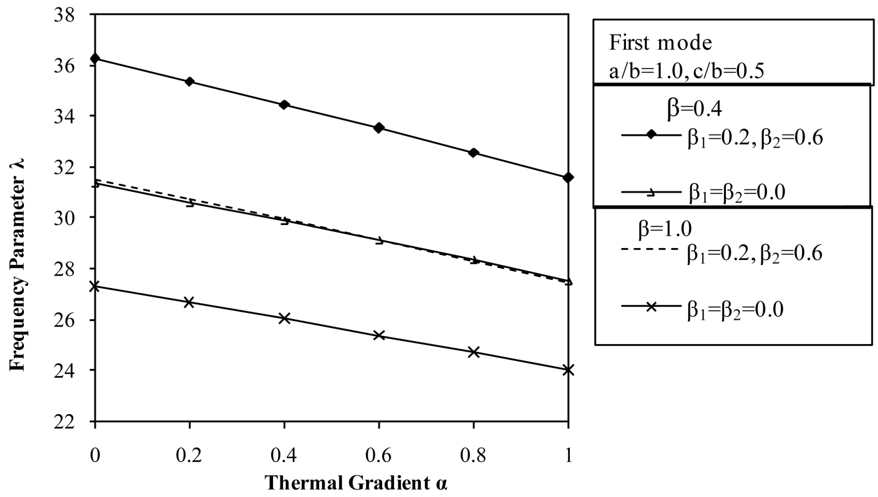
In this case the value of non-homogeneity constant $\beta = 0.4$ and two values of aspect ratio a/b = 0.75, 1.0 have been considered.

Now it can be easily observed from **Figure 5** that as aspect ratio c/b increases the frequency parameter decreases for both the modes of vibration. It is also noticed that frequency parameter also increases as taper constant increases. Furthermore when the value of aspect ratio a/b is increased from 0.75 to 1.0 then frequency parameter increases for both the modes of vibration.

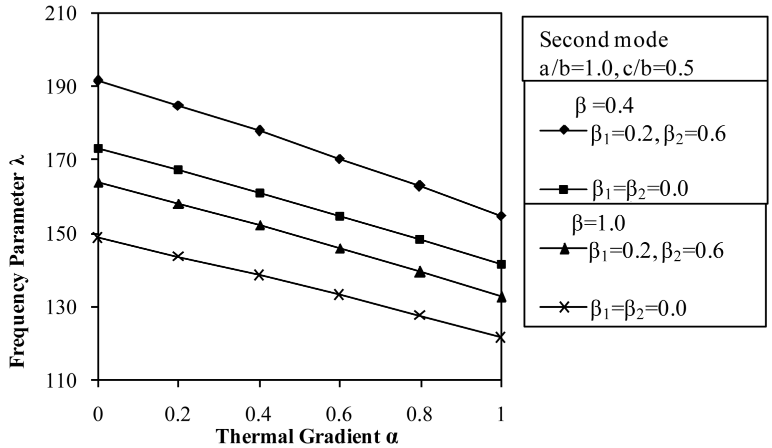
Figure 6 demonstrates the effect of non-homogeneity constant β which varies from 0.0 to 1.0, on the frequency parameter λ for different combinations of taper constants β_1 & β_2 and thermal gradient α as follows:

- $\beta_1 = \beta_2 = 0.0$, $\alpha = 0.0$.
- $\beta_1 = \beta_2 = 0.0$, $\alpha = 0.4$.
- $\beta_1 = 0.2, \beta_2 = 0.6$, $\alpha = 0.0$.
- $\beta_1 = 0.2, \beta_2 = 0.6$, $\alpha = 0.4$.

The behaviour of the frequency parameter is noticed and found that as non-homogeneity constant β increases the frequency parameter decreases for both the modes of vibration. In addition frequency parameter in-

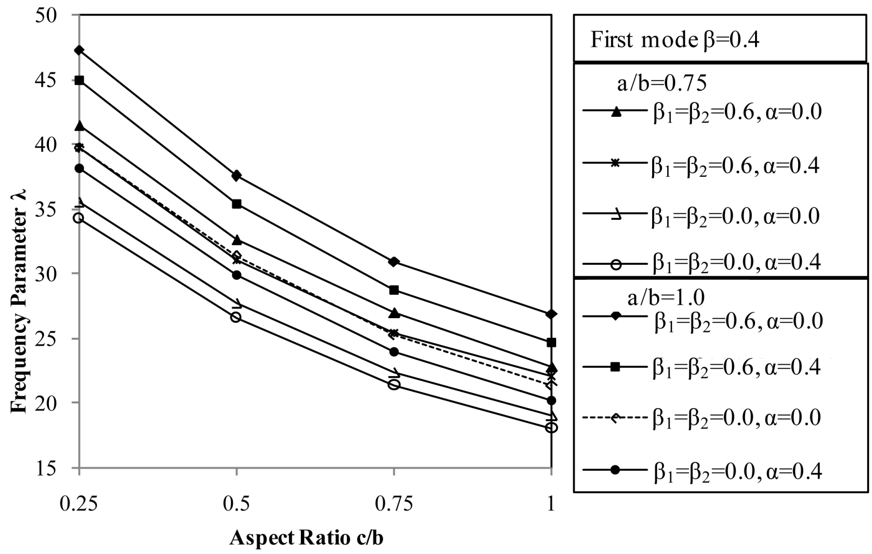


(a)

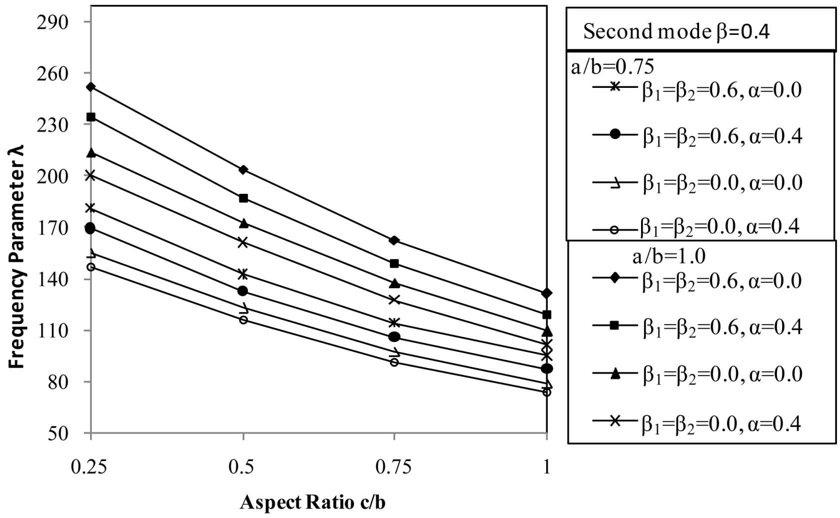


(b)

Figure 4. (a) & (b) Frequency parameter λ vs. thermal gradient α.

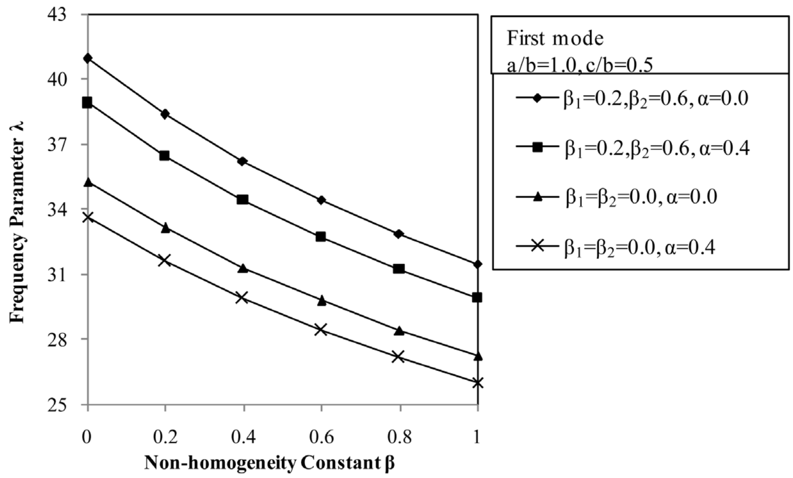


(a)

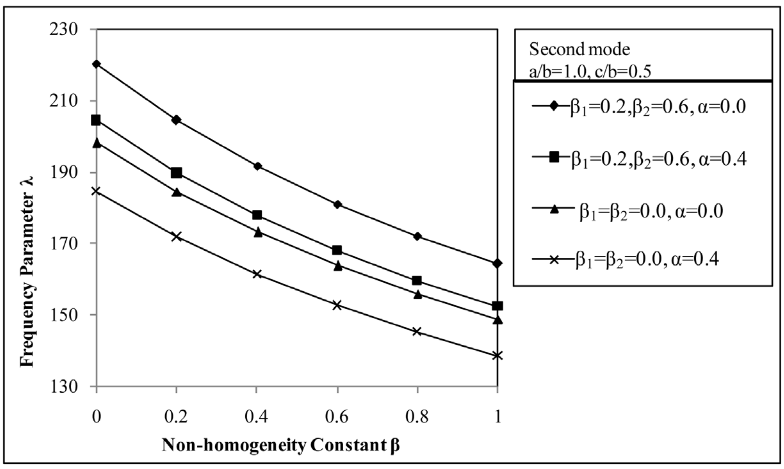


(b)

Figure 5. (a) & (b) Frequency parameter λ vs. aspect ratio c/b.



(a)



(b)

Figure 6. (a) & (b) Frequency parameter λ vs. non-homogeneity constant β.

creases for both the modes of vibration as taper constant increases.

6. Conclusion

Study of vibration of plates is an important area owing to its extensive range of engineering applications such as in aeronautical, civil and mechanical engineering. Rayleigh-Ritz method gives a perfect and computationally proficient scheme for finding the vibration characteristics of transverse vibration of trapezoidal plate. Thus the natural frequencies for a symmetric, non-homogeneous trapezoidal plate have been acquired by varying values of taper constants, thermal gradient, aspect ratio and non-homogeneity constant. Tables and graphs state that the frequency increases with the increase of taper constants and decreases with the increase of thermal gradient, aspect ratio and non-homogeneity constant. A design engineer can directly observe the presented plots of figures to have the knowledge about particular mode to finalize the design of the structure. The material should be selected such that the total cost should be minimum and within definite confines.

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