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Manpower Systems Operating under Heavy and Light Tailed Inter-Exit Time Distributions

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ABSTRACT

This paper considers a Manpower system where "exits" of employed personnel produce some wastage or loss. This system monitors these wastages over the sequence of exit epochs $\{t_0 = 0$ and t_k; $k = 1, 2,\cdots\}$ that form a recurrent process and admit recruitment when the cumulative loss of man hours crosses a threshold level Y, which is also called the breakdown level. It is assumed that the inter-exit times $T_k = t_{k-1} - t_k$, $k = 1, 2,\cdots$ are independent and identically distributed random variables with a common cumulative distribution function (CDF) $B(t) = P(T_k < t)$ which has a tail $1 - B(t)$ behaving like $t^{-\nu}$ with $1 < \nu < 2$ as $t \to \infty$. The amounts $\{X_k\}$ of wastages incurred during these inter-exit times $\{T_k\}$ are independent and identically distributed random variables with CDF $P(X_k < X) = G(x)$ and Y is distributed, independently of $\{X_k\}$ and $\{t_k\}$, as an exponentiated exponential law with CDF $H(y) = P(Y < y) = (1 - e^{-\lambda y})^n$. The mean waiting time to break down of the system has been obtained assuming $B(t)$ to be heavy tailed and as well as light tailed. For the exponential case of $G(x)$, a comparative study has also been made between heavy tailed mean waiting time to break down and light tailed mean waiting time to break down values. The recruitment policy operating under the heavy tailed case is shown to be more economical in all types of manpower systems.

KEYWORDS

Manpower System; Recruitment Policy; Inter-Exit Time; Wastage; Waiting Time to Breakdown; Heavy Tailed Inter-Exit Time Distribution and Light Tailed Distribution

1. Introduction

According to [1-3], the cumulative losses occurring over a period of time in any human resource organization can be minimized through a "Recruitment Policy". This paper considers a manpower system which monitors exit times of personnel who are the employees of the organization, the wastage incurred over a period of time and the cost of recruiting new people.

In [4], shock models pertaining to cumulative wastage, threshold level and their link with the breakdown status of devices have been surveyed. For more details on recruitment policies employed by various manpower organizations, the readers refer to [5-8].

This paper is organized as follows: Section 2 describes various features of a manpower system. Section 3 deals with survival aspects of the manpower system under study. Section 4 provides numerical illustrations which support the main results obtained in Section 3. Section 5 is devoted to providing some remarks and the future scope.

2. Description of the Manpower System

Assume that the organization under study monitors the sequence $\{t_0 = 0; t_1, t_2, \cdots\}$ of exit times (called renewal epochs or decision epochs) over a period of time $[0,t)$ where t is some positive and finite real value subject to the following features:

- The time durations $T_k = t_{k-1} - t_k \left(k = 1,2,\cdots\right)$ of inter-exit/inter-renewal epochs are independent and identically distributed (*iid*) random variables with a common cumulative distribution function (CDF) $B(t) = P[T_k < t]$ and probability density function (pdf) $b(t)\mathrm{d}t = \mathrm{d}B(t)$.
- Let the amount of wastage observed during T_k, be denoted by X_k such that the process $\{X_k; k = 1,2,\cdots\}$ forms a sequence of *iid* random variables with CDF $G(x) = P[X_k < X]$ and pdf $g(x)\mathrm{d}x = \mathrm{d}G(x)$.
- At each exit epoch t_k, a decision is also made based on the cumulative damage or wastage incurred as compared with a threshold level, say *Y*, to the occurrence of break down state of the manpower system. Independent of $\{X_k : k = 1,2,\cdots\}$ and $\{T_k : k = 1,2,\cdots\}$ sequences and other features, Y is assumed to vary continuously with CDF $H(x) = P[Y < x]$ and pdf $h(x)\mathrm{d}x = \mathrm{d}H(x)$ while the tail is $\overline{H}(y) = 1 - H(x)$
- The event of Breakdown state of the manpower system is assumed to occur at an exit epoch, say τ_h, provided the observed cumulative damage $D = X_1 + X_2 + \cdots + X_{\tau h}$ exceeds the random threshold level *Y*. It means that a few personnel must be recruited at this epoch τ_h to avoid the immediate breakdown status.

3. Distribution of Waiting Time till Breakdown

For any CDF $F_0(.)$ or the probability density function (PDF) $f(.) = \mathrm{d}F(.)$ of a non-negative random variable (rv) say X, use the notation $F_k(.)$ for the k^{th} convolution of F with itself and $F_0(.) = 1$, for the tail of $F_0(.)$ and the tail of the distribution of sum of k independent random variables by $\overline{F}_k(.) = 1 - F_k(.)$, $\mathrm{d}F_k(.) = f_k(.)$, the k-fold convolutions and

$$f^*(s) = \text{Laplace transform}(LT) \text{ of } f(.) = \int_0^\infty f(y)\mathrm{e}^{-sy}\mathrm{d}y = \int_0^\infty \mathrm{e}^{-sy}\mathrm{d}F(y) \text{ for } \operatorname{Re} s > 0 \tag{1}$$

where $\operatorname{Re} s$ means the real part of *s*.

Gupta and Kundu have discussed various applications of generalized exponential distributions inclusive of an "Exponentiated Exponential Distribution (EED)" [9,10]. Here, the random threshold level *Y* is assumed to follow the EED with CDF $H(y)$:

$$H(y) = P\{Y < y] = \left(1 - \mathrm{e}^{-\lambda y}\right)^n \tag{2}$$

$$\overline{H}(y) = 1 - \left(1 - \mathrm{e}^{-\lambda y}\right)^n = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \mathrm{e}^{-j\lambda y} \tag{3}$$

Let "τ" denote the waiting time to breakdown state of the system under study and let continuous function $W(t) = P(\tau < t)$ be the CDF of $\tau > 0$. Then, the distribution tail of τ is

$$\begin{aligned}
\overline{W}(t) &= P[\text{manpower system survives beyond } t] = P(\tau > t) \\
&= \sum_{k=0}^{\infty} P\big[\text{exactly } k \text{ exits in } [0,t)\big] P\left[\sum_{j=1}^{k} X_j < Y/k \text{ exits}\right] \\
&= 1 - B(t) + \sum_{k=1}^{\infty} \left[B_k(t) - B_{k+1}(t)\right] \left[\int_0^\infty g_k(x) \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \mathrm{e}^{-j\lambda x}\mathrm{d}x\right] \\
&= 1 - B(t) + \sum_{k=1}^{\infty} \left[B_k(t) - B_{k+1}(t)\right] \left[\sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left\{g^*(j\lambda)\right\}^k\right] \\
&= 1 - \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 - g^*(j\lambda)\right) \sum_{k=1}^{\infty} B_k(t) \left\{g^*(j\lambda)\right\}^{k-1} \\
&= \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 - g^*(j\lambda)\right) \sum_{k=1}^{\infty} \overline{B}_k(t) \left\{g^*(j\lambda)\right\}^{k-1}
\end{aligned} \tag{4}$$

Where $g^*(j\lambda) = \int_0^\infty g(y)\mathrm{e}^{-j\lambda y}\mathrm{d}y$.

3.1. Heavy Tailed Distribution B(t)

Following is some useful notation and terminology on "Heavy" tailed distributions that are usually characterized by extremely high variability as against with light tailed distributions. The term "heavy tailed distribution of a non-negative rv 'X'" refers to absence of all moments after the r^{th} moment of X for some $r \ge k > 0$.

Definition: The CDF $F(x)$ of X on $[0,\infty)$ is called heavy tailed if and only if (iff)

$$\int_0^\infty x^r \mathrm{d}F(x) = \int_0^\infty x^r f(x)\mathrm{d}x = \infty \text{ for all } r \ge k > 0$$

And the same $F(x)$ is called light tailed iff

$$\int_0^\infty x^k \mathrm{d}F(x) = \int_0^\infty x^k f(x)\mathrm{d}x < \infty \text{ for all } k > 0$$

For any two real valued and positive functions $f(x)$ and $f_1(x)$ defined on $[0,\infty)$, use the notational convention $f(x) \sim f_1(x)$ to denote $\lim_{x\to\infty} \frac{f(x)}{f_1(x)} = 1$.

Definition: If there exists some $v \in (-\infty,\infty)$ such that $\lim_{x\to\infty} \frac{f(tx)}{f(x)} = t^v$ for all $t \in (0,\infty)$ then $f(x)$ is said to be regularly varying at infinity with index $v \ne 0$; if $v = 0$, $f(x)$ is said to be slowly varying at infinity. If $f(x)$ is regularly varying at infinity with index $v \ne 0$ then there exists a slowly varying function $L(x)$ such that

$$f(X) = X^v \cdot L(X), X \to \infty$$

Thus the properties of regularly varying functions $f(x)$ can be deduced from those slowly varying functions $L(x)$ which will not be unique. One of the wider classes of heavy tailed distributions is called subexponential family of distributions, *denoted by the symbol* "$\mathbb{S}$".

Definition: A CDF $F(x)$ on $[0,\infty)$ is called regularly varying with index *v* if

$$\bar{F}(x) = 1 - F(x) = X^{-v} L(x), v > 0, x \to \infty$$

Where $L(x)$ is a function of slow variation. All moments of order $k < v$ are finite and all moments of order $k > v$ are infinite; further if $\bar{F}(x) > 0$ for all x and if $\bar{F}(x) \in S$ then 1)

$\lim_{x\to\infty} \frac{\bar{F}(x+y)}{\bar{F}(x)} = 1 \Rightarrow \bar{F}(x+y) \sim \bar{F}(x)$ as $x \to \infty$ uniformly over compact y-sets and 2)

$$\lim_{x\to\infty} \frac{\bar{F}_k(x)}{\bar{F}(x)} = k \Rightarrow \bar{F}_k(x) \sim k\,\bar{F}(x) \text{ as } x \to \infty \tag{5}$$

3.2. Heavy Tailed Mean of Waiting Time to Breakdown Distribution

This section is devoted to derive an asymptotic expression to (4) of the rv τ = waiting time to breakdown for which the inter-exit time *T* follows a heavy-tailed distribution with index $v\,(1 < v < 2)$ *i.e.*

$$1 - B(t) = P(T > t) \sim c\,(t/\theta)^{-v} \text{ for } t \to \infty \tag{6}$$

Where *c* and *θ* are positive constants. Obviously $E(T)$ is finite and $E(T^2) = \infty$ if $1 < v < 2$. The readers may refer to [11-13] and the references found there in for finding asymptotic waiting time distributions where the heavy tailed distribution (6) has been considered as the service time distribution.

It is observed that (6) is satisfied by the non-negative rv "T_θ" with the Pareto distribution: for $t > 0$,

$$P(T_\theta < t) = B(t,\theta) \cdot \left(1 - \delta\left(\frac{\theta}{\theta + t}\right)^v\right) \tag{7}$$

where *δ* is real with $0 < \delta \le 1$ and *θ* is a rv with Gamma PDF *m*(*θ*):

$$m(\theta) = \begin{cases} \dfrac{\beta^{2-\nu}}{\Gamma(2-\nu)} \theta^{1-\nu} e^{-\beta\theta} & \theta > 0,\ 1 < \nu < 2, \\ \Gamma(.) \text{ being the Gamma function} \end{cases} \tag{8}$$

Using (7) and (8), it can be shown that $B(t) = P(T < t) = \int_0^\infty m(\theta) B(t,\theta) \mathrm{d}\theta, \theta > 0, 1 < \nu < 2$ is simplified to

$$B(t) = 1 - \frac{\beta^{2-\nu}}{\Gamma 2-\nu} \int_0^\infty \left(\frac{\theta}{(\theta+t)^\nu} \right) \mathrm{d}\theta, 1 < \nu < 2 \tag{9a}$$

$$E(T) = \frac{2-\nu}{\nu-1} \frac{\delta}{\beta} \text{ and } B(0+) = 1-\delta \tag{9b}$$

Using (5), (6) and (12) in (4), it is proved that $W(t)$ follows a heavy tailed distribution since

$$\begin{aligned} \overline{W}(t) &= \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 - g^*(j\lambda)\right) \sum_{k=1}^{\infty} \overline{B}_k(t) \left\{ g^*(j\lambda) \right\}^{k-1} \\ &= \overline{B}(t) \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 - g^*(j\lambda)\right) \sum_{k=1}^{\infty} k \left\{ g^*(j\lambda) \right\}^{k-1} \\ &= \overline{B}(t) \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \frac{1}{1 - g^*(j\lambda)} \sim \text{constant } (t/\theta)^{-\nu} \text{ for } t \to \infty \end{aligned} \tag{10}$$

From (10), using the facts $W(0) = 1$, $E(\tau) = \int_0^\infty \overline{W}(t) \mathrm{d}t$ and (9), it can be shown that

$$E(\tau) = \frac{2-\nu}{\nu-1} \frac{\delta}{\beta} \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \frac{1}{1 - g^*(j\lambda)} \tag{11}$$

$$\text{where } \delta = 1 - \frac{1}{\sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \dfrac{1}{1 - g^*(j\lambda)}}$$

Let the common random amount of damage X measured during an inter-exit time be exponentially distributed with PDF $g(t) = \mu \cdot \mathrm{e}^{-\mu t}$ for the rest of the discussions. The LT of $g(t)$ is then $g^*(s) = \mu/(\mu+s)$. Thus it is simplified from (11) that the mean of τ is as derived below:

$$E(\tau) = \frac{2-\nu}{\nu-1} \left(\frac{\delta}{\beta} \right) \left\{ 1 + \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(\frac{\mu}{j\lambda} \right) \right\} \text{ since } \frac{1}{1 - g^*(j\lambda)} = \frac{1}{1 - \dfrac{\mu}{\mu + j\lambda}} = \left(1 + \frac{\mu}{j\lambda} \right) > 1 \tag{12}$$

and

$$\delta = 1 - \frac{1}{\sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 + \dfrac{\mu}{j\lambda} \right)} = \frac{\sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(\dfrac{\mu}{j\lambda} \right)}{1 + \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(\dfrac{\mu}{j\lambda} \right)} < 1$$

When $\nu = 3/2$, it is interesting to observe from (12) that

$$E(\tau) = \left(\frac{1}{\beta} \right) \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(\frac{\mu}{j\lambda} \right) \tag{13}$$

3.3. Light Tailed Exponential Distribution to Inter-Exit Times

For the case of exponential inter-exit time T with PDF $b(t) = \alpha \mathrm{e}^{-\alpha t}$ or $b^*(s) = \alpha/(\alpha+s)$, together with

$g^*(s) = \mu/(\mu + s)$, the PDF $w(t)\mathrm{d}t = \mathrm{d}W(t)$ of τ from (4) is derived as below:

$$w(t) = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 - g^*(j\lambda)\right) \sum_{k=1}^{\infty} b_k(t) \left\{g^*(j\lambda)\right\}^{k-1} = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} A(j,\mu,\alpha,\lambda) \mathrm{e}^{-tA(j,\mu,\alpha,\lambda)} \quad (14)$$

where

$$= A(j,\mu,\alpha,\lambda) = \alpha\left\{1 - g^*(j\lambda)\right\} = \alpha\left(\frac{j\lambda}{j\lambda + \mu}\right).$$

It is remarked that the expression obtained in (14) to the PDF of the rv, τ = waiting time to breakdown of the manpower system is a linear and convex combination of "*n*" non-identically and exponentially distributed random variables and $w^*(s) = LT$ of $w(t)$ is given by

$$w^*(s) = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(1 - g^*(j\lambda)\right) \sum_{k=1}^{\infty} \left(b^*(s)\right)^k \left\{g^*(j\lambda)\right\}^{k-1} = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \frac{\alpha\left\{1 - g^*(j\lambda)\right\}}{s + \alpha\left\{1 - g^*(j\lambda)\right\}} \quad (15)$$

and this result (15) agrees with that of [5]. Thus the expected value $E(\tau)$ is obtained from (14) or (15) as below:

$$E(\tau) = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \frac{1}{\alpha\left(1 - g^*(j\lambda)\right)} = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \frac{1}{A(j,\mu,\alpha,\lambda)}$$

$$= \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(\frac{1 + \frac{\mu}{j\lambda}}{\alpha}\right) = \frac{1}{\alpha}\left(1 + \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \frac{\mu}{j\lambda}\right)$$

Let $a(n) = \sum_{j=1}^{n} (-1)^{j+1} \binom{n}{j} \left(\frac{\mu}{j\lambda}\right)$. Then comparing the result (16) on $E(\tau)$ derived from the light tailed exponential distribution $B(t) = 1 - \mathrm{e}^{-\alpha t}$ assigned to the inter-exit times with mean = $\frac{1}{\alpha}$ with that of the result (12) on the same $E(\tau)$ derived from the heavy tailed inter-exit time distribution $B(t)$ satisfying the property

$$1 - B(t) = \frac{\beta^{2-\nu}\delta}{\Gamma(2-\nu)} \int_0^{\infty} \frac{\theta}{(\theta + t)^{\nu}} \mathrm{e}^{-\beta\theta} \mathrm{d}\theta, \; 1 < \nu < 2$$

It is concluded that

$E(\tau_{\text{heavy}}) = \frac{2-\nu}{\nu-1}\left(\frac{\delta}{\beta}\right)\left[1 + (a(n))\right]$ and $\delta = a(n)/\left[1 + (a(n))\right]$.

4. Better Recruitment Policy under Heavy and Light Tailed Cases

All these results relating to $E(\tau)$ obtained above have been considered for a numerical study with specific input values "ν = 1.2, β = 0.75, α =1.2, λ = 2.0, and μ = 0.8" allowing *n* to vary from 1 to 10 and the outcome of this exercise on the value of $E(\tau)$ computed for both light tailed and heavy tailed distributions of $B(t)$ has been reported in **Table 1**.

Inspection of the numerical values of the above **Table 1** reveals that each value of δ lies between 0 and 1 and increases with increase in n values from 1 to 10 as expected. Further the mean waiting time till breakdown is longer with heavy tailed inter-exit times than with that of the lighter tailed inter-exit time periods and the variations among the mean values of the heavy tailed environment is much higher as compared with that of a lighter tailed exponential distribution.

5. Remarks and Scope

One of the primary tasks of an efficient hiring process of a manpower system is to create a suitable recruitment policy. It specifies the objectives of recruitment and provides a framework for the implementation of a recruit-

Table 1. Input values for $\nu = 1.2$, $\beta = 0.75$, $\alpha = 1.2$, $\lambda = 2.0$, and $\mu = 0.8$.

n	$a = a(n)$	$\delta = a/(1 + a)$	$E(\tau_{light})$	$E(\tau_{heavy})$
1	0.4000	0.2857	1.1667	2.1333
2	0.6000	0.3750	1.3333	3.2000
3	0.7333	0.4231	1.4444	3.9111
4	0.8333	0.4545	1.5278	4.4444
5	0.9133	0.4774	1.5944	4.8711
6	0.9800	0.4949	1.6500	5.2267
7	1.0371	0.5091	1.6976	5.5314
8	1.0871	0.5209	1.7393	5.7981
9	1.1316	0.5309	1.7763	6.0351
10	1.1716	0.5395	1.8096	6.2483

ment programme by filling up vacancies with best qualified people.

This paper obtains few results relating to expected waiting time to break down *i.e.* $E(\tau)$ of a human resource management system or manpower system. To support the theoretical results obtained, a numerical study with specific input values "$\nu = 1.2$, $\beta = 0.75$, $\alpha = 1.2$, $\lambda = 2.0$, and $\mu = 0.8$" allowing n to vary from 1 to 10 has been carried out. Two values of $E(\tau)$ are computed: 1) for light tailed inter-exit time distribution $B(t)$ giving $E(\tau_{light})$ and 2) for a specific heavy tailed distribution $B(t)$ giving $E(\tau_{heavy})$. It is remarked that similar numerical investigation can also be carried out to other cases like the rv. Y follows exponentiated gamma law of [9,10] or the Weibull family of [14], or the inter-exit time follows any other light tailed and generalized exponential distribution and so on.

The numerical values of $E(\tau_{heavy})$ are found to be larger with larger variations than that of $E(\tau_{light})$ values. This fact ensures that the cost of recruitment with the case of heavy tailed inter-exit times would be smaller than the cost associated with a lighter case of the inter-exit times of employed personnel. Thus it is more advantageous in terms of recruitment costs if more people whose inter-exit time distribution follows a heavy tailed distribution are recruited in all types of manpower systems.

There is much scope to extend the analysis of this paper to the cases of heavy tailed distribution for the amount of wastages *i.e.* $G(x)$ or for the threshold level $H(y)$ or to both of $G(x)$ and $H(y)$.

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