Applied Mathematics, **2010, 1, 515-519**
10.4236/am.2010.16068 Published Online December 2010 (http://www.SciRP.org/journal/am)

A Mathematical Approach to Develop the Distribution of Greenhouse Gas Emissions*

Lev Sergeevich Maergoiz[1], Tatiana Yur'evna Sidorova[1], Rem Grigor'evich Khlebopros[1,2]
[1]*Siberian Federal University, Krasnoyarsk, Russia*
[2]*Krasnoyarsk Research Centre of Siberian Branch of Russian Academy of Sciences, Krasnoyarsk, Russia*
E-mail: {*bear.lion, tatiana-sidorova*}*@mail.ru, olikru@yandex.ru*
Received September 18, 2010; *revised October* 17, 2010; *accepted October* 21, 2010

Abstract

A mathematical algorithm of the distribution of greenhouse gas emissions is proposed as a way to tackle the topical issue of climate change and develop approaches to attaining an agreement among emitters of greenhouse gases (on the global scale, in a country, a region, a megalopolis).

Keywords: Mathematical Algorithm, Extremal Problems, Greenhouse Effect, the Principle of Differentiated Responsibilities

1. Introduction

Among the global issues that came to the fore in the 20th century is that human impact changes the Earths climate, leading to global warming. The general public and even some scientists still doubt the prevalence of human impact among factors influencing the Earths climate. An added complication is that if the prevalence of human impact on the global climate is recognized, urgent measures will have to be taken to control human impact, and this will cause a dramatic rise in the cost of living. Since the early 1990s numerous attempts have been made to overcome these difficulties at the international level, but none of them have been successful, mainly due to the lack of objective criteria for the solution of this problem. This study proposes a mathematical procedure for objectivizing these criteria.

2. Climate Change and Sustainable Development

There are two possible ways for humans to stabilize the surface temperature of the Earth: by regulating parameters of the greenhouse effect in the atmosphere and on the Earth. These parameters can be changed by varying not only atmospheric concentrations of greenhouse gases but also surface reflectivity by changing the amount of clouds at different heights. This idea was first proposed in the 20th century [2]. This approach to controlling the surface temperature of the Earth actually develops Vernadskys idea of the noosphere, as applied to issues of local and global climate control [3].

We can reduce our interference in natural processes by maintaining the contemporary state of the atmosphere. On the one hand, emissions of greenhouse gases (carbon dioxide) due to combustion of nonrenewable energy sources have to be considerably reduced. On the other hand, total energy production should be increased in order to maintain and improve the quality of life in developed countries and, what is even more important, to provide an opportunity for developing countries to attain a comparable standard of living. In order to reduce emissions of greenhouse gases due to combustion of carbon fossil fuel, both its percent in the energy budget and its actual amount should be decreased, by replacing it with renewable sources of carbon fuel, wind power, water power, and nuclear energy. It should be remembered, though, that the use of alternative energy sources will directly or indirectly increase the cost of power generation and, according to UNESCO estimates, must decrease the GDP by 1-2%. An important consideration is that the effect of this loss on developed and developing countries will be different: the use of alternative energy sources can delay the achievement of high life quality in developing countries for decades.

Let us discuss various ways to solve this problem. The first was proposed by Dirk Solte [4]. The simplest, most democratic, and equitable way to switch from the contemporary levels of emissions to the levels of emissions

*This article is written on the basis of the authors' preprint [1].

equal to the threshold Vr, at which the effect of humans on the global temperature becomes actually imperceptible, is to set a quota, $Vr : N$, where N is the global human population. The quota for each country is proportional to its population: $(Vr : N)n$, where n is the population of a given country. The difference between the actual level of emissions and the quota for a country may be either positive or negative. If this difference is positive, the country will have to buy quotas from the countries that have a negative difference (industrially undeveloped countries emit much smaller amounts of greenhouse gases than their quota allows). The proposed algorithm allows a nearly instant attainment of the maximal level of greenhouse gas emissions necessary for the stabilization of the global temperature, and the countries are divided into three categories: the countries that buy quotas (developed countries), the counties that gradually reduce the amount of the quotas they sell (developing countries), and those preferring to live off the environmental endowment (selling the same or increased amounts of the quotas). Although this way seems to be simple, democratic, and equitable, it is actually not simple, democratic, or equitable. The first and most significant drawback of this approach is that the expected effect is too instantaneous and, like any sudden revolution, can lead to numerous social and international catastrophes. The second drawback is that this algorithm does not take into account a nations history. Thirdly, no account is taken of the influence of geographic conditions: the quotas for the people living in high-latitude areas and for those living in the equatorial zone cannot be equal, as the former have to heat their homes and other buildings.

The second approach, whose implementation is being attempted now, is to get different countries, gradually and to a greater or lesser extent, to reduce their emissions. The legal basis for international control and reduction of the human impact causing the greenhouse effect is currently provided by the UN Framework Convention on Climate Change accepted in 1992 [5] and an addition to it, the Kyoto Protocol adopted in 1997 [6]. One of the basic principles of the Convention is that of differentiated responsibilities. This principle states that the global nature of climate change calls for the widest possible cooperation by all countries, specifically pointing out that their participation should be determined by their capabilities. Thus, highly developed countries are supposed to take more serious measures and spend much more money than less developed ones. However, international community has not reached an agreement on the amounts of emissions to be reduced as the subjective approach to determining them does not suit any country in the world. It is the main defect of this approach.

Thus, in our opinion, the most topical issue today is objectivization of the establishment of quotas.

3. A Mathematical Algorithm of Solving problem

The problem of the distribution of greenhouse gas emissions is solved using the algorithm having tested for distribution of monetary resource in problems of collective investment management [7, 8].

3.1. Problem statement

Assume N groups of greenhouse gas emitters (on the global scale, in a country, a region, a megalopolis) negotiate on a certain admissible quantity V of greenhouse gas emissions (in weight units) during a fixed time period. Concentrate on the problem of the distribution of this value among all groups of emitters taking into consideration the size of the population in every group. In mathematical terms this is sum partitioning of the value V:

$$V = \sum_{k=1}^{N} V_k, \tag{1}$$

where V_k is an admissible quantity of emissions for the group with number k. Let S_k be population of the same group, $k = 1, 2, \cdots, N$ and

$$S = \sum_{k=1}^{N} S_k$$

be population of all groups. Denote by $r = V/S$, $r_k = V_k/S_k$ the mean value (density) of emissions per capita of all population and for the group with number k, where $k = 1, \cdots, N$ respectively. By (1) it follows the relation

$$\sum_{i=1}^{N} r_k S_k = rS.$$

Introduce the dimensionless values $s_k = S_k/S$ (part of population in the group with number k), $\lambda_k = r_k/r$ (coefficient of proportionality), $k = 1, \cdots, N$. Then taking into account the previous equality we find

$$\sum_{k=1}^{N} \lambda_k s_k = 1, \quad \sum_{k=1}^{N} s_k = 1. \tag{2}$$

Suppose that emitters reach to the following agreement: conditional rating of every group is defined by the value of the corresponding coefficient of proportionality. Moreover, taking into consideration the principle of differentiated responsibilities for climate change climate groups differ from each other by the introduced rating, and group indexing is given in ascending order of this value, e.g.

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n < 1 < \lambda_{n+1} < \cdots < \lambda_N. \tag{3}$$

Here inequality $\lambda_n < 1$ implies that n groups for

$n < N$ agree that their value (density) of emissions per capita of population be less than the mean density *r*. Mathematical approach to the choice of coefficients is based on the following extremal problem.

Problem B. *It is necessary to find values of parameters* $\{\lambda_k\}_1^N$ *, such that the functional*

$$\Phi(\lambda) = \sum_{i=1}^{N-1} \left(\lambda_{i+1} - \lambda_i\right)^2, \quad (4)$$

attains the minimum provided that Equality (2) holds and the following additional linear relation between coefficients is fulfilled:

$$\sum_{i=1}^{N} d_i \lambda_i = b. \quad (5)$$

Here $d_1, d_2, \cdots, d_N, b$ are fixed numbers satisfying natural restrictions ensuring the condition: the Inequality (3) is true.

Relation (5) can be a result of agreement among emitters. For example, the equalities $\lambda_1 / \lambda_N = \gamma$, or $\lambda_N = \gamma$, where $\gamma \in (0,1)$ or, correspondingly, $\gamma \in (1, 1/s_N)$, are used in [7,8] for the distribution of the monetary resource in problems of collective investment. In geometrical terms the proposed optimal principle (see (4)) implies that desired vector

$$\alpha = \left(\alpha_1, \cdots, \alpha_{N-1}\right), \alpha_i = \lambda_{i+1} - \lambda_i, i = 1, \cdots, N-1$$

has the smallest length. Its coordinates are differences in emission densities per capita for groups with adjacent numbers. This approach to the choice of positive parameters $\alpha_i, i = 1, 2, \cdots, N-1$ is of great “psychological” significance. The smaller their values, the easier it is to come to the conclusion of the contract if emitters have agreed with the principle of division into groups, which is reflected in (3).

The obtained solution of the mathematical problem under consideration provides a way to define the admissible quantity of emissions for the group with number *k*. In previous notations the following formula is correct

$$V_k = \lambda_k r S_k = \lambda_k s_k V, k = 1, \cdots, N. \quad (6)$$

3.2. Another Variant of the Problem Statement

The conditional rating of every group can be determined based on another criterion, e.g. its living area. In this case the previous notations have the following meaning: S_k is the living area of the group with number *k*, where $k = 1, 2, \cdots, N$, $S = \sum_{k=1}^{N} S_k$ is the total living area; $r = V/S, r_k = V_k / S_k$ is, respectively, the mean quantity of emissions per area unit of the total living area and for the group with number *k*, where $k = 1, \cdots, N$; $s_k = S_k / S$ is the portion of the territory of this group; $\lambda_k = r_k / r$, $k = 1, \cdots, N$. Coefficients $\{\lambda_k\}_1^N$ are chosen realized by solving the same extremal problem.

3.3. A solution of Problem A

It is known (see [7]) the following solution of above mentioned problem provided that

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_N, \quad (7)$$

which differs from Inequality (3) by the absence of indication to a fixed number of parameters smaller than 1. The problem whose statement has been changed in this way well be called by Problem A.

A solution of Problem A. *In the notations of* (2), (5) *denote by*

$$D = \sum_{i=1}^{N} d_i, D_j = \sum_{i=j+1}^{N} d_i, P_j = \sum_{i=j+1}^{N} s_i, j = 1, \cdots, N-1, \quad (8)$$

and also

$$D \neq 0, b, A_j := \frac{P_j D - D_j}{D - b} > 0, j = 1, \cdots, N-1,$$

$$\sum_{j=1}^{N-1} A_j P_j < \sum_{j=1}^{N-1} A_j^2. \quad (9)$$

Then the functional $\Phi(\lambda)$ (*see* (4)) *attains the minimum at the values of parameters* $\{\lambda_k\}_1^N$ *in the form*

$$\lambda_1 = 1 - \sum_{j=1}^{N-1} \frac{A_j P_j}{\|A\|^2}, \lambda_k = 1 + \sum_{j=1}^{k-1} \frac{A_j\left(1 - P_j\right)}{\|A\|^2} - \sum_{j=k}^{N-1} \frac{A_j P_j}{\|A\|^2}, \quad (10)$$

where

$$\|A\|^2 = \sum_{j=1}^{N-1} A_j^2, k = 2, \cdots, N,$$

moreover for $k = N$ *the negative term in* (10) *vanishes.*

Hence, by (3), (7) we also conclude that the solution of Problem B (in comparison with Problem A) must satisfy the additional inequality $\lambda_n < 1 < \lambda_{n+1}$ (see also (10)).

A solution of Problem A is a solution of Problem B if and only if the additional condition is fulfilled for parameters in (8)-(10)

$$\sum_{j=1}^{n-1} A_j\left(1 - P_j\right) < \sum_{j=n}^{N-1} A_j P_j, n > 1; \sum_{j=1}^{n} A_j\left(1 - P_j\right) > \sum_{j=n+1}^{N-1} A_j P_j,$$
$$n < N-1. \quad (11)$$

Remarks. 1. Problem B is meaningful for $N > 2$.

2. By (7) and (2) it follows that $\lambda_1 < 1, \lambda_N > 1$ (see also (10)).

Let us prove that there is a collection of parameters $d_1, d_2, \cdots, d_N, b$ in the linear relation (5) that satisfies conditions (9), (11), e.g. problem B has a nonempty set of solutions. This assertion is reduced to determination of a vector $A=(A_1,\cdots,A_{N-1})$ with positive coordinates satisfying inequalities in Formulas (9), (11). Indeed, then assuming $b=0, D_0:=D=1, D_N:=0$, by (8)-(9), we determine desired parameters $\{d_j\}_1^N$ in (5) using formulas

$$D_j = P_j - A_j,\ j=1,\cdots,N-1; d_j = D_{j-1}-D_j,$$
$$j=1,\cdots,N. \tag{12}$$

1. Let $n=1$ in Inequality (3). Taking into account Remarks for the solution of Problem B, it is sufficient to find relation (5) which ensures realization of inequality $\lambda_2>1$. Therefore (see (10)), Inequality (11) is transformed into

$$A_1(1-P_1)=A_1s_1>\sum_{j=2}^{N-1}A_jP_j.$$

In particular, this inequality is correct for

$$A_1=2t/s_1,A_j=t/P_j,\ j=2,\cdots,N-1,\forall t>0.$$

Then assuming

$$B=2P_1/s_1+1, C=4/s_1^2+\sum_{j=2}^{N-1}1/P_j^2,$$

in the notations of (10) we have with these values of coordinates of vector $A=(A_1,\cdots,A_{N-1})$:

$$\sum_{j=1}^{N-1}A_jP_j-\|A\|^2=tB-Ct^2<0,\forall t>B/C, \tag{13}$$

i.e. the last inequality in (9) holds for any fixed $t>B/C$. So there exists a vector A with above mentioned properties. We do the same with $n=N-1(N>2)$.

2. Let $1<n<N-1$. Inequality (11) holds, e.g., for

$$A_j=\frac{r}{1-P_j},\ j=1,\cdots,n-1; A_n=t, A_j=\frac{s}{P_j},$$
$$j=n+1,\cdots,N-1,$$

where $(n-1)r=(N-1-n)s=t,\forall t>0$. By analogy with the case 1 (see (13)), we find a number $t_0>0$ such that for all $t>t_0$, the chosen values of parameters $A_1,\cdots,A_{N-1}$ satisfy the last inequality in (9).

3.4. Examples

For solution of Problem B it is necessary to find an admissible variant of relation (5). We illustrate a possible way to choose it for a case of three groups $(N=3)$.

Example 1. Assume in the notations of subsection 3.1 $V=270$ milliard tons, $N=3, S=6$ milliard peoples, $S_1=S_2=S_3$, $n=2$. Then $s_j=1/3$, $j=1,2,3$, $P_1=2/3$, $P_2=1/3$ (see (8)), $\lambda_2<1$. By analogy with the case 1 in 3.2 (see (13)), we find parameters A_1, A_2. Inequality (11) has the form $A_1s_1<A_2s_3$, consequently, $A_1<A_2$. For example, vector $A=(1,2)$ satisfies this inequality and the inequality $2A_1/3+A_2/3-\|A\|^2<0$ (see (13)). By using formula (12) we obtain a suitable relation of the form (5): $4\lambda_1+4\lambda_2-5\lambda_3=0$. By (10) we conclude: $\lambda_1=11/15,\lambda_2=14/15,\lambda_3=4/3$. Based on this and (6) we find desired parameters $V_1=66$, $V_2=84$, $V_3=120$ milliard tons of emissions.

Taking into consideration the another variant of the problem statement (see 3.2) consider the following example.

Example 2. Assume in the notations of subsection 3.1 $V=270$ milliard tons, $N=3, S=150$ million km^2, $S_2=3S_1, S_3=2S_1$, $n=2$. Then $s_1=1/6$, $s_2=1/2$, $s_3=1/3$, $P_1=5/6, P_2=1/3, \lambda_2<1$. Inequality (11) is transformed into $A_1<2A_2$, and the last inequality in (9) is $5A_1/6+A_2/3-\|A\|^2<0$. Vector $A=(1,1)$, for example, satisfies these inequalities. In a similar way, we find the equality of the form (5) $7\lambda_1+3\lambda_2-4\lambda_3=0$ and desired parameters $\lambda_1=23/30$, $\lambda_2=29/30$, $\lambda_3=7/6$. Finally, from (6) we obtain $V_1=34{,}5$, $V_2=130{,}5$, $V_3=105$ milliard tons of emissions.

3.5. An Alternative Version of Problem B

Consider the equality of the form (5)

$$(1-\delta_n)\lambda_n+\delta_n\lambda_{n+1}=1,\delta_n\in(0,1). \tag{14}$$

It guarantees fulfilment of the inequality (see (3)) $\lambda_n<1<\lambda_{n+1}$, but it does not coordinate with condition $D\neq b$ (in the notations of relations (5), (8), (9)), which is necessary for solution of Problems A, B. Therefore consider another statement of an extremal problem, consistent with (14).

Problem C. *For a suitable choice of parameter* δ_n *in* (14) *it is necessary to find values of parameters* $\{\lambda_k\}_1^N$*, such that the functional*

$$\Phi_1(\lambda)=\lambda_1^2+\sum_{i=1}^{N-1}(\lambda_{i+1}-\lambda_i)^2, \tag{15}$$

attains the minimum provided that Equalities (2), (3), (14) are fulfilled.

For a case of three groups $(N=3)$ it is not difficult to prove, using elementary tools of mathematical analysis, that Problem C is solvable, e.g. for $\delta_1=s_1/2+s_2+s_3$, $\delta_2=s_3/2$, if, respectively, $n=1$, $n=2$. In particular, for $n=2$ and for the above mentioned value of δ_2 desired parameters are as follows:

$$\lambda_1 = \frac{4(s_1/s_3)^2 + 1}{(1+s_1)^2 + 4(s_1/s_3)^2 + 1};$$

$$\alpha_1 = \frac{1-\lambda_1}{1+s_1}; \alpha_2 = \frac{2s_1\alpha_1}{s_3}; \lambda_{i+1} = \lambda_i + \alpha_i,$$

where $i = 1, 2$. Then application of this algorithm under the assumptions of Example 1 in 3.4 yields the result close to the result of Example 1: $\lambda_1 = 45/61$, $\lambda_2 = 57/61$, $\lambda_3 = 81/61$.

4. Conclusions

The proposed objectivization of the emission reduction by different countries (or limitation of the emission buildup rates for developing countries) can be based on three approaches.

Approach 1. Following Dirk, we recognize the right of each citizen of Earth to have an emission quota, but we would distribute quotas of emission reduction. However, social conditions do not allow us to attain our ultimate goal too quickly and physical conditions do not allow us to do this too slowly. The time necessary for emissions to reach a threshold level, at which the effect of humans on the global temperature becomes actually imperceptible, must be such that atmospheric concentrations of greenhouse gases should not be able to reach threshold levels that would cause a transition from "a cold climate" to "a warm one" and, then, if concentrations of greenhouse gases are further increased, to Venus-like atmospheric conditions. Citizens of different countries, due to historical reasons, get different rates of emission reduction (or emission increase for industrially undeveloped countries). Thus, actual per capita emissions will vary widely. The resultant difference in quotas must correspond to differences in geographic conditions of nations.

Approach 2. Theoretically, the distribution of quotas could be proportional to the area occupied by each state. If this is taken as the sole principle of distribution, it will become absolutely inequitable.

Approach 3. Another possible approach is to take into account the rights of a citizen of Earth and those of the proprietor of the area. As the land area amounts to about one-third of the Earth's surface, quotas should be determined proportionally to the area of each country, while the two-thirds occupied by the Global Ocean should be distributed proportionally to the population size of each country, taking into account historical and geographic differences. Once an agreement is reached on the minimal and maximal gas emissions and the number of groups, the approach proposed in this study can provide a basis for using the objective algorithm of determining the quotas of emission reduction for different countries, for any actual level of emissions.

5. References

[1] L. S. Maergoiz, T. Yu. Sidorova and R. G. Khlebopros, "Matematicheskii Algoritm Raspre-Deleniya Vybrosov Parnikovykh Gazov (A Mathematical Algorithm of the Distribution of Greenhouse Gas Emissions)," in Russian, Preprint, SFU+KRC SB RAS, 2010, p. 22.

[2] M. I. Budyko, "Effect of Solar Radiation Variation on Climate of Earth," *Tellus*, Vol. 21, No. 5, 1969, pp. 611-1969.

[3] V. I. Vernadsky, "Zhivoye Veshchestvo i Biosfera (Living Matter and the Biosphere)," In Russian, Moscow, 1994.

[4] D. Solte, "Understanding the Worlds's Crisis: An Opportunity for Global Action for a Sustainable Future," *Terra Media Verlag*, 2009.

[5] "United Nations Framework Convention on Climate Change, 1992," *Byulleten Mezhdu-Narodnykh Dogovorov* (*The Bulletin of International Agreements*), in Russian, No. 12, 1996, p. 324.

[6] "Kyoto Protocol to the United Nations Framework Convention on Climate Change," *Byulleten Mezhdunarodnykh Dogovorov* (*The Bulletin of International Agreements*), in Russian, No. 5, 2005, p. 323.

[7] E. A. Galkova and L. S. Maergoiz, "On One Linear Problem of Joint Investment," in Russian, *Siberian Journal of Industrial Mathematics*, Vol. 9, No. 3, 2006, pp. 26-30.

[8] E. A. Galkova, L. S. Maergoiz and R. G. Khlebopros, "Applications of the Optimality Principle in Tasks of Joint Investment Management," *Vestnik Novosibirskogo gos. Universiteta* (*Bulletin of the Novosibirsk University*), *Social-Economic Series*, in Russian, Vol. 8, No. 1, 2008, pp. 138-143.

