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APPLICATION OF ADOMIAN'S APPROXIMATION TO BLOOD FLOW THROUGH ARTERIES IN THE PRESENCE OF A MAGNETIC FIELD

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ABSTRACT. The present investigation deals with the application of Adomian's decomposition method to blood flow through a constricted artery in the presence of an external transverse magnetic field which is applied uniformly. The blood flowing through the tube is assumed to be Newtonian in character. The expressions for the two-term approximation to the solution of stream function, axial velocity component and wall shear stress are obtained in this analysis. The numerical solutions of the wall shear stress for different values of Reynold number and Hartmann number are shown graphically. The solution of this theoretical result for a particular Hartmann number is compared with the integral method solution of Morgan and Young [17].

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1. Introduction

There are many frontier problems which exist in physics, engineering, biology, medicine, astrophysics and in many other disciplines. These problems are formulated by nonlinear ordinary or partial differential equations or by the systems of them subject to certian boundary conditions. The exact solutions of these problems are not always possible due to their nonlinear character. In order to solve them we need some simplications which change the physical problems to mathematically tractable ones; but the solutions of the simplified problems deviate much from the actual solutions of the original problems. As a result we take the advantage of traditional numerical techniques which result in massive computations, to obtain solutions of the problems up to desired accuracy.

Recently, a powerful method known as decomposition method which has been developed by Adomian [1-6], can provide analytic approximations to a wide class of nonlinear ordinary and partial differential equations or systems of them. This method gives an accurate and computable solution of the problem for a sufficiently small number of terms and demands to be parallel to any modern supercomputer. The advantages of this method are avoidance of simplifications which convert the physical problems to mathematically tractable ones whose solutions are not consistent with those of the original problems. Theoretical applications of decomposition method to fluid mechanics have been discussed by Adomian [4]; but specific problems in this field have yet not been studied.

The basic equations of motion in fluid mechanics are represented by the Navier-Stokes equations which are the nonlinear partial differential equations and govern the flow field of air round aircraft, in ramjet, blood circulation in the cardiovascular system of human body and in many other fields. The main object of this paper is to study the specific problem of blood flow with the help of this method under the influence of an externally applied magnetic field.

It has been reported by Barnothy [7] that the biological systems, in general, are affected by the application of external magnetic field. Gold [8] has obtained an exact solution of one-dimensional steady flow of an electrically conducting fluid through a non-conduction circular tube under the influence of a uniform transverse magnetic field. The corresponding unsteady problem has been studied by Gupta and Bani Singh [9] considering an exponentially decaying pressure gradient. Ramchandra Rao and Deshikachar [10] have studied the physiological-type flow in the presence of a transverse magnetic field. Sud and Sekhon [11] have used the finite-element method to study the blood flow through the human arterial system in the presence of a steady magnetic field.

In all the above works the effects of different types of magnetic field on flow characteristics in the tubes of uniform circular cross-section have been studied; but the corresponding problem in the presence of a constriction are more inportant from the physiological standpoint of view. Deshikachar and Ramchandra Rao [12] have studied the steady blood flow through a channel of variable cross-section in the presence of a transverse magnetic field and the corresponding unsteady problem has been investigated by Ramchandra Rao and Deshikachar [13]. Mc Michael and Deutsch [14] have analysed the steady flow problem in a circular tube of variable cross-section under the influence of an axial magnetic field and the same problem in unsteady case has been investigated by Deshikachar and Ramchandra Rao.

The present investegation gives an application of decomposition method to a steady two-dimensional blood flow through a constricted artery in the presence of a uniform transverse magnetic field.

2. Mathematical Model

Consider a steady, laminar and axially symmetric flow of blood through a locally constricted straight artery of infinite length under the influence of an external transverse magnetic field which is applied uniformaly. Blood flowing through the tube is supposed to be conducting and Newtonian in character. The assumptions of constant fluid density and viscosity are used here. The appropriate equations governing the flow field in the tube are the momentum equations and these equations, after introducing the electro-magnetic force, in cylindrical polar co-ordinates $(\bar{x}, \bar{r}, \bar{\theta})$ are

$$\vec{U} \cdot \bar{\nabla} \vec{U} \;=\; \frac{1}{\rho} \cdot \vec{\nabla} \vec{P} + \nu \vec{\nabla}^2 \vec{U} + \frac{1}{\rho}\left(\vec{I} \times \vec{B}\right) \tag{1}$$

where $\vec{U}$ is the velocity vector of the field, P the pressure, ν the kinametic viscosity, ρ the density of the fluid, $\vec{I}$ the current density, $\vec{B}$ the magnetic field and the operator $\bar{\nabla}^2$ is given by

$$\bar{\nabla}^2 \;=\; \frac{\partial^2}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \cdot \frac{\partial}{\partial \bar{r}} + \frac{1}{\bar{r}^2} \cdot \frac{\partial^2}{\partial \bar{\theta}^2} \tag{2}$$

The current density and magnetic field are expressed by the Maxwell's equations and Ohm's law, namely

$$\vec{I} \;=\; \sigma_e \left[\vec{E} + \mu_e \left(\vec{U} \times \vec{B}\right)\right] \tag{3}$$

$$\bar{V} \vec{B} \;=\; 0 \tag{4}$$

$$\bar{V} \times \vec{B} \;=\; 0 \tag{5}$$

where $\vec{E}$ is the electric field, σ_e conductivity of the fluid and μ_e is the magnetic permeability.

In the present investigation, it is assumed that the effects of the induced magnetic field and the electric field produced due to the motion of electrically conducting fluid are very small and no external force is applied. With these assumptions and assumption of axially symmetric flow of fluid, the governing equations of motion of the fluid are the Navier-Stokes equations in cylindrical polar coordinates

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{r}} \;=\; -\frac{1}{\rho}\,\frac{\partial \bar{p}}{\partial \bar{x}} + \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \cdot \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial^2 \bar{u}}{\partial \bar{x}^2}\right) - \frac{B_0^2}{\rho} \cdot \bar{u} \tag{6}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{r}} \;=\; -\frac{1}{\rho} \cdot \frac{\partial \bar{p}}{\partial \bar{r}} + \nu\left(\frac{\partial^2 \bar{v}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \cdot \frac{\partial \bar{v}}{\partial \bar{r}^2} - \frac{\bar{v}}{\bar{r}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\right) \tag{7}$$

and the continuity equation :

$$\frac{\partial}{\partial \bar{x}}\left(\bar{r}\bar{u}\right) + \frac{\partial}{\partial \bar{r}}\left(\bar{r}\bar{v}\right) \;=\; 0 \tag{8}$$

where $(\bar{u}, \bar{v})$ are the components of the fluid velocity in the axial and radial directions respectively, $B_0(= \mu_e H_0)$ is the electromagnetic induction and H_0 is the transverse component of magnetic field.

The geometry of the constriction is described by

$$\frac{\bar{R}(\bar{x})}{R_0} = 1 - \frac{\bar{\Sigma}}{R_0}\bar{f}(\bar{x}) \tag{9}$$

where R_0 is the radius of the normal tube, $\bar{R}(x)$ the radius of the tube in the stenotic region and $\bar{\Sigma}$ the maximum height of stenosis.

The boundary conditions are

$$\bar{u} = \bar{v} = 0 \ at \ \bar{r} = \bar{R}(\bar{x}) \tag{10}$$

$$\frac{\partial \bar{u}}{\partial \bar{r}} = 0 \ at \ \bar{r} = 0 \tag{11}$$

$$\int_0^{\bar{R}(\bar{x})} \bar{r}\bar{u}d\bar{r} = \bar{Q}/2\pi \tag{12}$$

where $\bar{Q}$ is the constant volumetric flux across any cross-section of the tube.

It is convenient to write the system of equations from (6) to (12) in the non-dimensional forms with the help of the following transformations

$$\begin{aligned} u &= \bar{u}/U_0, \ v = \bar{v}/U_0 \\ r &= \bar{r}/R_0, \ x = \bar{x}/R_0, \ p = \bar{p}/\rho U_0^2 \end{aligned} \tag{13}$$

where (u, v) are the dimensionless velocity components, U_0 is the characteristic velocity and p is the non-dimensional fluid pressure.

The momentum equations (6) and (7) are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{R_e}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2}\right] - M^2 u \tag{14}$$

in the axial direction and

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{R_e}\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial x^2}\right] \tag{15}$$

in the radial direction where R_e and M are the Reynolds number and Hartmann number defined by

$$\begin{aligned} R_e &= U_0 R_0/\nu \\ M^2 &= B_0^2 \sigma_e R_0^2/\mu \end{aligned} \tag{16}$$

Similarly, the dimensionless continuity equation (8) is

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{17}$$

and the geometry of constriction takes the form

$$\eta(x) \;=\; 1 - \Sigma f(x) \tag{18}$$

where

$$\begin{array}{lll} \eta(x) & = & \bar{R}(\bar{x})/R_0 \\ f(x) & = & \bar{f}(\bar{x})/R_0 \\ \Sigma & = & \bar{\Sigma}/R_0 \end{array} \tag{19}$$

The corresponding non-dimensional boundary conditions are

$$u = v = 0 \; at \; r = \eta; \tag{20}$$

$$\frac{\partial u}{\partial r} = 0 \; at \; r = 0 \tag{21}$$

$$\int_0^{\eta(x)} rudr \;=\; -\frac{1}{2} \tag{22}$$

Next we introduce the stream function ψ defined by

$$u \;=\; -\frac{1}{r}\cdot\frac{\partial\psi}{\partial r}, \qquad v \;=\; -\frac{1}{r}\cdot\frac{\partial\psi}{\partial x} \tag{23}$$

Then the continuity equation (17) is satisfied identically and using (23) elimination of p between (14) and (15) gives the following governing equation

$$R_e\left[\frac{1}{r}\cdot J - \frac{2}{r^2}\cdot\nabla^2\psi\cdot\frac{\partial\psi}{\partial x}\right] \;=\; \nabla^4\psi - M^2 r\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial\psi}{\partial r}\right) \tag{24}$$

where J is the Jaccobian defined by

$$J \;=\; \frac{\partial(\nabla^2\psi,\psi)}{\partial(r,x)} \;=\; \begin{vmatrix} \frac{\partial}{\partial r}(\nabla^2\psi) & \frac{\partial\psi}{\partial r} \\ \frac{\partial}{\partial x}(\nabla^2\psi) & \frac{\partial\psi}{\partial x} \end{vmatrix} \tag{25}$$

and the operator ∇^2 is given by

$$\nabla^2 \;=\; \frac{\partial^2}{\partial r^2} - \frac{1}{r}\,\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2} \tag{26}$$

The boundary conditions in terms of ψ are

$$-\frac{1}{r}\,\frac{\partial\psi}{\partial r} = 0, \qquad \psi - \frac{1}{2} \; at \; r = \eta \tag{27}$$

$$-\frac{\partial}{\partial r}\left(\frac{1}{r}\,\frac{\partial\psi}{\partial r}\right) = \psi = 0 \; at \; r = 0 \tag{28}$$

3. Method of Solution

The equation (24) is a non-linear partial differential equation and the exact solution of this equation is not always possible. This equation can be solved by using traditional numerical techniques which result in massive numerical computations. Recently, a modern powerful method known as decomposition method has been developed by Adomian [1-6] and applied here to obtain analytic approximations to this non-linear equation. Let $L = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\,\frac{\partial}{\partial r}$ and the equation (24) becomes

$$L^2\psi \;=\; R_e N\psi - \frac{\partial^4\psi}{\partial x^4} - 2\frac{\partial^2}{\partial x^2}\,(L\psi) + M^2 L\psi \tag{29}$$

where

$$N\psi \;=\; \frac{1}{r}\cdot J - \frac{2}{r^2}\cdot\frac{\partial\psi}{\partial x}\cdot\nabla^2\psi \tag{30}$$

If we operate both sides of (29) by the inverse operation L^{-2} [2], then we get

$$\psi \;=\; \psi_0 + L^{-2}\left[R_e N_\psi - \frac{\partial^4\psi}{\partial x^4} - 2\frac{\partial^2}{\partial x^2}(L\psi) + M^2 L\psi\right] \tag{31}$$

Here ψ_0 is the solution of the hogoneous equation

$$L^2\psi_0 \;=\; 0 \tag{32}$$

and it is given by

$$\psi_0 \;=\; \frac{1}{16}\;A(x)r^4 + B(x)L_1^{-1}r\;log\;r + \frac{1}{2}C(x)r^2 + F(x) \tag{33}$$

The integration constants A, B, C and F involved in (33) are to be determined from the given boundary conditions (27), (28) and $L_1^{-1} = \int(.)dr$.

Next we decompose ψ and $N\psi$ into the following forms

$$\psi \;=\; \sum_{n=0}^{\infty}\lambda^n\psi_n \tag{34}$$

$$N_\psi \;=\; \sum_{n=0}^{\infty}\lambda^n P_n \tag{35}$$

where P_n are Adomian's special polynomials which are to be discussed later. The parameter λ used in (34) and (35) is not a perturbation parameter; it is only used for grouping the terms of different orders. Then the parameterixed form of (31) as [2]

$$\psi \;=\; \psi_0 + \lambda L^{-2}\left[R_e N_\psi - \frac{\partial^4\psi}{\partial x^4} - 2\frac{\partial^2}{\partial x^2}(L\psi) + M^2 L\psi\right] \tag{36}$$

Now we substitute (34), (35) into (36) and them comparing the like-power terms of λ on both sides of the resulting expression we get

$$\psi_{n+1} \;=\; L^{-2}\left[R_e P_n - \frac{\partial^4 \psi_n}{\partial x^4} - 2\frac{\partial^2}{\partial x^2}(L\psi_n) + M^2 L\psi_n\right] \tag{37}$$

where $n = 0, 1, 2, \cdots$. Once the component ψ_0 is determined, the other components of ψ such as, ψ_1, ψ_2, ψ_3 etc. can be easely determined from (37). The decomposition refered to above is called regular decomposition of ψ.

If we further take parameterized decomposition of ψ_0 given by

$$\psi_0 \;=\; \sum_{n=0}^{\infty} \lambda^n \psi_{0,n} \tag{38}$$

we mean the double decomposition [3]. Substitution of (34), (35), (38) into (36) gives the double decomposition components of ψ and these are given by the relation

$$\psi_{n+1} \;=\; \psi_{0,n+1} + L^{-2}\left[R_e P_n - \frac{\partial^4 \psi_n}{\partial x^4} - 2\frac{\partial^2}{\partial x^2}(L\psi_n) + M^2 L\psi_n\right] \tag{39}$$

n being zero and any positive integer.

Since the expression for ψ_0 contains the constants A, B, C and F therefore, the parameterized decomposition forms of all these constants are

$$\begin{aligned} A &= \sum_{n=0}^{\infty} \lambda^n A_n \\ B &= \sum_{n=0}^{\infty} \lambda^n B_n \\ C &= \sum_{n=0}^{\infty} \lambda^n C_n \\ F &= \sum_{n=0}^{\infty} \lambda^n F_n \end{aligned} \tag{40}$$

If we substitute (38) and (40) into (33) and then if we compare the like-power terms of λ on both sides of the resulting expression we get

$$\psi_{0,n+1} \;=\; \frac{1}{16}\, A_{n+1} r^4 + B_{n+1} L_1^{-1} r \; log \; r + \frac{1}{2}\, C_{n+1} r^2 + F_{n+1} \tag{41}$$

The relations (39) and (41) together give the components of ψ. The constants involved in each ψ_n will be determined by their respective boundary conditions.

The polynomials P_0, P_1, $\cdots$ P_n are Adomian's polynomials [1, 2]. They are defined in such a way that $P_0 \equiv P_0(\psi_0)$, $P_1 \equiv P_1(\psi_0, \psi_1)$, $P_2 \equiv P_2(\psi_0, \psi_1, \psi_2)$, $\cdots$ $P_n = P_n(\psi_0, \cdots \psi_n)$. In order to determine these polynomials, we substitute (34) and (35) into (30) and then comparison of the terms of like power of λ

on both sides of the resulting equation gives the following set of Adomian's polynomials

$$\begin{aligned} P_0 &= \frac{1}{r}\cdot\frac{\partial(\nabla^2\psi_0,\psi_0)}{\partial(r,x)} - \frac{2}{r^2}\cdot\frac{\partial\psi_0}{\partial x}\cdot\nabla^2\psi_0 \\ P_1 &= \frac{1}{r}\left[\frac{\partial(\nabla^2\psi,\psi_0)}{\partial(r,x)} + \frac{\partial(\nabla^2\psi_0,\psi_1)}{\partial(r,x)}\right] \\ &= \frac{2}{r^2}\left[\frac{\partial\psi_0}{\partial x}\cdot\nabla^2\psi_1 + \frac{\partial\psi_1}{\partial x}\cdot\nabla^2\psi_0\right] \\ & \quad \cdots\ \cdots\ \cdots\ \cdots \\ & \quad \cdots\ \cdots\ \cdots\ \cdots \end{aligned} \tag{42}$$

Again substitution of (34) into the boundary conditions (27) and (28) gives the boundary conditions for the respective components ψ_0 ψ_1, etc. as

$$\left.\begin{aligned} -\frac{1}{r}\cdot\frac{\partial\psi_0}{\partial r} &= 0,\ \psi_0 = -\frac{1}{2} \quad at \quad r=\eta \\ -\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial\psi_0}{\partial r}\right) &= \psi_0 = 0 \quad at \quad r=0 \end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned} -\frac{1}{r}\cdot\frac{\partial\psi_n}{\partial r} &= \psi_n = 0 \qquad at \quad r=\eta \\ -\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial\psi_n}{\partial r}\right) &= \psi_n = 0 \quad at \quad r=0 \end{aligned}\right\} \tag{44}$$

for any positive integer.

4. Solution of the Problem

Before proceeding to the solutions we have to find out the inverse operator L^{-2} and for that we consider the following equation for ψ

$$L_\psi = F \tag{45}$$

which, on solving, gives

$$\psi = \left[L_1^{-1} r \left(L_1^{-1} r^{-1}\right)\right] F \tag{46}$$

remembering that the boundary condition terms vanish and L_1^{-1} is an one-fold indefinite integral. From the relation (46) it is obvious that the inverse L^{-1} is identified as

$$L^{-1} = \left[L_1^{-1} r \left(L_1^{-1} r^{-1}\right)\right] \tag{47}$$

and hence we get

$$L^{-2} = L_1^{-1}\left[r L_1^{-1}\left\{r^{-1} L_1^{-1}\left(r L_1^{-1} r^{-1}\right)\right\}\right] \tag{48}$$

Using the boundary conditions (43) in (33), we have the expression for ψ_0 as

$$\psi_0 \;=\; \frac{1}{2\eta^4}\left(r^4 - 2\eta^2 r^2\right) \tag{49}$$

The expression for ψ_1 can be obtained from (39) and (41) by putting $n = 0$ and this expression involves the operator L^{-2} given by (48). Performing the operation of this inverse operator, we get

$$\begin{aligned}\psi_1 \;&=\; \alpha(x)r^{10} + \beta(x)r^8 + \gamma(x)r^6 \\ &\quad + \frac{1}{16}\,A_1 r^4 + B_1 L_1^{-1}\,r\,log\,r + \frac{1}{2}C_1 r^2 + F_1\end{aligned} \tag{50}$$

where A_1 B_1 C_1 and F_1 are integration constants to be obtained by satisfying the boundary conditions (44) obtained by putting $n = 1$ and these constants are found to be

$$\begin{aligned} B_1 \;&=\; F_1 \;=\; 0 \\ A_1 \;&=\; -16\eta^2\left(4\alpha\eta^4 + 3\beta\eta^2 + 2\gamma\right) \\ C_1 \;&=\; 2\eta^4\left(3\alpha\eta^4 + 2\beta\eta^2 + \gamma\right)\end{aligned} \tag{51}$$

The expressions for α, β and γ are given by

$$\alpha \;=\; \left(R_e/960\eta^{11}\right)\left(20\eta_1^3 - 13\eta\eta_1\eta_2 + \eta^2\eta_3\right) \tag{52}$$

$$\begin{aligned}\beta \;&=\; \left(R_e/144\eta^9\right)\left(4\eta_1 - \eta^2\eta_3 + 11\eta\eta_1\eta_2 - 16\eta_1^3\right) \\ &=\; \left(1/576\eta^8\right)\left(15\eta^2\eta_2^2 + 20\eta^2\eta_1\eta_3 - 180\eta\eta_1^2\eta_2 + 210\eta_1^4 - \eta^3\eta_4\right)\end{aligned} \tag{53}$$

$$\begin{aligned}\gamma \;&=\; \left(R_e/48\eta^7\right)\left(12\eta_1^3 - 9\eta\eta_1\eta_2 + \eta^2\eta_3 - 8\eta_1\right) \\ &=\; \left(1/96\eta^6\right)\left(\eta^3\eta_4 - 12\eta^2\eta_1\eta_3 - 9\eta^2\eta_2^2 + 72\eta\eta_1^2\eta_2 - 60\eta_1^4 + 80\eta_1^2 - 16\eta\eta_2\right) \\ &=\; +\frac{M^2}{48\eta^4}\end{aligned} \tag{54}$$

where η_1, η_2, η_3 and η_4 are the derivatives of η with respect to x indicating the orders according to their suffices. The resulting expression for ψ_1 is found to be

$$\begin{aligned}\psi_1 \;&=\; \alpha r^{10} + \beta r^3 + \gamma r^6 \\ &\quad -\eta^2\left(4\alpha\eta^4 + 3\beta\eta^2 + 2\gamma\right)r^4 \\ &\quad +\eta^4\left(3\alpha\eta^4 + 2\beta\eta^2 + \gamma\right)r^2\end{aligned} \tag{55}$$

If we consider two-term approximation of the solution ψ we obtain from (34)

$$\psi \;=\; \psi_0 + \psi_1 \tag{56}$$

where ψ_0 and ψ_1 are given by (49) and (55) remembering that $\lambda = 1$. Convergence of solution ψ has now been well established [1, 2]. The axial velocity component is found to be

$$u \;=\; -\left[\frac{1}{r}\cdot\frac{\partial\psi_0}{\partial r} + \frac{1}{r}\cdot\frac{\partial\psi_1}{\partial r}\right] \tag{57}$$

The wall shearing stress is defined by

$$T = -\frac{1}{4}\left(\frac{\partial u}{\partial r}\right)_{r=n} \left(1+\eta_1^2\right) \tag{58}$$

which, on substitution of u from (56), gives

$$T = (1/\eta^3)\left[1+2\eta^6\left(6\alpha\eta^4+3\beta\eta^2+\gamma\right)\right]\left(1+\eta_1^2\right) \tag{59}$$

5. Numerical Discussion

For numerical discussion the function $f(x)$ is described by

$$f(x) = \frac{1}{2}\left(1+cos\,\frac{\pi x}{L_0}\right), -L_0 \leq x \leq L_0 \tag{60}$$

Then the geometry of constriction (18) takes the following form

$$\eta(x) = 1-\frac{1}{2}\Sigma\left(1+cos\,\frac{\pi x}{L_0}\right), -L_0 \leq x \leq L_0 \tag{61}$$

The variations of the wall shear stress (58) along the length of the constricted artery are shown graphically for different values of Reynolds number and Hartmann number it .shows that the maximum value of the solution for each Reynolds number occurs just ahead of the throat of stenosis and negetive distribution of the solution is observed over some length of the tube in the diverging section. This negetive behaviour of the wall shear stress indicates separation which involves circulation with back flow near the wall. As a result of this back flow a low shear exists at the wall and a high velocity core surrounded by the separated region is formed. With the increase of Reynolds number the negetive behaviour of the solution increases showing the enlargement of circulation which is physiologically unfavourable. It shows the variations of the wall shear stress with x for different values of Hartmann number. It is seen that the negetive behaviour of the solution observed in the diverging section of the tube decreases with increasing Hartmann number. As a result the circulation diminishes indicating the favourable physiological condition. Therefore, it can be concluded that the effect of an external transverse magnetic field applied uniformly favours the condition of blood flow.

The was shear stress is compared with that of Morgan and Young [17] for the Hartmann number equal to unity. The solution is in good agreement with Morgan and Young [17] in the diverging section whereas same deviations are observed in the converging region of the tube.

The theoretical result has been explained numerically for two-term approximation of the solution ψ. The result can be improved by considering three-term approximation or more to the solution.

The advantage of decomposition method is to give analytical approximate solution of nonlinear ordinary or partial differential equation which is rapidly convergent [2, 15, 16]. The speed of convergency depends upon the choice of operator which may be a highest-ordered differential operator or a combination of linear operators or a multidimensional operator. This method does not take the help of any simplification for handling the nonlinear terms. Since the decomposition parameter is used only for grouping the terms, therefore, the non-linearities can be handled easily in the operator equation and accurate approximate solution may be obtained for any physical problem.

References

1. Adomian G., *Nonlinear Stochastic Operator Equations*, Academic Press (1986).
2. Adomian G., *Nonlinear Stochastic Systems Theory and Applications to Physics*, Kluwer Academic Publishers (1989).
3. Adomian G., *Solving Frontier Problems of Physics : The Decomposition Method*, Kluwer Academic Publishers (1994).
4. Adomian G., *Application of the Decomposition Method to the Navier-Stokes Equations*, Jr. Math. Anal. Appl. Vol. **119**, p. 340-360 (1986).
5. Adomian G., *Nonlinear Transport in moving fluids*, Appl. Math. Lett. Vol. **6**, No. 5, p. 35-38 (1993).
6. Adomian G. and Rach R. : *Analytic Solution of Nonlinear Boundary value Problems in several Dimensions*, Jr. Math. Anal. Appl. Vol. **174**, No. 1, p. 118-137 (1993).
7. Barnothy M. F. (ed), *Biological effects of Magnetic fields*, Vols. **1** and **2**, Plenum Press (1964-1969).
8. Gold, R. R., *Magnetohydrodynamics pipe flow*, Part-1, J. Fluid Mech. Vol. **13**, p. 505-512 (1962).
9. Gupta S. C. and Bani Sinha, *Unsteady magnetohydrodynamic flow in a circular pipe under a travsverse magnetic field*, Phys. Fluids, Vol. **13**, p. 346-352 (1970).
10. Ramchandra Rao A. and Deshikachar K. S., *Physiological-type flow in a circular pipe in the presence of a transverse magnetic flow*, Indian Inst. Sci. Vol. **68**, p. 247-260 (1988).
11. Sud V. K. and Sekhon G. S., *Blood flow through the human arterial system in the presence of a steady magnetic field*, Phys. Med. Bio. Vol. **34**, p. 795-805 (1989).
12. Deshikachar K. S. and Ramchandra Rao A., *Effect of a magnetic field on the flow and blood oxygenation in channels of variable cross-section*, Int. J. Engng. Sci. Vol. **23**, p. 1121-1133 (1985).
13. Ramchandra Rao A. and Deshikachar K. S., *MHD Oscillatory flow of blood through channels of variable cross-section*, Int. J. Engng. Sci. Vol. **24**, p. 1615-1628 (1986).
14. Mc Michacl. J. M. and Deutsch S., *Magnetohydrodynamics of laminar flow in slowly varying tubes in an axial magnetic field*, Phys. Fluids. Vol. **27**, p. 110-118 (1984).
15. Cherruault Y. Kybernetes, *Convergence of Adomian's Method*, Vol. **18**, No. 2, p. 31-39 (1989).
16. Cherruault Y., *New results for the convergence of Adomian's Method applied to integral equations*, Math. Comp. Modelling, Vol. **16**, No. 2, p. 85-93 (1992).
17. Morgan B. E. and Young, D. F., *An integral method for the analysis of flow in arterial stenoses*, Jr. Math. Biol. Vol. **36**, p. 39-53 (1974).

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