Retraction Notice

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Retraction initiative (multiple responses allowed; mark with **X**):

☐ All authors
☐ Some of the authors:
X Editor with hints from
- O Journal owner (publisher)
- O Institution:
- **X** Reader:
- O Other:

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☐ Unreliable findings
- O Lab error O Inconsistent data O Analytical error O Biased interpretation
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History
Expression of Concern:
☐ yes, date: yyyy-mm-dd
☐ no

Correction:
☐ yes, date: yyyy-mm-dd
☐ no

Comment:

The paper does not meet the standards of "Advances in Linear Algebra & Matrix Theory".

This article has been retracted to straighten the academic record. In making this decision the Editorial Board follows COPE's Retraction Guidelines. Aim is to promote the circulation of scientific research by offering an ideal research publication platform with due consideration of internationally accepted standards on publication ethics. The Editorial Board would like to extend its sincere apologies for any inconvenience this retraction may have caused.

Editor guiding this retraction: Prof. Mubariz Garayev and Prof. Qingwen Wang (EiC of ALAMT)

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Generation of Pascal Triangle Using Matrices

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Abstract

Pascal's triangle can be generated in many ways. In this paper, we generate the numbers in the Pascal triangle by applying a small perturbation technique in matrices.

Keywords

Pascal Triangle, Perturbation, Matrices

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1. Introduction

The great French mathematician Blaise Pascal introduced the concept of Pascal's triangle during the 17th century and applied it to study the probability theory. The Pascal's triangle is an unending equilateral triangle. The generation of numbers in this triangle is obtained by the simplest technique. Each number in this triangle is the sum of the two numbers directly above it. Although the creation of Pascal's triangle is simple, it has connections throughout many areas of mathematics such as algebra, probability, number theory, combinatorics and fractals. Pascal's triangle has many interesting features, however, it is primarily applied to write any binomial expansion.

There are some interesting patterns associated with the Pascal's triangle. The sum of all elements in the n^{th} row is equal to 2^n. From the inner diagonals, we obtain a sequence of triangular numbers 1, 3, 6, 10, … There is a technique to obtain Fibonacci numbers from this triangle.

There are several methods to obtain numbers in the Pascal's triangle and related numbers. V. E. Hoggatt (1967) discussed the binomial coefficients and Fibonacci numbers [1]. Marjorie Bicknell and V. E. Hoggatt, Jr. (1973) discussed the multinomial coefficient triangle and the convolution triangle formed from sequences [2]. Boris A. Bondarenga (1990) discussed the history of the Pascal Triangle and the binomial coefficients, and also described Pascal triangle of s^{th} order, Pascal pyramids and Hyper pyramids and triangle associated with the Fibonacci and other analog of the binomial coefficients [3]. Bing Cheng Li (1992) studied the three-dimensional moments that had been widely used in computer vision, but until now obtaining 3D moments had always needed much computation, which has not been resolved well [4]. In this paper, he proposed a fast and simple algorithm for calculating 3D moments and Pascal triangle transform (PTT) method is used to calculate monomials with one variable. The calculation of monomials is extended to those with three variables. Finally, sequential and parallel algorithms that need no multiplications are provided for calculating 3D moments. We also concluded that the numbers in the Pascal's triangle could be obtained through a pattern of tossing coins. Gragory S. Call and Daniel J. velleman (1993) discussed the Pascal's matrices while working on a probability problem involving repeated flips of an unfair coin. In this paper, we provide a method of generating the numbers in the Pascal's triangle by using the matrices [5].

2. Main Result

As usual, we denote by $C^{m\times n}$ the set of all $m\times n$ matrices over the field of complex numbers C.

Definition 1. Let $A=\left(a_{ij}\right)\in C^{m\times n}$, choose $\varepsilon>0$ and let $k\in\{0,1,2,\cdots,mn\}$. We define an (ε,k)-*perturbations* of *A* to be a matrix $B=\left(b_{ij}\right)\in C^{m\times n}$, where *k* distinct entries of *B*, say $b_{i_1j_1},b_{i_2j_2},\cdots,b_{i_kj_k}$ are given by $b_{i_kj_k}=a_{i_kj_k}+\varepsilon$, and all the remaining entries of *B* are equal to the corresponding entries of *A*. we use the convention $\varepsilon>0$-*perturbations* of *A* to mean the matrix *A* itself.

Lemma 1. Let $A=\left(a_{ij}\right)\in C^{m\times n}$, and let $\varepsilon>0$. Then for each $k\in\{0,1,2,\cdots,mn\}$ the number of all possible (ε,k)-*perturbations* of *A* is the same as the number of ways of selecting *k* objects from "*mn*" distinct objects without regard to the order.

The proof of the lemma is straight forward and hence omitted. The lemma and the binomial theorem, we get the following result:

$$\binom{mn}{k}=\frac{(mn)!}{(k)!(mn-k)!}$$

Theorem 1. Let $A=\left(a_{ij}\right)\in C^{m\times n},x\in C$ and $\varepsilon>0$. Then for each $k\in\{0,1,2,\cdots,mn\}$, the number of all possible (ε,k)-*perturbation* of *A* is the binomial coefficient of the term x^k in the binomial expansion of $(1+x)^{mn}$. By changing the positive integer $p:=mn$ over the set of positive integers *N*, we generate the Pascal's triangle.

Remark 1. Let $A=\left(a_{ij}\right)\in C^{m\times n}$, and let $\varepsilon>0$. Then for every $k\in\{0,1,2,\cdots,mn\}$, denote by A_k the number of all possible (ε,k)-*perturbation* of *A*. It then follows that

$$\sum_{k=0}^{mn}A_k=2^{mn}$$

Example 1. Let $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\in C^{2\times 2}$, and let $\varepsilon>0$. Then $A_0=1$, And $A_1 = 4$ since the $(\varepsilon,1)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b \\ c & d \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon \\ c & d \end{pmatrix}, \begin{pmatrix} a & b \\ c+\varepsilon & d \end{pmatrix}, \begin{pmatrix} a & b \\ c & d+\varepsilon \end{pmatrix}$$

Also $A_2 = 6$, since the $(\varepsilon,2)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b+\varepsilon \\ c & d \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b \\ c+\varepsilon & d \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b \\ c & c+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a & b+\varepsilon \\ c+\varepsilon & d \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon \\ c & d+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b \\ c+\varepsilon & d+\varepsilon \end{pmatrix}$$

Moreover $A_3 = 4$, since the $(\varepsilon,3)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b+\varepsilon \\ c+\varepsilon & d \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon \\ c & d+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b \\ c+\varepsilon & d+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon \\ c+\varepsilon & d+\varepsilon \end{pmatrix};$$

Finally $A_4 = 1$, since the $(\varepsilon,4)$-perturbations of A is $\begin{pmatrix} a+\varepsilon & b+\varepsilon \\ c+\varepsilon & d+\varepsilon \end{pmatrix}$.

So,

$$\sum_{i=1}^{4} A_i = 1+4+6+4+1 = 16 = 2^4$$

Example 2. Let $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \in C^{2\times 3}$, and let $\varepsilon > 0$ then, $A_0 = 1$,

And $A_1 = 6$ since the $(\varepsilon,1)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b & c \\ d & e & f \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c \\ d & e & f \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d & e & f \end{pmatrix},$$
$$\begin{pmatrix} a & b & c \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a & b & c \\ d & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a & b & c \\ d & e & f+\varepsilon \end{pmatrix}.$$

Also $A_2 = 15$ since the $(\varepsilon,2)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c \\ d & e+\varepsilon & f \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b & c \\ d & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d & e & f \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c \\ d & e+\varepsilon & f \end{pmatrix},$$
$$\begin{pmatrix} a & b+\varepsilon & c \\ d & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d & e & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a & b & c \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a & b & c \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b & c \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}$$

Moreover $A_3 = 20$ since the $(\varepsilon,3)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d & e+\varepsilon & f \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d & e+\varepsilon & f \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix},$$

$$\begin{pmatrix} a+\varepsilon & b & c \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d & e+\varepsilon & f \end{pmatrix},$$
$$\begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a & b+\varepsilon & c \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a & b & c+\varepsilon \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b & c \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}.$$

Moreover also $A_4 = 15$ since the $(\varepsilon, 4)$-perturbations of A are

$$\begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d & e & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b & c \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b+\varepsilon & c \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a & b & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}.$$

Moreover also then $A_5 = 6$ since the $(\varepsilon, 5)$-perturbations of A are

$$\begin{pmatrix} a & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix},$$
$$\begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d & e+\varepsilon & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e & f+\varepsilon \end{pmatrix}, \begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f \end{pmatrix}.$$

Finally $A_6 = 1$ since the $(\varepsilon, 6)$-perturbations of A is $\begin{pmatrix} a+\varepsilon & b+\varepsilon & c+\varepsilon \\ d+\varepsilon & e+\varepsilon & f+\varepsilon \end{pmatrix}$.

So,

$$\sum_{i=1}^{6} A_i = 1+6+15+20+15+6+1 = 64 = 2^6 .$$

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