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An Inventory Model for Deteriorating Items with Exponential Declining Demand and Time-Varying Holding Cost

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ABSTRACT

In the present paper, a total optimal cost of an inventory model with exponential declining demand and constant deterioration is considered. The time-varying holding cost is a linear function of time. Shortages are not allowed. The items (like food grains, fashion apparels and electronic equipments) have fixed shelf-life which decreases with time during the end of the season. A numerical example is presented to demonstrate the model and the sensitivity analysis of various parameters is carried out.

KEYWORDS

1. Introduction

The main objective of the proposed model is to develop an inventory model for a deteriorating item having a time-dependent exponential declining demand rate and time-varying holding cost. The literature review of the present paper is discussed as follows:

Generally, deteriorating items refer to the items that become damaged, decayed, spoiled, lost of utility or lost of its marginal value, evaporative, devaluation, invalid, degradation and so on through time. The process of deterioration is happened in two categories of items. The first category refers to the items that become damaged, spoiled, decayed, loss of utility, evaporative or expired through time like food grain, food stuffs, fruits, flowers, vegetables, films, medicines and so on, while the other category refers to the items that loss their parts or their total values through time because of the introduction of new technology or the alternatives like fashion and seasonal goods, electronic equipments, computer chips and mobile phones and so on. For the first category, the items have a short natural life cycle whereas in the second category, the items have a short market life. The literature surveys by Raafat [1], Wee [2], Shah and Shah [3], Goyal and Giri [4] and Li *et al.* [5] discuss the up to date review on deteriorating inventory model.

Harris [6] was the first researcher who studied the inventory model. Wilson [7] generalized Harris' model and gave a formula to obtain (EOQ) economic order quantity. Whitin [8] established the inventory model on fashion items deteriorating at the end of the stroage period. Then Ghare and Schrader [9] concluded in their study that the consumption of the deteriorating items was closely relative to a negative exponential function of time. They established the classical EOQ model with constant deterioration rate and without shortages. Dave and Patel [10] studied firstly the inventory model on deteriorating items with linear increasing demand when shortages were not allowed. Some of the recent work in this field has been established by Chung and Ting [11], Wee [12] and

Mishra *et al.* [13]. Chang and Dye [14] established an inventory model with time-varying demand and partial backlogging. On the basis of demand variations, Ouyang and Cheng [15] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Abad [16,17], Shah and Raykundaliya [18] studied the case of partial backlogging in their inventory model. Singh and Pattnayak [19] also studied an inventory model for deteriorating items with linear demand, variable deterioration and partial backlogging. Tripathy and Mishra [20], Singh and Pattnayak [21,22], Amutha and Chandrasekaran [23] also studied an inventory model on deteriorating items. Usually, in inventory models, inventory holding cost and the demand rate are considered to be constant. But sometimes in reality, we may observe that inventory holding cost and demand rate for physical goods are time-independent. This paper examines an inventory model with linearly varying holding cost. The objective function is to minimize the total cost of an inventory system based on inventory holding cost, ordering cost and deterioration cost. In reality, the demand rate and the inventory holding cost for physical goods may be time-dependent. As time is an important factor in the inventory system, we consider that demand rate and inventory holding cost are time-dependent. We have developed an EOQ inventory model for deteriorating items with exponentially declining demand when the deterioration rate follows a constant rate with time-dependent linear holding cost and shortages are not allowed.

In the present paper, an effort has been made to analyze an EOQ model for deteriorating items by considering the time-dependent exponential declining demand rate and time-dependent linear inventory holding cost. Shortages are not allowed. The proposed model is based on the inventory items like fashion and seasonal goods, electronic equipments, computer chips and mobile phones and so on, as they experience fluctuations in the demand rate. The mathematical model has been derived under exponential declining demand rate.

The rest part of the paper is arranged as follows: In Section 1, we review the literature on the effects of deterioration and time-dependent demand rate and we position our model in relative to previous work. Section 2 details the model assumptions. In Section 3, we formulate the model as a cost minimization problem. In Section 4, a numerical example is given to illustrate the model. Sensitivity analysis of various parameters is taken in Section 5. In Section 6, we sum with a conclusion of this work and suggest directions for future research.

2. Assumptions

The following assumptions are made in developing the model.
1) The inventory system considers a single item only.
2) The demand rate is deterministic and is an exponential declining function of time.
3) The deterioration rate is considered as constant.
4) The inventory system is considered over a finite time horizon.
5) Lead time is zero.
6) Shortages are not allowed.

3. Mathematical Formulation and Solution of the Model

We consider the inventory model of deteriorating items with declining demand rate. As the inventory reduces due to demand rate as well as deterioration rate during the interval $[0,T]$, the differential representing the inventory status is governed by

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t), \ 0 \le t \le T \tag{1}$$

where $\theta(t) = \theta$ and $D(t) = Ke^{-\gamma t}$.

The solution of Equation (1) with boundary condition $I(t) = 0$ is as follows:

$$I(t) = \frac{K}{(\theta - \gamma)}\left[e^{(\theta-\gamma)T-\theta t} - e^{-\gamma t}\right], \ 0 \le t \le T. \tag{2}$$

Thus, the initial order quantity is obtained by putting the boundary condition $I(0) = I_0$ in Equation (2). Therefore,

$$I_0 = I(0) = \frac{K}{(\theta - \gamma)}\left[e^{(\theta-\gamma)T} - 1\right]. \tag{3}$$

The ordering cost is $(OC) = A_0$.

The total demand during the cycle period $[0,T]$ is

$$\int_0^T D(t)\,dt = \int_0^T Ke^{-\gamma t}\,dt = \frac{K}{\gamma}\left[1 - e^{-\gamma T}\right].$$

The number of deteriorated units is

$$I_0 - \int_0^T D(t)\,dt = \frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T} - 1\right] + \frac{K}{\gamma}\left[e^{-\gamma T} - 1\right]$$

The deterioration cost (DC) for the cycle $[0,T]$ $= C_d \times$ (the number of deteriorated units)

$$= C_d\left[\frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T} - 1\right] + \frac{K}{\gamma}\left[e^{-\gamma T} - 1\right]\right]$$

$$= C_d Ke^{-\gamma T}\left[\frac{1}{(\theta-\gamma)}\left[e^{\theta T} - e^{\gamma T}\right] + \frac{1}{\gamma}\left[1 - e^{-\gamma T}\right]\right]. \tag{4}$$

The total inventory holding cost (IHC) for the cycle $[0,T]$ is

$$= \int_0^T (h + \alpha t) I(t)\,dt$$

$$= \int_0^T (h + \alpha t)\left[\frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T-\theta t} - e^{-\gamma t}\right]\right]dt$$

$$= \frac{Ke^{-\gamma T}}{\theta\gamma(\theta-\gamma)}\left[(h + \alpha T)(\theta-\gamma) - h\left(\theta e^{\gamma T} - \gamma e^{\theta T}\right)\right.$$

$$\left. + \frac{\alpha}{\theta\gamma}\left[\left(\theta^2 - \gamma^2\right) - \left(\theta^2 e^{\gamma T} - \gamma^2 e^{\theta T}\right)\right]\right]. \tag{5}$$

Total variable cost = ordering cost (OC) + deterioration cost (DC) + inventory holding cost (IHC).

So, the total variable cost per unit time $TC(T)$ is

$$= \frac{A_0}{T} + \frac{C_d Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta-\gamma)}\left(e^{\theta T} - e^{\gamma T}\right) + \frac{1}{\gamma}\left(1 - e^{-\gamma T}\right)\right]$$

$$+ \frac{Ke^{-\gamma T}}{\theta\gamma(\theta-\gamma)T}\left[(h + \alpha T)(\theta-\gamma) - h\left(\theta e^{\gamma T} - \gamma e^{\theta T}\right)\right.$$

$$\left. + \frac{\alpha}{\theta\gamma}\left[\theta^2 - \gamma^2 - \theta^2 e^{\gamma T} + \gamma^2 e^{\theta T}\right]\right]. \tag{6}$$

Our aim is to find minimum variable cost per unit time.

The necessary and sufficient conditions to minimize $TC(T)$ for a given value T are respectively $\frac{\partial TC(T)}{\partial T} = 0$ and $\frac{\partial^2 TC(T)}{\partial T^2} > 0$.

Now $\frac{\partial TC(T)}{\partial T} = 0$ gives the following equation in T:

$$\frac{\partial TC(T)}{\partial T} = \frac{Ke^{-\gamma T}}{\theta^2 \gamma T(\theta - \gamma)} \Big[\alpha\theta(\theta - \gamma) - h\theta^2 \gamma \big(e^{\gamma T} - e^{\theta T}\big)$$

$$\alpha\theta\big(\gamma e^{\theta T} - \theta e^{\gamma T}\big) - \theta\gamma(\theta - \gamma)(h + \alpha T)$$

$$+ h\theta\gamma\big(\theta e^{\gamma T} - \gamma e^{\theta T}\big) - \alpha\big(\theta^2 - \gamma^2 - \theta^2 e^{\gamma T} + \gamma^2 e^{\theta T}\big) \Big] \qquad (7)$$

$$+ \frac{C_d K e^{-\gamma T}}{(\theta - \gamma)T} \Big[\theta e^{\theta T} - \gamma e^{\gamma T} - \gamma\big(e^{\theta T} - e^{-\gamma T}\big) - \theta + \gamma \Big] - \frac{TC}{T} = 0$$

provided that it satisfies the following condition $\dfrac{\partial^2 TC(T)}{\partial T^2} > 0$.

Again,

$$\frac{\partial^2 TC(T)}{\partial T^2} = \frac{Ke^{-\gamma T}}{T^2} \Big[\theta(\theta - \gamma)\big(h\gamma^2 T + \alpha T^2 \gamma^2 - 2\alpha\gamma T\big)\theta^2 \gamma\big(e^{\gamma T} - e^{\theta T}\big)(2\gamma hT - \alpha T + 2h)$$

$$+ \theta\big(\gamma e^{\theta T} - \theta e^{\gamma T}\big)(\gamma T - 2\alpha + 2h\gamma) + \alpha(\gamma T + 2)\big(\theta^2 - \gamma^2 - \theta^2 e^{\gamma T} + \gamma^2 e^{\theta T}\big)$$

$$- hT\theta^2 \gamma\big(\gamma e^{\gamma T} - \theta e^{\theta T}\big) + 2\theta\gamma(\theta - \gamma)(h + \alpha T) \Big] + \frac{C_d K e^{-\gamma T}}{(\theta - \gamma)T^2} \Big[(\theta - \gamma)\big((\theta - \gamma)Te^{\theta T} + \gamma T + 2\big)$$

$$+ 2\gamma\big(e^{\theta T} - e^{-\gamma T}\big) - 2\gamma^2 Te^{-\gamma T} - 2\theta e^{\theta T} + 2\gamma e^{\gamma T} \Big] + \frac{2(TC)}{T^2}$$

Since $\dfrac{\partial^2 TC(T)}{\partial T^2} > 0$ for the optimal value of T obtained from (7), it implies that Equation (6) has a unique optimal solution.

4. Numerical Example

In this section, we provide a numerical example to illustrate the above model.

Example 1: For the numerical illustration of the developed model, the values of various parameters in proper units can be taken as follows:

$$A_0 = 500, \quad K = 250, \quad \gamma = 0.02, \quad \theta = 0.8, \quad h = 0.5, \quad \alpha = 0.2 \text{ and } C_d = 1.$$

Solving Equation (7) with the above parameters, we obtain $T^* = 1.21537$. On substitution of the optimal value T^* in Equations (6) and (3), we obtain the minimum total cost per unit time $TC^* = 703.055$ and the economic order quantity $I_0^* = 506.575$ respectively.

5. Sensitivity Analysis

We now study the effect of changes in the values of the system parameters A_0, K, γ, θ, h, α and C_d on the optimal cost and number of reorder. The sensitivity analysis is performed by changing each of the parameters by 50%, 25%, −25% and −50% taking one parameter at a time and keeping the remaining parameters unchanged.

The analysis is based on the Example -1 and the results are shown in the **Table 1**. The following points are observed.

1) T^*, TC^* & I_0^* increase with increase in value of the parameter A_0. Here T^*, TC^* & I_0^* are moderately sensitive to change in A_0.

2) T^* decreases while TC^* & I_0^* increase with increase in value of the parameter K. Here T^*, TC^* & I_0^* are moderately sensitive to change in K.

3) T^* & I_0^* increase while TC^* decreases with increase in value of the parameter γ. Here T^*, TC^* & I_0^* are lowly sensitive to change in γ.

4) T^* & I_0^* decrease while TC^* increases with increase in value of the parameter θ. Here T^*, TC^* & I_0^*

Table 1. Sensitivity analysis.

Parameter	% Change in parameter	T^*	TC^*	I_0^*
A_0	+50	1.40557	893.468	638.848
	+25	1.3176	801.698	575.227
	−25	1.09229	594.818	430.865
	−50	0.935077	471.714	344.152
K	+50	1.04469	833.742	605.219
	+25	1.11906	771.785	558.399
	−25	1.34828	624.717	447.691
	−50	1.55256	531.163	377.697
γ	+50	1.22198	700.578	507.256
	+25	1.21866	701.818	506.912
	−25	1.2121	704.291	506.238
	−50	1.20885	705.524	505.901
θ	+50	0.995753	837.37	474.175
	+25	1.09322	771.429	489.626
	−25	1.37297	631.905	524.729
	−50	1.5835	557.627	542.943
h	+50	1.14468	753.981	462.205
	+25	1.17827	729.033	482.984
	−25	1.25668	675.914	533.659
	−50	1.30311	647.439	565.16
α	+50	1.19686	710.769	494.719
	+25	1.20595	706.946	500.52
	−25	1.22516	699.095	512.915
	−50	1.23533	695.061	519.553
C_d	+50	1.10828	782.717	440.295
	+25	1.15773	744.117	470.213
	−25	1.28385	659.001	551.955
	−50	1.36729	611.206	610.626

are highly sensitive to change in θ.

5) T^* & I_0^* decrease while TC^* increases with increase in value of the parameter h. Here T^*, TC^* & I_0^* are moderately sensitive to change in h.

6) T^* & I_0^* decrease while TC^* increases with increase in value of the parameter α. Here T^*, TC^* & I_0^* are lowly sensitive to change in α.

7) T^* & I_0^* decrease while TC^* increases with increase in value of the parameter C_d. Here T^*, TC^* & I_0^* are moderately sensitive to change in C_d.

6. Conclusions

In the proposed paper, an inventory model is developed which investigates the optimal order quantity of the on-hand inventory due to an exponential declining demand rate. The items (like food grains, fashion apparels and electronic equipments etc.) have fixed shelf-life which decreases with time during the end of the season and the storage period in which the demand, deterioration, and holding cost depend upon the time and shortage is not

allowed. This model is solved analytically by minimizing the total inventory cost. Finally, the proposed model has been verified by the numerical example along with sensitivity analysis.

In the future study, it is hoped to further extend the proposal model into several situations such as shortages being allowed and the consideration of multi-item problem. The suggested model can be extended for items having linear increasing demand, stock dependent demand, price dependent demand or power demand. This model can be extended to a three-parameter Weibull distribution or Gamma distribution deterioration. Furthermore, it may also take partial backlogging into account when determining the optimal replenishment policy.

Acknowledgements

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Notations

The following notations have been used in developing the model.

1) A_0: the fixed ordering cost per order..
2) $I(t)$: the inventory at time t, $0 \le t \le T$.
3) $D(t)$: the exponential demand rate where $D(t) = K e^{-\gamma t}$, $K > 0$, $\gamma > 0$, $K \gg \gamma$, are constants, K denotes constant demand rate and γ denotes the rate of change of demand rate.
4) $\theta(t)$: the constant deterioration rate of an item where $\theta(t) = \theta$ $(0 < \theta < 1)$ and $0 < \gamma < \theta$.
5) $h(t)$: the time-varying holding cost per unit time where $h(t) = h + \alpha t$, $h > 0$ and $\alpha > 0$.
6) C_d: the cost of each deteriorated unit.
7) T: the length of the ordering cycle.
8) I_0: the economic order quantity.
9) TC: the total cost per unit time.
10) T^*: the optimal length of the cycle.
11) I_0^*: the optimal economic order quantity.
12) TC^*: the minimum total cost per unit time.