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Archimedes' <Book> to Eratosthenes in the Palimpsest and Archimedes in Heron's Metrikon

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Abstract

It is argued that even with some new readings made by the publication of the *Letter to Eratosthenes* in the *Archimedes Palimpsest*, with the wonderful discovery of his so-called "mechanical method" (*a certain way of theorizing in mathematical things by means of mechanical entities*) at the beginning of the twentieth century, some important historical-philological and philosophical-epistemological issues still remain, which have already discussed in part in my writings. We produce some important testimonies taken from Metrikon by Heron of Alexandria in favour of our translations and interpretations of Archimedes' lexicon, not without placing under investigation at the same time the personality and the importance of Hero in the history of philosophical, scientific and technological Greek-Hellenistic thought in line with Archimedes and the tradition of Italic thought of science (The quotations of the Greek texts of Archimedes, Heron and Pappus are my translations).

Keywords

***Methodos* (Method), *Ephodos* (Methodics), *Tropos* (Way), *Theorein* (To Theorize), *Deiknunai* (To Prove), *Tomai* (Partitions), Italic Thought of Science**

1. Introduction

Archimedes died in 212 B.C. with the destruction of Syracuse and was born in 287 B.C., moving from ancient testimonies, according to historians.

His scientific production is immense and extraordinary, but in the sixth century A.C., it is already almost entirely forgotten. Only in the ninth and tenth century the Byzantine culture knows a flourishing rebirth. Three

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manuscripts of the works of Archimedes, the so-called Codes A, B and C, are produced

The first two survived the destruction and plundering of Constantinople by the Crusaders, in April 1202, in which hundreds of thousands of volumes were destroyed (this contained more books than men) and disappeared, only to reappear in the West with the Renaissance, and then disappeared again permanently after various vicissitudes.

Instead, the code C knows a particular story. Between the end of the eleventh century and the beginning of the twelfth century a scribe in possession of the code C decided to use it to write a Eucologion and to do this he scraped and tampered it.

For centuries no one had spoken of it. Only in 1906 Heiberg, a Danish scholar, had information that some mathematician writings were hidden in the manuscript-Eucologion, he went to Constantinople, and succeeded in large part in deciphering what there was written, publishing it, thus making it know to the world.

We know works by Archimedes disappeared in their original Greek language which was well-known but they were only translated into Latin. With the discovery of the letter to Eratosthenes, it seems to thin the air of mystery that has surrounded the mathematical research in Archimedes (Boscarino, 2010).

The manuscript with the disturbing events of the two world wars disappeared again. It reappeared in 1998 at an auction sale in New York and bought by a wealthy American just two million dollars.

Fortunately he delivered it to a research center in Baltimore, where a group of scholars using modern technologies restored the old manuscript, which was published in 2011 for scholars and for a wider reading public. The scholar Reviel Netz carefully studied the place that the code C would have had in the Archimedean production and studied the innovations that it brought in particular.

In his view, three novelties can be found in the rereading of the preface of the Letter to Eratosthenes.

The first concerns the title of the Letter.

The title that the great scholar Heiberg gave: *Archimedes*' *Method of Mechanical Theorems, to Eratosthenes*, (1913), in light of new more sophisticated observations, should be amended as follows: *Archimedes*' *book to Eratosthenes, concerning Mechanical Theorems*; *Method* (Netz, 2011).

Glimpsing a point of punctuation in the title, Netz so interprets:

The work as transmitted in Codex C carried two titles, *one an elaborate description* (*perhaps to be thought of as catalogue description rather than an actual title*), *the other the title itself—the single work* <*ephodos*>.[1]

The second novelty concerns the interpretation of a term used by Archimedes, when addressing to Eratosthenes he says that the new *mechanic tropos* would make him proficient in mathematical discoveries. Actually Archimedes, with this expression reread, not only would turn to Eratosthenes, but to anyone who wants to apply to mathematical questions.

The third novelty concerns the re-reading of a verb, with which, Archimedes, would not say that Eudoxus discovered (*ekseureken*) first the theorems mentioned in the letter, but that just published them (*eksenenke*). We should think that perhaps instead Democritus already possessed the proofs, so the growth of Greek mathematics should be rethought. Netz then concludes:

Most likely, *I think*, *Archimedes was simply projecting backwards his own scientific practice into that of Archimedes. … Apparently*, *he imagined Democritus doing the same*.[2]

I think Netz still moves, although in the novelties that he considers making the new reading of the Archimedean Palimpsest, inside old prejudices and misinterpretations.

Suffice it to note that he still translates during his speech the two Greek terms in the letter, the one in the title, *ephodos*, the other in the course of the entire letter, *tropos*, with the same term <*method*>, while he is experiencing then some discomfort in notes 89 and 91, about their correct translation (see p. 317, Vol. II, *The Archimedes Palimpsest*, Cambridge University Press, 2011), for which <*ephodos*>, in his opinion, would be better translated with the term <*approach*>, and <*tropos*> should be more properly translated with the term <*method*>.

Besides are we sure that the true title of the work was "*Ephodos*", as it seems to believe Netz, considering the other part of the title only "*work of cataloging*", when he headlines the first part of it rightly "<*The book*> *of Archimedes to Eratosthenes concerning mechanical theorems*", which is the name that Archimedes himself gives to his work in the course of the letter?

Thus we read in the Letter:

Seeing that you, as I say, are zealous and in an excellent way master of philosophy and that you also know

[1]*The Archimedes Palimpsest,* Cambridge, 2011, II, p 296.

[2]Ibidem p.297

how to evaluate in mathematical things the <observation> that is presented to you, decided to write to you and in the same <book> to expose the properties of a certain <way> (*tropos*) … *in order to have the ability to <observe>* (*theorein*) *in mathematical things by means of mechanical beings*.[3]

Or should we not suppose instead that the term "*Ephodos*" is a later addition, given that during the Letter-foreword and work Archimedes never use it, but only he uses the term *tropos*, which more than a simple method, a mere way of discovering (*euriskein*), seems to indicate not only a way of discovering but also a way of building the theory, which is his "*theorein in mathematical things*", then of proving (*deiknunai*), through mechanical entities?

And what about the important and one of the oldest testimonies on this work, that of Heron, I am referring to his *Metrikon*, where the work of Archimedes is denominate <*En to ephodikò*>, which seems rather to send to a book which discusses issues of method, of methodology, that then is the good translation that Heiberg gives in his first German translation, in the 1906-1907 in *Bibliotheca mathematica* of the Archimedean work, of *ephodos = methodenlehere = doctrine of the method*? (Heiberg, 1906-1907)

That the title of the work could be "<*book*> on mechanical theorems to Eratosthenes" we can deduce from an indication that it is given by two propositions, prop. 14 and prop. 15, of the Book II of Heron's *Metrikon*, or by the two expressions used by him about the two famous theorems proved by Archimedes in his <*book*>, on the so-called cylindrical nail and the common segment of two cylinders intersecting in a cube, with their bases placed on the cube.

Heron writes in the book II proposition 14: *Archimedes proved in the Ephodikon* while in the proposition 15 he writes just referring to <book>: *The same Archimedes in the same book proves* (Heronis Alexandrini, 1903).

2. The Propositions of the <*Book of Archimedes on the Mechanical Theorems*>: Simple Heuristic Results or True Theorems, Rigorous Proofs

On the myth of an Archimedes who with his mechanical *tropos* limited himself only to "discover" but not to "prove", in his propositions (Heath, 1912; Boyer, 1982; Frajese, 1974) forcing the interpretation of his expression, *without proof*, during the letter-preface, we have said in other writings quoted by us.

We will limit ourselves only to register as in a recent Italian translation, made it is said on the new text delivered to us by reading the Palimpsest, just to adapt the Archimedean propositions to this presumed interpretation, not only they commit trivial writing errors, manifest forced translations, far from the Greek text, but also they neglect or consider interpolations the most numerous expressions that contradict the presumed interpretation of the expression of the letter-preface quoted by us, the *without proof* (Acerbi, 2013).

If in the letter-preface Archimedes, referring to his mechanical propositions, says of them twice that are to be placed out of a proof context, of geometric way, (*gheometroumenas*), many more times he refers to them deeming <proofs>. See conclusions of the propositions, 2, on the sphere (*as it was to be proved*), 3, concerning the spheroid, indicating the canonical *how it is to be proved*, with the initials *OI*, 4, concerning the rectangle conoid, with the same *OI*.

We read then as in the expression of the Letter-preface the mechanical propositions are placed by the same Archimedes together with the geometrical propositions indiscriminately inside a procedure of proof. Archimedes indeed writes, after finding the proofs in relation to the statements before sent to Eratosthenes: *In this book* (*once* again Archimedes uses the term "book" to indicate his writing; our?) *therefore I send you the written proofs of theorems*.[4]

You can read again in the proposition 12 on the so-called cylindrical nail, even after using the canonical <*theoreitai = we theorize*> according to the mechanical way, as Archimedes refers to it by saying: *Proved these things we will come back to this proof geometrically*.[5]

They argue that all these expressions are possible interpolations, as in contradiction with the expression, *koris-apodeikseos*!

But why is it not to consider instead wrong the interpretation given to this expression, in comparison with the many expressions used by Archimedes and that we reported in which he calls his mechanical propositions <*apodeikhseis =proofs*) not considering them mere discoveries, but also theorems, proved otherwise, *out of a purely geometrical proof context* (*koris*), which seems the true meaning of his <*koris-apodeikseos*>to us?

[3]Ib. p. 71, 33-36, 1-8.
[4]Ib. p. 71, 30-32.
[5]Ib. p. 111, 22-24.

Metrikon's Heron, an ancient and authoritative witness of Archimedean things, seems not to corroborate the presumed epistemological dualism in the Archimedean production, dividing between works of simple discovery, heuristics works, and works of rigorous proofs. In fact, when he quotes the Archimedean discoveries in his Metrikon, he uses the term <to prove = *deiknunai*) indiscriminately, when these are referring both to works of geometric type and mechanical one, in our case to the Archimedean work, *En to ephodikò*.

In fact, in the book I, prop. 34, he writes, referring to a work, written and theorized geometrically, relatively to the measurement of an ellipse: it was proved in the *Conoides* of Archimedes, while in the same book, prop. 35, this time referring to a work, written and theorized in a mechanical way, regarding the measurement of a parabola, he writes: it was proved in the *Conoides* of Archimedes (Heronis, 1903).

To <*theorein*> Archimedes does not give the mere meaning of <discover, investigate >, as it seems to interpret Heath (Heath, 1912), contrasting it to the authentic, geometric <to prove> (*deiknùnai*), as just when he frames his mechanical propositions inside the aforementioned < *it is theorized with this so that*...>, he then concludes with the already mentioned also < *how it was be to prove*>.

So Archimedes, like then Heron, considers the mechanical propositions, such as geometric proofs, <proofs>!

That Archimedes is convinced to have built a fruitful mechanical theory, with premises (*prolambanomena*) and consequences (*theoremata*), to be delivered, by publishing, to future mathematicians, because they can enrich it with new theorems, and therefore it has nothing to reproach itself in terms of its proved ability, it can be inferred from his latest remarks of his letter, where he writes: *I support in fact that some of today and tomorrow will discover thanks to the way that I showed from proofs* (*apo-deikhthentos*; our translation of the Greek term; in Acerbi the term disappears) *other theorems we do not yet see*.[6]

To Archimedes his mechanical theory or *mechanica*l *tropos* allows not only to discover and prove geometric theorems, but also mechanical theorems, related the centers of gravity. He is "the founder of rational mechanics", whose practice Pappus attributes to Heron and his followers.

3. The *punctum dolens* of the Mechanical Propositions: The Use of Infinitesimal Sections

But they say: Archimedes, both in his mechanical theorems and in the strictly geometric ones, especially in the proposition 14, sums "*infinitesimal sections*", improperly using his assumption 11 (Prop. 1 of his work *Conoids and Spheroids*). For this he would judge his geometrical proposition 14 and all other mechanical propositions "*without proof* = *koris-apodeikseos*" (Acerbi, 2013).

Meanwhile Archimedes relatively to the two theorems discovered (cylindrical nail and intersection of two cylinders with the basics in a cube), at a time when he is very pleased to have found the way or *tropos* of equaling a solid figure included by plans to a solid figure included by curved surfaces, clearly affirms: <*Here in this book I inform you the written proofs* (*prop*. 12, 13, 14, 15) *of these theorems*>, as well as at the end of the letter, with respect to the two theorems, of which he had sent to Eratosthenes only the statements, he writes: *at the end of the book we write the proofs by geometric way of those theorems of whom we sent before the statements*.[7]

So with regard to the statements of the two theorems, of which he asked to Eratosthenes to give the proofs about these, Archimedes is clearly certain to give the proofs about these in the rest of his book.

Then Archimedes widens his remarks, saying that many of his statements with proofs were found by him, as for the two aforesaid, before in a mechanical way, or outside a theoretical-proved context of geometrical way, not excluding a theoretical-proved context of mechanical way.

The sections or better the partitions (*tomai*) that Archimedes uses both in the mechanical context and in the geometric context are in the mechanical case weight-sections, so *magnitudes, or parts*, of which he is not interested in the form, in the geometric case still magnitudes-sections, weightless, but still always parts, according to the theory of the magnitudes of Euclidean way, of which still he is not interested in the forms, not so "*indivisible magnitudes*", which can be considered as such only conceptually (Boscarino, 2010, 2011), but that such these are not, even less "*infinitesimal sections*", of which we are not yet interested in the form, but only in their numerability and their numerical ratios between homogeneous magnitudes

The "*whole*" (*ta panta*)" of the assumption 11 of the book does not refer to a supposed "*actual infinity*" or a supposed "*potential infinite*", but to "*how many magnitudes you want*", that is, if you want, a concept of static

[6]Ib. p. 76, 6-10.
[7]Ib. p. 73, 18-22.

nature, in which the partitions (*tomai*) neither grow nor diminish, but are from time to time given, determined, and of finite number and of finite magnitudes.

Terms such as *actual infinite* or *potential infinite* as the term *indivisible magnitudes*, all of Aristotelian matrix cannot be attributed arbitrarily to Archimedes, which among other never uses them, without thereby degrading the Archimedean speech to inconsistent and absurd proves, which among other things would offend not only his genial rationality but also his extraordinary legacy, that he first believes to leave to the mathematicians of the future, together with his geometrical and mechanical way to investigate and prove quadratures and cubatures.

Archimedes' *mathema* is *physis*: his composing (*sugkeisthai*) in geometrical proposition 14 as in the other mechanical propositions is filling by putting together (*sunpleròo*), making geometric forms full (lines, surfaces and volumes), imagined empty, with sections (partitions)-magnitudes (*tomai-meghetes*) (Netz's wonder and fascination, 2007, leave perplexed when he glimpses the word *meghetos* through violet rays in the missing part of prop. 14 in that rebuilt by Heiberg, while it was logical that there was, given the Archimedean assumptions), in turn these imagined homogeneous, in which the ratio weight-geometric figure is equal to 1, while for their partitions, as magnitudes, and for their numerability, since equal in multiplicity (*isois to plethos*), are measurable.

4. What Is Archimedes' Image in the *Metrikon*: A Platonic or an Inventor of Methods (*Methodoi, Ephodoi*) of Measurements

Archimedes is the most present figure in *Metrikon*, while Euclid is not nominated there, if not for his theorems, as well as Hipparchus is not nominated there, if not for his theorems <*On the chords in the circle*> (see prop. 22 and 24, Book I) and Apollonius, about his theorems of his work, that we know for a quotation by Pappus (See prop.10, prop. 13 and prop. 15, L.III., Heronis, 1903).

Eudoxus is nominated twice only in the introduction to the Book I; only once Dionisidorus, about his theorem for measuring the volume of the loop (see prop. 13, Book II), and Plato, about the five solid said of Plato, Book II. (Heronis, 1903).

In Book I, which deals with the measurement of superficies, Archimedes is mentioned in the introduction, and in prop. 25, about the measurement of the circle, in the propositions 30, 31, 32, about the measurement of segment circle, in the prop. 34, about the measurement of the ellipse, the prop. 35, about the measurement of the parable, in prop. 37, about the measurement of the surface of an isosceles cone, in prop. 38, about the measurement of the surface of the sphere, in prop. 39, about the measurement of the surface of the segment of a sphere (Heronis, 1903).

In Book II, which deals with the measurement of the volume of solids, Archimedes is quoted in the prop. 11, about the measurement of the volume of the sphere, in the prop. 12, about the measurement of the volume of the spherical segment, in prop. 14, about the measurement of the cylindrical nail and in prop. 15, about the measurement of the intersection of two cylinders with the bases in a cube (Heronis, 1903).

In Book III, which deals with the division of surfaces and bodies, according to a given ratio, Archimedes is quoted in the prop. 17, about the division of a spherical surface with a plan, according to a given ratio, of the surfaces of spherical segments, and in prop. 23, about the division of a given sphere with a plan according to a given ratio of spherical segments.

The quoted works are: *Measurement of the circle*, *On Conoids and Spheroids*, *On the Sphere and Cylinder*, *En to ephodikò*, as I said, the work was lost over the centuries, and found itself with the Palimpsest, *On Plinths and Cylinders*, work instead lost (Heronis, 1903).

Heron's work and particularly his work *Metrikon*, the object of our attention, has been variously interpreted, not to say of the great historical difficulties that created the historical position of our personage, who, apparently after many diatribes, is to be placed around the first century after Christ.

The opinions of historians were among the most varied about his work, deemed not always of certain attribution, sometimes manipulated and with interpolations, usually simply syncretistic work by erudition, collection, of value only encyclopaedic and confused testimony of certain technical and scientific culture of the first imperial age.

If Boyer (1982) seems to underestimate his work, his Metrikon, considering it most of Babylonian matrix and Kline (1991) more of Egyptian matrix (2) that Greek, of type, it is called, "classic", coarsely of practical nature, otherwise we find instead opening and attention for a number of reasons, in the historic Heath, who writes:

The Metrica is the most important from our point of view because it seems, more than any the others, to have *preserved its original form. It is also more fundamental in that it gives the theoretical basis of formulae used, and is not a mere application of rules to particular examples. It is also more akin to theory in that it does use concrete measures, but simple numbers or units which may then in particular cases be taken to be feet, cubits, or any other unit of measurement* (Heath, 1981).

It seems to us questionable, however, his observation about the Archimedean tradition, held by him now very cloudy, at the time of Heron, when he writes:

The preface to Book II is interesting as shoving how vague the tradition about Archimedes had already become (Heath, 1981).

It seems to us even more positive the opinion of Gino Loria on the role played by Hero in the history of Greek mathematics and for a strong presence in all his work of high aspects of rigor, elegance and originality, for which he writes:

Heron appears not as a Newton, creator of infinitesimal calculus, but rather as a Euler who reorders supplementing and perfecting the fundamental doctrines of it. The pages that Heron devotes to such exposure (*Calculate the area of a triangle whose sides are known*)*, for elegance and rigor of argument and importance of results can compete among the most beautiful ones written during the golden age of Greek geometry and do not substantially differ from those found in the DIOPTRA on the same theme.The general impression that the Metrika produce in the reader would say that they present numerous chiaroscuros*, *which perhaps can be explained by admitting many disfigurements of the text by the scribes*: *from passages of unquestionable originality and permanent value you pass*, *without transition*, *to pages of pure and simple compilation*; *from ingenious reasoning to simple numerical applications of methods of others. If then the Metrika cannot stand the comparison with the masterpieces of ancient geometry*, *they contain many pages that today still could usefully be introduced into a discussion of elementary geometry*; *for which they explain and confirm the high regard their author has enjoyed in ancient times* (Loria, 1914, our translation from Italian).

Today the figure of Heron is increasingly object of study and appreciation (Vitrac, 2008b, and another online).

The proofs of Archimedes are referred to, because they allow easy numerical operations of measurements, in order to serve for useful construction works, because, as Heron writes, was the utility that permitted to develop geometry since its beginnings, forcing it to expand more and more its field of application and study.

The investigation (*epìnoia*) of Eudoxus and Archimedes, but above all the comprehension (*synesis*) of Archimedes, namely the ability of Archimedes to go beyond the apparent divisions of different geometric forms, up to grasp the intimate numerical relationship, such as to know how to submit to measure the different forms that the nature provides, have impressed an excellent leap forward in the science of measurement and division.

Heron then, in that anticipating our progressive and cumulative modern conception of science, wants to be the heir of passed science, and he himself an investigator of geometric things and of measurements, building adequate theories. In short, his work wants to be not only applicative but also theoretical science. He writes: *Now*, *since the said study is essential, so we have decided to collect everything that our predecessors wrote and that we ourselves have also investigated theoretically* (*prosetheorésamen*) (Heronis 1903; Vitrac, 2008a).

The picture that emerges from the reading of the Heronian work about Archimedes is certainly that of an author interested in works of proofs, but is especially the image wider of an author interested in physical operations of measurements, and for this inventor of measurement methods, not stopping at methods of measurements of regular, perfect forms, according to the Platonic image, passed down by Plutarch, but going beyond, to the measurement of the most irregular, empirical forms, not just ideals, inventing appropriate methods for them, though not rigorous, approximate.

Read in this regard what he writes both in the introduction to the Book II and in the conclusion at the same book:

After the measurement of the rectilinear surfaces and not, after we have to pass to solid bodies, of which in the previous book we measured both the flat surfaces as the spherical ones, and also the conical and cylindrical ones, and then the irregular ones, the investigations (*epi-noìas*) *of which, as things of not common opinion* (*paradòksous*)*, they who tell about the succession traced them back to Archimedes. Whether they are of Archimedes or they are of someone else, it is necessary to present these in writing, so that this Treaty does not contain gaps about anything for those who intend to deal with these things.*

After measuring the irregular bodies, we believe we make at least a brief reference on the measurement of those non-regular, what would be the roots of trees and pieces of marble, as some report (*istorousi*) *that Archimedes has discovered* (*epinenoekenai*) *a method* (*methodos*) *for these things. If in fact the body to be measured* *is easy to transport, we'll get a tank having the form of a rectangular parallelepiped, able to contain what is to be measured, we will fill it of water and will dive into it the irregular body.*

Now it is clear that consequently a portion of water will be released from the bathtub; when that body will be extracted from the bathtub, from this an amount equal to the volume of the immersed body will miss. If now we measure the space of the bathtub remained empty, we get the searched volume of the immersed body. But the same measurement can be achieved in another way. It covers, in fact, the irregular body of wax or clay, so that, after covered, appears rectangular; let's measure it under this new aspect, then we remove the wax or clay, added, and we place it in the new form of parallelepiped, we measure the resulting volume and subtract it to the measured volume, so the volume of the body will result. This method (methodos) is recommended in the case of bodies which cannot be transported (Heronis, 1903).

The interpretation then that someone wants to give of the introduction to the Book III, once again in Platonic key, it seems to us a forcing (Guillaumin, 1997). To us instead it seems an interpolation or a result of later manipulation, so in it the style appears confused and vague, compared to introductions of the other books, clear and sober, where intrusions of ethical and ethical-political nature do not appear.

The Archimedes by Heron appears in short more a Pythagorean, interested in the mathematical number, ratio of magnitudes, than in the ideal, existing in itself, metaphysical number,, of Platonic ascendance, which was what divided the Pythagoreans from the Platonists according to testimony of Aristotle (Boscarino, 1991).

From Heron among other things we learn that Archimedes, in search of a method for measuring the circle, more rigorous than that of the ancients, had to cross in his investigations different methods, if he testifies that before in his work *On Plintides and cylinders* (Heronis, 1903), about the ratio circumference/diameter gives a certain ratio correcting it then introducing one more accurate ratio, that is then what is accepted in his own time, or the ratio Circumference = 3d + 22/7d (d = diameter), in which it is to note that the famous Greek pi is not considered a mere number, but a ratio between magnitudes, a mathematical number, not an ideal number.

In short yet in Heron as in the Archimedes the *mathema* is not conceived as pure abstraction in the Aristotelian sense, divorced from the relationship with the physical world, or pure ideal, metaphysical existence in the Platonic sense, but the relationship between physical entities, magnitudes.

Not for nothing in his measurements Hero puts next to the numbers the monads, the units of measure, which, if in their ideal reality, give to the speech of Heron a theoretical feature, as stated by Heath (1981), they can from time to time take an applicative and building feature in the various physical contexts.

That Heron's measurements of Archimedean ascendancy serve for works of building you can see it over and over by many conclusions that Heron draws from those, as in the cases of prop. 25, L. I, in which he writes:

If you have to build a circle, in the case where it is given a space portion formed straight or in any way, which is equal to that, we take the area of the space portion, and it is of 154 *monads, from this we take* 1/11 *of* 14, *which is* 196, *and again we take the square root of this, which is* 14 *monads. So great we will indicate the diameter of the circle.*

or of prop. 15, L. II, in which he writes:

This proposition is usable in the case in which it is to be built in this way vaults, that occurs mostly in bathrooms and sources, if the entrances and the windows are in all four sides, and where it is not possible that the places are covered by beams> and of other still (Heronis, 1903).

5. Archimedes, Heron and the Tradition of Italic Thought of Science

Today the figure Heron is increasingly object of study and revaluation, and to do that we must overcome prejudices and mediations, in order to restore a more truthful image of Heron, in our opinion, i.e. not only of person of certain mathematical and philosophical-speculative thickness, but also of a person inserted in a vast cultural trend, in a tradition of thought, that bearing in mind the unity of knowledge, resisted, in the course of ancient history, to its disintegration as to its alteration in hierarchical sense, with which other hierarchies were to justify, we mean the social and political ones.

Our interpretation may become the reply to what in acute and disquieting way, in the face of historical superficial, obtuse judgments and preconceptions about Heron, L. Geymonat poses in his writing of the history of mathematics, when, concluding his presentation of Heron, about that he calls "*Heron's the operational turning point*" towards the Euclidean spirit, marked by geometric purism, asks himself:

Is it possible to explain such an important change, produced in Greek science, referring only to the new prac-

tical requirements surfaced in the school of mechanics of Alexandria? *Or have we to admit that it represents something more complicated and deep*? *Does it reveal the presence*, *in the history of Hellenic mathematics*, *of a trend different from the official one*, *or neglected because of the reputation achieved by the latter one*, *but spite of that full of an actual vitality of its own* ? (Geymonat, 1965)

Well we have to admit, as we have tried to demonstrate in many of our writings, that since the seventh-sixth century B.C in the history of Greek science two traditions of thought have faced on how to conceive science and mathematics, in particular, in our case of the *Metrikon* (Boscarino, 1999, 2014, 2015).

If on one hand in the Italic thinkers such as Pythagoras, Parmenides, Archytas, Eudoxus, Democritus, Archimedes, the unity of knowledge was kept fixed, in which the science-philosophy was combined with different types of knowledge as we call them today (mathematics, physics, astronomy, etc.), not separating them from their practical, technical use, not creating a hierarchy between them, for perfection, on the other hand, we refer to Plato and Aristotle, the knowledge that has been separated by creating between them a hierarchy of values about the subject that they treated (Boscarino, 1999).

So if for Plato the objects of mathematics were pure, perfect objects, existing eternally in themselves in a supercelestial world, objects of a higher science, geometry itself even higher of arithmetic, compared to which even in higher was dialectic, the highest knowledge of all, against those for Plato there were sensitive objects considered impure, imperfect and existing in a transient, ephemeral way, with which the lowest knowledges, represented by the various arts, were concerned. Not differently the things were for Aristotle, who believed that mathematical objects were separated and existing in a perfect way only in an abstract world, which can never represent the sensible reality, but only, as also for Plato, imitate it, ape it, but not innovate it, enrich it with new objects, or create new machines (Boscarino, 2012).

For the first instead the *mathesis*, the mathematical knowledge, derives from the multiform, confused and sensible world some properties, changes them into ideas, the properties as such, or into elements and composes them then, according to mathematical logic, which is not the logic of common sense, which is that of Aristotle, in a theory, or in a new observation; thereby it also creates a new physical world, beyond the mere sensible world, in which a philosophy with its elements and principles is combined with a science with its own set of physical properties, with its precise geometric forms, related to one another by numeric ratios, which capture the inner essence of it.

Physical reality, no longer being, nor a mere, fixed and absolute metaphysical reality, existing in a supercelestial world, then inaccessible to human action (Plato), or a mere abstract reality, to which still, beyond the sensible world correspond essential unchangeable forms, therefore making again useless and powerless the human action (Aristotle), becomes instead a precise theoretical context, in which elements, geometric forms, laws, and precise numerical ratios are related, through which we can now change it, invent new tings, use its own laws to create machines that help and strengthen the human action.

Let us read what Heron writes in this regard in one of his passages at the beginning of his *Pneutikon*:

About the actions to air (*pneumatikespragmateias*) *estimated worthy of study by the ancient philosophers and mechanics*, *the first of which showed the power* (*dynamin*) *of this from the rational point of view and the seconds from the point of view of the same energy* (*energheias*) *of sensible things*, *we think it is necessary that even ourselves put in order that was transmitted by the ancients*, *adding our own discoveries*; *so those who want later to turn that towards the mathematics could find some use in these* (Heronis, 1903).

Here for Heron natural being does not consist only of essential forms or of merely potentialities, as the ancient philosophers thought, in which we can place Plato and Aristotle, but also of artificial entities, produced by the *energheia*, that internal natural strength and external human one, thanks to which we can produce new objects and machines not occurring naturally.

The otherwise philosophers, those that Heron called the mechanics, then not only innovate on a different concept of physical reality with the concepts of element and principle, as in the case of the concept of *vacuum*, in Hero, when they alter the meaning of these, from the point of epistemological and philosophical view, so they build, not a static theory of forms, but a new dynamic theory, rich not only of old discoveries to keep by transmitting them to the memory of the men, but also of possible future discoveries mixing them more and more with the *mathesis*.

In short, we find in Heron what at the beginning of the scientific revolution in the sixteenth century Newton had advocated refer to Pappus, which among other things recognized in Heron him who had mixed well philosophy, mathematics and mechanics in his concept of science.

Let us read before the testimony of Pappus on Heron and his followers:

The mechanics Heron's followers say that a part of the mechanics is rational, the other party applied, and that the rational part is composed of geometry, arithmetic, astronomy, and of the stages of physical things, while the applied part includes the art of working copper, the art of building, the art of making with wood, the art of painting, and the manual operation of these arts (Ver Eecke, 1933).

And yet of the same Pappus:

The mechanical theory, or my son Ermodorus, being useful to many and important things that occur in life, rightly deserves the greatest favor among philosophers and is the ambition of all mathematicians, because first it deals closely the physiology of the material elements in the world.... Some say the Syracusan Archimedes recognized the cause and demonstration (*ton logon*) *of all these things*; *this indeed until our days was the only* (*monos*) *that owned skilled nature and understanding for all things, as also Geminus the mathematician stated in this way in his book The ordering of mathematics. Carpus of Antioch instead says somewhere that Archimedes of Syracuse has composed only one book of mechanics, that one concerning the Sferopea and that he has not deigned himself to compose others of the same kind....*

That wonderful man, to tell the truth celebrated by most for the mechanics, endowed with high nature, to the point of continuing to be extremely lauded by all people, also collected in writing so remarkable (*σπουδαίως*) *theories that appeared very short* (*τὰβραχύτατα*) *on emerging issues of geometry and arithmetic, and it is certain that he loved the sciences* (*ἐπιστήμας*) *that we have mentioned so far as to impose himself not to introduce anything from outside* (*μηδὲν-ἔξωθεν*).

But the same Carpus and some others have sided with geometry in a rational manner and to the benefit of certain techniques; *geometry in fact does not degrade at all, when, applying to these, tends to increase structurally many techniques*; *and on the contrary it appears to raise these techniques and so honored and decorated as befits to the same* (ibid).

Thus what Newton wrote early in his first preface of 1686 to masterwork *Mathematical Principles of Natural Philosophy*:

Since the ancients (*as it is said by Pappus*) *had the utmost account of the mechanics in the investigation of natural things, and the moderns, abandoned the substantial forms and the occult qualities, tried to subject the phenomena of nature to mathematical laws, I thought it appropriate in this treaty to cultivate mathematics regarding philosophy* (Newton, 1997).

In the mathematical scientific rationality of Pythagoras, of Archytas, of Eudoxus, of Democritus, of Euclid, of Archimedes, who only refers the last two in his writings, then the different types of knowledge are mixed without a hierarchical position, such as philosophy, mathematics and mechanics, while in the scientific rationality not mathematical these are broken, creating a separation and a hierarchy of values which then ultimately serve the purpose of social conservation, common and religious sense and of the tradition.

Not for nothing, according to the testimony of Plutarch, Plato condemns the mixture of mechanics and geometry in Archytas and Eudoxus, when on these Plutarch writes:

The initiators of mechanics, science followed with interest and well known to all, were Eudoxus and Archytas, who communicated a great charm to geometry through the elegance of its proceedings. They gave the support of visual and mechanical to problems that did not offer possible solutions with a procedure only logical and verbal. For example in the solution of the problem of two proportional medium straight lines, necessary element to the composition of many figures, both scientists resorted to mechanical means, using the proportional means that some instruments derived from curved lines and segments. Plato was indignant for this approach and polemized with the two mathematicians, almost as they destroyed and corrupted all that was good in geometry; *in fact in this way it abandoned abstract concepts to fall in the sensible world, and is also used widely objects requiring a rough manual labor. Mechanics was thus separated and broke away from geometry*; *for a long time philosophy ignored it and it became one of military arts.* (Plutarch'Lives, Marcellus)

Not for nothing Aristotle still writes in his *De Caelo*:

It is good to be convinced of the truth of the ancient doctrines heritage for the excellence of our fathers according to which there is something immortal and divine …

The ancients assigned the heaven and the upper region to the goods, considering them as the only immortal region, and the same discourse that now we are doing certifies that the heaven is incorruptible and engendered,... (I, 284a).

In the end Aristotle concludes:

only in this way we will be able to profess theories which agree with what oracular science tells us about the divine (I, 284b).

6. Conclusions: Archimedes and Heron, Enlightenment Thinkers, Beyond Their Time

For Aristotle, therefore, scientific rationality not mathematics must move in a circle; as the tradition, common sense with its words, which equals the real with the sensitive, and religion with its beliefs, attest, is found by the scientific and philosophical reason and vice versa. Scientific research shows all that the common language says, without the rational knowledge, and religion believes without the mathematical reasoning. Indeed it cannot profess theories that conflict with these.

Instead, in Heron we find a scientific, mathematical, open, progressive, modern rationality, from the beginning of his work, *Metrikon*, when he writes:

The primitive geometry, *as we learn from the ancient narrative*, *took care of the measurement and the division of land*, *and this was called geometry*. *Then becoming more and more useful to men*, *its scope was extended*, *so the discussion of measures and divisions was extended to solid bodies*; *and since the first theorems discovered were not enough*, *further investigation were therefore required*, *that still up today some things remains to find*, *although Archimedes and Eudoxus have treated the object of study so well* (Heronis, 1903).

As mentioned in the introduction to the *Metrikon* only Archimedes mentioned is, with Eudoxus, of Italic tradition. In short, we can consider Heron and Archimedes, the precursors of modern thought, indeed the authors of Enlightenment thought beyond their time.

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