Adaptive Tropospheric Delay Modelling in GPS/INS/Pseudolite Integration for Airborne Surveying

Jianguo Jack Wang and Jinling Wang
School of Surveying and Spatial Information Systems, University of New South Wales, Sydney, NSW 2052 Australia

Abstract. Integrated GPS/INS systems have been used for geo-referencing airborne surveying and mapping platforms. However, due to the limited constellation of GPS satellites and their geometric distribution, the accuracy of such integrated systems cannot meet the requirements of precise airborne surveying. This problem can be addressed by including additional GPS-like ranging signals transmitted from the ground-based pseudolites (PLs). As GPS measurement geometry could be strengthened dramatically by the PL augmentation, the accuracy and reliability of an integrated system can be improved, especially in the vertical component. Nevertheless, some modelling challenges exist in PLs augmentation. As PLs are relatively close to receivers, the unit vectors from a PL to he reference and rover receivers can be significantly different. PL tropospheric delay modelling errors cannot be effectively mitigated in a differencing procedure. Furthermore, PL signals propagate through the lower troposphere, where it is very difficult to accurately model the signal delay due to temporal and spatial variations of meteorological parameters. In this paper, an adaptive PL tropospheric delay modelling method is developed to reduce such modelling errors by estimating meteorological parameters in a model. The performance of this adaptive method is evaluated with field test data. The testing results have shown that the PL tropospheric delay modelling error can be effectively mitigated by the proposed method.

Key words: Pseudolite, tropospheric delay, integrated navigation, adaptive modelling, airborne surveying

1. Introduction

GPS aided positioning systems have been widely used for airborne surveying, which can reduce the number of ground control points dramatically (Abdullah et al., 2002; Kwon and Jekeli, 2001; Lewis, 1996). Whilst GPS pseudo-range positioning is utilized for real time navigation during aerial image acquisition, carrier phase differential GPS (DGPS) positioning is applied for determining the camera exposure positions. However, the accuracy of camera exposure positions is normally much worse than that of the DGPS solutions, due to a number of factors, such as a low GPS positioning data rate, the lever arm between the GPS antenna and the camera exposure centre, and airplane attitude variations etc. Therefore integrated GPS/INS systems are introduced for direct geo-referencing, which needs both very accurate position and attitude information for the platform (Cramer, 2003; Grejner-Brzezinska, 1997; Kwon and Jekeli, 2001).

In integrated GPS/INS systems, accurate DGPS measurements are used to estimate and correct INS errors. High data rate INS measurements then provide accurate position and attitude information between the GPS updates, which can also be used for GPS cycle slip detection and repair. However, due to the limited constellation of GPS satellites and their geometric distribution, the accuracy of GPS positioning in the vertical coordinates is much worse than that of the horizontal components. Integrated GPS/INS systems cannot meet the requirement of direct geo-referencing for precise airborne surveying under undesirable operating conditions.

Ground-based PLs can be used to transmit additional GPS-like ranging signals, which can strengthen the measurement geometry dramatically for airborne GPS based geo-referencing systems. As a result, positioning accuracy and reliability can be improved. PL signals can also improve integer ambiguity resolution, which is the key for carrier phase based precise GPS positioning (Lee et al., 2002; Wang, 2002).

Unlike the case in GPS, the unit vectors from a PL to the reference and rover receivers can be significantly different as it is relatively close to these receivers. GPS tropospheric delay can be mitigated in the differencing
procedure, which is due to the fact that signal propagation paths from a satellite to a ground reference station and a user receiver are largely the same. However, the paths from a ground based PL to a reference and to a user receiver can be significantly different. Accordingly PL tropospheric delay modelling errors cannot be effectively mitigated in the differencing procedure.

In carrier phase differential GPS/PL processing, the residual tropospheric delay may be the largest remaining error source. This is primarily due to spatial variations of atmospheric parameters and dense water vapour in the lower troposphere, where PL signals propagate through. The accuracy of common modeling methods using the measured or standard meteorological data is poor because such data is inaccurate and thus, not representative for the entire environment of a site (Bernese, 1999). The variations in atmospheric condition can easily change the PL differential tropospheric delay by the order of ± 20 cm per kilometre (Barltrop et al., 1996). Even more momentous for airborne applications, tropospheric delay is also strongly dependent on the vertical differences. Without real time meteorological data to represent the ambient atmosphere in the tropospheric models, we turn to develop an estimation method for the meteorological parameters instead.

The impact of the local meteorological parameters for the GPS and PL tropospheric delay estimation is demonstrated in Fig. 1. It compares the carrier phase double differences between two GPS satellites (SV28 & SV7) and between a PL (SV12) and the reference GPS satellite (SV28), with a short baseline flight test data set. Fig. 2 shows the aeroplane trajectory of the data used for producing Fig. 1. Different relative humidity values were applied in both GPS and PL tropospheric delay models. Based on the results shown in Fig. 1, it is noted that the double differenced PL tropospheric delay is much more sensitive to local meteorological data than that of GPS. Therefore, advanced PL tropospheric delay estimation strategies should be applied in the PL augmented GPS/INS system to achieve high accuracy positioning results.

This paper introduces an adaptive tropospheric delay estimation method for an integrated airborne GPS/INS system with the PL augmentation. It is well known that, comparing with the GPS/PL measurement update rate (normally 1 Hz), local meteorological data are slowly varying parameters. The proposed method estimates these parameters for a PL troposphere model with the measurements from previous epochs. Then the estimated parameters are applied in the PL tropospheric delay model for subsequent epochs. The quality of parameter estimation can be continuously monitored with the GPS measurements. The test results prove that the PL tropospheric delay modelling error can be effectively mitigated using this adaptive method, which is demonstrated by comparing the positioning results of an integrated system with and without the proposed method.

The paper is organized as follows. PL tropospheric delay models are explained first. The methodology of the adaptive PL tropospheric delay modelling method is then proposed. Field test data are processed to evaluate the proposed modelling method. Test results demonstrate the effectiveness of the proposed methods for the airborne GPS/PL/INS integrated system.

2. Pseudolite Tropospheric Delay Models

Tropospheric delay is defined as the integration of local refractivity along a signal path (Hofmann-Wellenhof et al., 2000). However, as it is impossible to measure local meteorological data or refraction index continuously along a signal path, many existing models estimate the tropospheric delay with the meteorological data at the observing site. This data set does not reflect the troposphere condition along the signal path, thus causing
modelling biases in the estimation. The proposed method estimates the meteorological data with the GPS and PL measurements, instead of using the meteorological parameters measured only at the reference point, allowing the modelling bias to be reduced using the proposed adaptive estimation method.

By assuming that the neutral atmosphere is both horizontally stratified and azimuthally symmetric, PL tropospheric delay models are normally a function of local refraction index (Biberger et al., 2003; Bouska and Raquet, 2003; RTCA, 2000), consisting both dry (hydrostatic) and wet components. They are caused by the atmosphere gases in the hydrostatic equilibrium and by those gases not in hydrostatic equilibrium (primarily water vapor) respectively. Both components can be expressed by:

\[ \Delta \sigma_{\text{dry}} = 10^{-6} \cdot N \cdot R \cdot F(h, h_{PL}, h_{s0}) \]  

\[ \Delta \sigma_{\text{wet}} = 10^{-6} \cdot N \cdot \rho \cdot \lambda \cdot f \cdot 10^{\frac{7.4475(T-273)}{T-38.3}} \]  

where **“”denotes the parameters for dry or wet components, respectively;** the receiver height \( h \) and PL height \( h_{PL} \) indicate the importance of the elevation for troposphere modeling while \( R \) is the slope distance between the receiver and PL; the \( h_{s0} \) is defined as the upper boundaries for dry or wet troposphere refraction. It is the empirically fixed height in a model (which is 42,700m for the hydrostatic component and 12,000m for the wet component) or as a function of temperature in other models (Hofmann-Wellenhof et al., 2000). The refraction index \( N \) is determined by local meteorological data with (Zhang and Barton, 2005):

\[ N_{\text{dry}} = 77.6 \cdot \frac{P}{T} \]  

\[ N_{\text{wet}} = 22770 \cdot \frac{f}{10} \cdot 10^{\frac{7.4475(T-273)}{T-38.3}} \]  

where \( T \) is temperature (K), \( f \) is the relative humidity (%) and \( P \) is the atmospheric pressure (mb) measured at the station.

The accuracy of common PL tropospheric delay modeling methods using the measured meteorological data is poor because the measured data cannot be representative for the entire environment of a site (Bernese, 1999). The variation of atmospheric condition can easily change PL tropospheric delay by more than a cycle of the carrier over a few kilometres.

### 3. Adaptive Tropospheric Delay Estimation

GPS satellites transmit radio signals that can be inverted to measure the atmospheric refractivity. Similarly, the radio signals transmitted by PLs can also be inverted to measure local atmospheric refractivity. The proposed adaptive tropospheric delay estimation method is based on the assumption that local meteorological parameters, such as temperature, pressure and relative humidity, change slowly, comparing with the GPS/PL measurement data rate (1 Hz or higher). As PL tropospheric delay modeling is very sensitive to the local meteorological parameters while GPS is not, the meteorological parameters in a PL troposphere model can be estimated with a set of GPS/PL measurements. Then the estimated parameters can be applied in the model for subsequent PL tropospheric delay estimation.

#### 3.1 Methodology

For a short baseline, single-differenced PL tropospheric delay can be estimated using the double difference of GPS/PL carrier phase measurements:

\[ \nabla \Delta \Phi = \nabla \Delta \rho + \Delta \sigma_{\text{dpl}}(p,T,f) - \Delta \sigma_{\text{dps}} + \nabla \Delta \Phi_{\text{dl}} + \nabla \Delta \Phi_{\text{dmp}} \]  

where \( \rho \) is the geometric distance between a PL/GPS satellite and a receiver, \( \lambda \), \( N \), \( \Delta m \) and \( \Delta d \) are the signal wavelength, integer ambiguity, signal multipath and tropospheric delay, respectively. The integer ambiguity \( N \) in Equation (3) should be close to an integer, and the term \( \Delta m \) is a fraction of one cycle, consisting of multipath, measurement noise and other bias. It is noted that the separation of the integer ambiguity term \( N \) from the multipath term \( \Delta m \) in kinematic situations is very difficult, although it is feasible for the PL multipath estimation in the case of static observation environment (Wang, 2002; He et al., 2005). These two terms are lumped together as \( \Delta \Phi_{\text{dmp}} \) in Equation (4).

A function \( F \) can be established with the meteorological parameters and the double differenced integer ambiguity and multipath as variables as follows:

\[ F(p,T,f,\Delta m) = \nabla \Delta \Phi - \nabla \Delta \rho - \nabla \Delta \sigma_{\text{dpl}}(p,T,f) + \nabla \Delta \sigma_{\text{dps}} - \nabla \Delta \Phi_{\text{dmp}} \]  

where \( p \), \( T \), \( f \) are the atmospheric pressure, temperature and relative humidity at sea level, respectively; \( \Delta m \) is the lumped double differenced multipath and integer ambiguity. As GPS tropospheric delay is modelled accurately for the satellites with high elevation angles, \( \Delta \sigma_{\text{dps}} \) is treated as an error-free term for the reference GPS satellite. It is not sensitive to the changes of meteorological parameters as shown in Fig. 1.

The function \( F \) equals to zero if all the terms in Equation (4) is correct. If all the four variables in the function \( F \) are constant or change slowly, they can be estimated with a small set of GPS/PL measurements. Then the estimated meteorological parameters can be applied for PL tropospheric delay modelling in the subsequent epochs. The estimated term \( \Delta \sigma_{\text{dmp}} \) will be used for measurement modeling during the differential
GPS/PL positioning procedure. This is the principle of the proposed adaptive PL tropospheric delay estimation method.

GPS and PL carrier phase integer ambiguity remains constant unless a cycle slip occurs; and the multipath from a static PL to a reference receiver’s antenna is constant. Previous research shows that PL multipath is usually more severe than GPS multipath, and has different characteristics compared to GPS (Choi et al., 2000; Dai et al., 2001). Consequently the term $ND_{mp}$ in above equation mainly comprises a constant component. Furthermore, local meteorological parameters ($p$, $T$ and $f$) should be estimated as slowly varying parameters (Barltrop et al., 1996), comparing with the GPS/PL measurement update rate. Therefore, all the four variables ($p$, $T$, $f$, $ND_{mp}$) in the function $F$ are constant or changing slowly.

### 3.2 Estimation Method

Based on the previous analysis on the PL tropospheric delay models (Wang et al., 2004), the model proposed by Bouska and Raquet (2003) performs well for the whole range of elevation angles. Therefore, this model is used in this adaptive tropospheric delay estimation approach, which is presented as:

$$d_{\text{dry}} = 2 \times 10^{-4} \cdot N_c \cdot R_{\text{pl}} \cdot \left(1 - \left(1 - \frac{h_* - h_{\text{pl}}}{h_c - h_{\text{pl}}}\right)^2\right) \frac{h_* - h_{\text{pl}}}{h_c - h_{\text{pl}}} \left(5\right)$$

$$h_* = h_{\text{wp}} - h_{\text{pl}} \text{ and } N_c(h = 0) = N_c^0$$

where $d_{\text{dry}}$ is the hydrostatic or wet PL tropospheric delay, $R_{\text{pl}}$ is the slope distance between a receiver and PL, $h_{\text{wp}}$ is a fixed scaled height for the model as in Equation (1). These heights are empirically defined as the upper boundaries for the hydrostatic and wet tropospheric refraction. The $N_c$ and $N_c^0$ in Equation (5) are the hydrostatic or wet refraction index component at the height of PL and at sea level, respectively; and both of them are decided by Equation (2).

Applying the above PL tropospheric delay model, the term of single-differenced PL tropospheric delay $\Delta d_{\text{dry}}$ in Equation (4) is expressed as:

$$\Delta d_{\text{dry}} = \left(c_{\text{dry}} - C_{\text{dry}}^0\right) \cdot N_{\text{dry}} \cdot \left(c_{\text{dry}} - C_{\text{dry}}^0\right) \cdot N_{\text{dry}}$$

$$= 77.6 \cdot \left(c_{\text{dry}} - C_{\text{dry}}^0\right)^2 \frac{P}{T} + 22770 \cdot \left(c_{\text{dry}} - C_{\text{dry}}^0\right) \cdot \frac{7.4475 \cdot (273^T - 273^T)}{r-38.5}$$

where $C_{\text{dry}} = 2 \cdot 10^{-5} \cdot R_{\text{dry}} \cdot \left(1 - \left(1 - \frac{h_* - h_{\text{pl}}}{h_c - h_{\text{pl}}}\right)^2\right) \frac{h_* - h_{\text{pl}}}{h_c - h_{\text{pl}}}$

and $C_{\text{dry}}^0 = 2 \cdot 10^{-5} \cdot R_{\text{dry}} \cdot \left(1 - \left(1 - \frac{h_* - h_{\text{pl}}}{h_c - h_{\text{pl}}}\right)^2\right) \frac{h_* - h_{\text{pl}}}{h_c - h_{\text{pl}}}$

Merging Equation (6) into Equation (4), the function $F(x)$ can be expressed with the three meteorological parameters $p$, $T$, and $f$, and term $ND_{mp}$, double differenced integer ambiguity and multipath, as a variable vector $x$. The common least squares method can be used to estimate the vector $x$ after the function $F(x)$ is linearised. However, test results show that the solutions are not stable. This is due to high degree nonlinearity of the function. Therefore, a nonlinear least squares (NLSQ) function lsqnonlin is used to estimate the above variables (Coleman, 2006). This function, which uses a subspace trust region method and is based on the interior-reflective Newton method, can be expressed as:

$$F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \end{bmatrix} \text{ and } \min_x |F(x)|^2 = \sum_{i=1}^n F_i(x)^2 \left(7\right)$$

The function vector $F(x)$ is evaluated by a small set ($n$ epochs) of GPS/PL measurements, as shown in Equation (7). Then the NLSQ function lsqnonlin can find the minimum of the sum of squares of the functions, and return a vector of the estimated values $x$. The optimal size of the GPS/PL measurement set used in Equation (7) is investigated in the following test section.

### 4 Flight Test Results

#### 4.1 Parameter Analysis

The flight test data used in Fig. 2 was processed with the proposed adaptive troposphere delay estimation method. L1 carrier phase GPS/PL measurements with different size of data set (ten, twenty, thirty and sixty epochs respectively) were processed to estimate the four variables in the $F$ function in Equation 4.

The estimated meteorological parameters ($p$, $T$, and $f$), and double differenced multipath and integer ambiguity ($ND_{mp}$) in different sets of 10 epochs’ period are listed in Table 1. The estimation results with twenty, thirty and sixty epochs’ data set are listed in Table 2, and compared with the averaged ten epochs’ results. The term “Resn” in the tables is the quadratic form of the residuals from the NLSQ estimation. The unit of Resn and $ND_{mp}$ are the wavelength of the L1 (cycles). According to the results in Table 1, the meteorological parameters and the term $ND_{mp}$ is relatively stable when estimated with ten epochs’ data, though they varies from time to time.
Table 1. The estimated parameters in different periods

<table>
<thead>
<tr>
<th>Epochs</th>
<th>P (KPa)</th>
<th>T (K)</th>
<th>$f$ (%)</th>
<th>NDmp (Cycles)</th>
<th>Resn (Cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–10</td>
<td>1015.74</td>
<td>292.34</td>
<td>66.5</td>
<td>-28.22</td>
<td>0.00679</td>
</tr>
<tr>
<td>11–20</td>
<td>1023.24</td>
<td>295.45</td>
<td>89.9</td>
<td>-28.62</td>
<td>0.0150</td>
</tr>
<tr>
<td>21–30</td>
<td>1015.37</td>
<td>292.23</td>
<td>84.8</td>
<td>-28.14</td>
<td>0.00438</td>
</tr>
<tr>
<td>31–40</td>
<td>1027.15</td>
<td>297.30</td>
<td>98.9</td>
<td>-28.71</td>
<td>0.02758</td>
</tr>
<tr>
<td>41–50</td>
<td>1018.50</td>
<td>293.74</td>
<td>90.1</td>
<td>-28.55</td>
<td>0.01058</td>
</tr>
<tr>
<td>51–60</td>
<td>1018.13</td>
<td>293.55</td>
<td>88.2</td>
<td>-28.57</td>
<td>0.03794</td>
</tr>
<tr>
<td>Average</td>
<td>1019.69</td>
<td>294.10</td>
<td>86.4</td>
<td>-28.47</td>
<td>0.01705</td>
</tr>
</tbody>
</table>

Table 2. The estimated parameters in different periods

<table>
<thead>
<tr>
<th>Epochs</th>
<th>P (KPa)</th>
<th>T (K)</th>
<th>$f$ (%)</th>
<th>NDmp (Cyc.)</th>
<th>Resn (Cyc.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–20</td>
<td>1020.02</td>
<td>294.11</td>
<td>91.0</td>
<td>-28.54</td>
<td>0.0302</td>
</tr>
<tr>
<td>1–30</td>
<td>1018.50</td>
<td>293.40</td>
<td>85.3</td>
<td>-28.40</td>
<td>0.0441</td>
</tr>
<tr>
<td>1–60</td>
<td>1019.15</td>
<td>294.55</td>
<td>93.3</td>
<td>-28.57</td>
<td>0.1632</td>
</tr>
</tbody>
</table>

However, as shown in Table 2, the estimation results with twenty, thirty and sixty epochs’ data are relatively stable, and are similar to the average of the ten epochs’ data. The results of the estimated meteorological parameters with twenty or more epochs data sizes are much stable than that of the standard parameters. This indicates that reliable meteorological parameters for PL tropospheric delay modeling can be estimated by the proposed adaptive estimation method using a set of GPS/PL measurements with a proper window size (e.g. twenty epochs).

4.2 Positioning Performance

The effects of the PL tropospheric delay modelling error in airborne positioning are demonstrated by applying the proposed adaptive estimation method in an integrated airborne GPS/PL/INS system. The positioning results of the integrated system with the proposed adaptive PL tropospheric delay estimation method are compared with the positioning results using the standard meteorological condition. As there were seven GPS satellites available with a strong geometry during the test, the GPS positioning results were used as the reference.

Fig. 4 Positioning difference with and without adaptive estimation.

The positioning differences between the GPS-only solutions and the PL augmented airborne GPS/INS solutions, with and without applying the proposed adaptive PL tropospheric delay estimation method, are shown in Fig. 4. The dashed lines in the figure are the positioning differences with the standard meteorological condition, and the solid lines are the one with the adaptive method. The GPS and/or PL ambiguities are fixed. The dashed lines with the value of 0.05 meters indicate that the PL ambiguity cannot be resolved, and its measurements are not accurate enough for augmentation. The positioning difference using the adaptive PL tropospheric delay estimation method (solid line) is close to zero and relatively stable. This proves that the proposed method can effectively mitigate the PL tropospheric delay modelling error and augment the integrated system.
5. Concluding Remarks

In order to effectively augment an airborne GPS/INS integrated system with the PL measurements, the PL tropospheric delay must be precisely estimated. This is because that the differencing procedure in processing GPS and PL measurements cannot effectively mitigate the PL tropospheric delay modelling error. At the same time, the PL tropospheric delay cannot be precisely modelled even if the simultaneously measured meteorological data are used. This is primarily due to spatial variations of atmospheric parameters and dense water vapour in the lower troposphere, the measured data is not representative in a PL tropospheric delay model. In contrast, GPS measurements have little problem about these. Therefore, the GPS measurements can be used to estimate the meteorological data which are then used in PL tropospheric delay model.

The proposed adaptive PL tropospheric delay estimation method for an airborne GPS/INS/PL system is based on the fact that local meteorological parameters ($p$, $T$ and $f$) can be estimated as slowly varying parameters comparing with the GPS/PL measurement update rate; And the term $(ND_{mp})$ presenting the double differenced multipath and integer ambiguity in Equation (4) comprises mainly a constant component if no cycle slips occur. A NLSQ function lsqnonlin can effectively estimate these parameters, with relative stable results using a small set of GPS and PL measurements. Then the estimated parameters can be applied in the PL tropospheric delay model for subsequent epochs.

The test results have demonstrated that the proposed method can effectively mitigate the PL tropospheric delay modelling error by estimating the meteorological parameters in a PL tropospheric delay model in real time. The estimated parameters are stable if the size of the data set is used for the estimation is twenty epochs or more. The test results have demonstrated that the proposed method can effectively mitigate PL tropospheric delay modelling error, and improve positioning solutions of the integrated GPS/INS/PL system.

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