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Online Stochastic Modelling for Network-Based GPS Real-Time Kinematic Positioning

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Abstract. Baseline length-dependent errors in GPS RTK positioning, such as orbit uncertainty, and atmospheric effects, constrain the applicable baseline length between reference and mobile user receiver to perhaps 10-15km. This constraint has led to the development of network-based RTK techniques to model such distance-dependent errors. Although these errors can be effectively mitigated by network-based techniques, the residual errors, attributed to imperfect network functional models, in practice, affect the positioning performance. Since it is too difficult for the functional model to define and/or handle the residual errors, an alternative approach that can be used is to account for these errors (and observation noise) within the stochastic model. In this study, an online stochastic modelling technique for network-based GPS RTK positioning is introduced to adaptively estimate the stochastic model in real time. The basis of the method is to utilise the residuals of the previous segment results in order to estimate the stochastic model at the current epoch. Experimental test results indicate that the proposed stochastic modelling technique improves the performance of the least squares estimation and ambiguity resolution.

Key words: Stochastic Modelling, GPS, and Network-Based RTK

1. Introduction

Carrier phase-based GPS has become an essential technique for a wide range of precise positioning applications, such as kinematic surveying and vehicle navigation and guidance. The 'relative' positioning concept, in which one GPS receiver's position is derived with the aid of observations made at a stationary reference station, has been used in most implementations of the technique. The current state-of-the-art implementation for high precision GPS positioning is the so-called Real-Time Kinematic (RTK) hardware/software system that can deliver centimetre-level positioning accuracies of a moving (i.e. kinematic) user receiver, *if the integer carrier phase ambiguities are correctly resolved*. However, the impact of baseline length-dependent GPS errors, such as orbit uncertainty, and atmospheric effects, constrains the applicable baseline length between reference and mobile user receiver to perhaps 10-15km. These constraints have led to the development of several network-based RTK techniques, including the Virtual Reference Station (VRS) approach, and the Area Correction Parameter techniques (Wübbena et al., 2001)

The key concept in using multiple reference stations (i.e., a network) to support GPS carrier phase-based positioning is: (a) to generate so-called "correction terms" representing the distance-dependent errors; and (b) to interpolate and apply them to mobile user receiver measurements, so as to significantly diminish the residual error terms, thus making it possible to perform medium or long-range (tens to hundred kilometre receiver separations) positioning. The correction message data are generated based on the pre-determined coordinates of the reference stations. Integer ambiguities among the reference stations must be correctly resolved and station-dependent errors, such as multipath, measurement noise, and antenna phase centre variation, should be mitigated in the correction terms. In addition, the generated correction message data with respect to each reference stations have to be interpolated or modelled for the user's location. Over the past decade, a number of interpolation methods have been proposed. These include *Linear Combination*

Model, *Distance-Based Linear Interpolation Method*, *Linear Interpolation Method*, *Lower-Order Surface Model* and *Least-Square Collocation* (Fotopoulos & Cannon, 2001). However, Dai et al (2001) demonstrated that the performances of all of these methods are similar.

In order to obtain optimal estimates from the least-square solution, both a *mathematical model*, also called a functional model, and a *stochastic model* should be correctly defined. The functional model describes the relationship between measurements and unknown estimates. On the other hand, the stochastic model represents the statistical characteristics of the measurement that is mainly provided by the covariance matrix for the measurements. Whilst the mathematical models for the network-based GPS RTK positioning are sufficiently investigated and well documented (e.g., Han and Rizos, 1996; Raquet, 1997; Wanninger, 1995; Wübbena et al., 1996), stochastic modeling is still an issue under investigation. In the case of network-based GPS RTK, the positioning performance is largely affected by the residual biases due to imperfect network mathematical models (Musa et al., 2004). The residual biases contribute to the noise terms and make it difficult to define a functional model that can deal with them. Hence, they should be taken into account within the stochastic model.

In this paper a stochastic modelling method that can be efficiently implemented for network-based GPS RTK positioning will be introduced. This technique determines the covariance matrix of observations at the current epoch based on the estimation residuals from the previous positioning results. Experimental test results will be presented to demonstrate the application of the stochastic modelling method.

2. Online stochastic modelling

A stochastic model for the network-based GPS RTK has to take into account uncertainties due to: (a) residual vectors at reference stations; (b) residual interpolation; (c) measurements at the mobile receiver. An online stochastic model reflecting all the uncertainties can be derived using the residuals from Kalman Filter solutions (Wang, 2000).

Since the true values of the model errors (measurement noise or process noise) are unknown, stochastic modelling has to be based on the filtering residuals of the measurements and state corrections, which are generated in the process of parameter estimation. A problem here is that the parameter estimation process itself relies on the estimated measurement covariance matrix R and process covariance matrix Q. The process covariance matrix can be fixed to the appropriate values to neglect the impact of the dynamic model on the GPS RTK solutions. Then, Kalman filtering results are in fact identical to the least-squares solutions (see e.g., Wang, 2000). An *adaptive* procedure can be used to estimate the matrix R online.

The basic idea of the adaptive method is that the residuals collected from the previous segment of positioning results are used to estimate the covariance matrices of the measurement noise and process noise for the current epoch. Preset default covariance matrices are needed to seed the adaptive estimation process. Within a segment, the covariance matrices for each epoch are assumed to be the same. Therefore, the formulation of an adaptive stochastic modelling method includes two critical steps, namely: (a) to derive suitable formulae for use in estimating the covariance matrices, and (b) to determine the optimal segment (window width), which is application-dependent (Wang, 2000; Dai, 2002).

Suppose the measurement filtering residuals are:

$$v_{z_k} = z_k - H_k \hat{x}_k \tag{1}$$

where z_k is the measurement vector and H_k is the measurement design matrix. Equation (1), obviously, is the optimal estimator of the measurement noise level because the estimated values $\hat{x}_k$ (not the predicted values $\bar{x}_k$) of the state parameters are used in their computations. In order to obtain the covariance matrix of the measurement filtering residuals, equation (1) is further derived as

$$v_{z_k} = z_k - H_k(\bar{x}_k + G_k d_k) = (E - H_k G_k) d_k \tag{2}$$

where G_k is the gain matrix and d_k is the innovation vector. By applying the error propagation law to equation (2), after extensive computations, one obtains

$$Q_{v_{z_k}} = R_k - H_k Q_{\hat{x}_k} H_k^T \tag{3}$$

In equation (3), if the covariance matrix $Q_{v_{z_k}}$ is computed using the measurement filtering residuals from the previous m epochs, the covariance matrix R_k can be estimated as (Wang, 2000)

$$\begin{aligned} \hat{R}_k &= \hat{Q}_{v_{z_k}} + H_k Q_{\hat{x}_k} H_k^T \\ &= \frac{1}{m} \sum_{i=0}^{m-1} v_{z_{k-i}} v_{z_{k-i}}^T + H_k Q_{\hat{x}_k} H_k^T \end{aligned} \tag{4}$$

which can be used in the computation of epoch $k+1$. In equation (4), m is called *the width of moving windows*. It is noted that the covariance matrix $\hat{R}_k$ estimated with equation (4) is always positive definite because it is the sum of the two positive definite matrices. Equation (4) requires some extra computations for both v_{z_k} and $H_k Q_{\hat{x}_k} H_k^T$, which are not generated by the standard Kalman filtering process. Fortunately, the amount of these additional computations is small and leads no significant time delay in data processing.

The initial covariance matrix is determined by using the previous covariance matrix. Based on the residuals at the previous epochs, the covariance matrix of the observations can be estimated in real-time using Equation (4).

3. Testing results

3.1 The Experiments

Field experiments were carried out on 21st October 2004 at Olympic Park in Sydney. The objective of these experiments was to test performance of the proposed stochastic modelling method. Three reference stations were used in the experiments. Whilst two of them are Continuously Operating Reference Stations (CORS), which belong to *"SydNet"* - a network of GPS reference stations in the Sydney metropolitan area (Rizos et al., 2003), the other one is a permanent GPS station on the roof of the Electrical Engineering Building at The University of New South Wales. Since MGRV was selected as a master reference station, the baseline length between reference and mobile receiver was approximately 32km. The locations of the reference stations and the trajectory of the mobile receiver are illustrated in Figures 1 and 2. During the fifteen minutes of the experiment, seven satellites were tracked. The data interval was one second. In addition, one more GPS station was installed in the experimental area to generate a reference trajectory using short-range data processing.

The acquired data was processed in the post-mission mode using in-house GPS kinematic processing software. However, it should be noted that all the algorithms used are applicable for real-time implementation. The network-based GPS positioning begins with generating network corrections to mitigate baseline distance-dependent errors (see Musa et al., 2004). The generated L1 and L2 carrier phase corrections for the test data are depicted in Figure 3. It can be seen from the figure that the baseline distance-dependent errors are at the few centimetre-level. This is due to the factor that the baseline length in this experiment was about 32km and the test site is geographically located in a middle latitude region where the ionospheric delay effect is relatively small. In the data processing, the initial standard deviation for the pseudo-range and carrier phase measurements was defined as 0.3m and 0.05 cycles, respectively. The width of the moving window for estimating the measurement covariance matrix was set to ten epochs and the initial estimate was calculated using measurement residuals of the least squares from the first ten epochs with ambiguities being fixed.

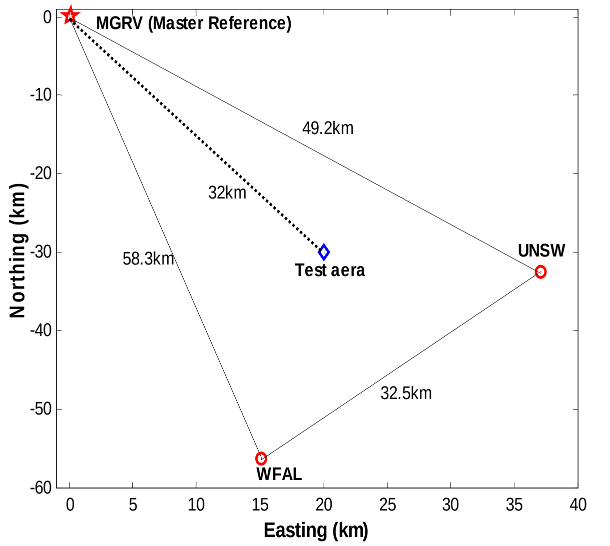

Fig. 1 Configuration of the reference stations and the roving station

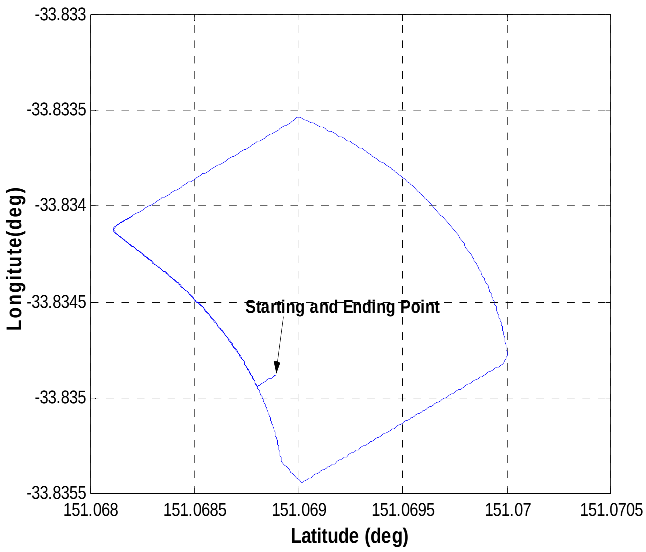

Fig. 2 Trajectory of mobile receiver

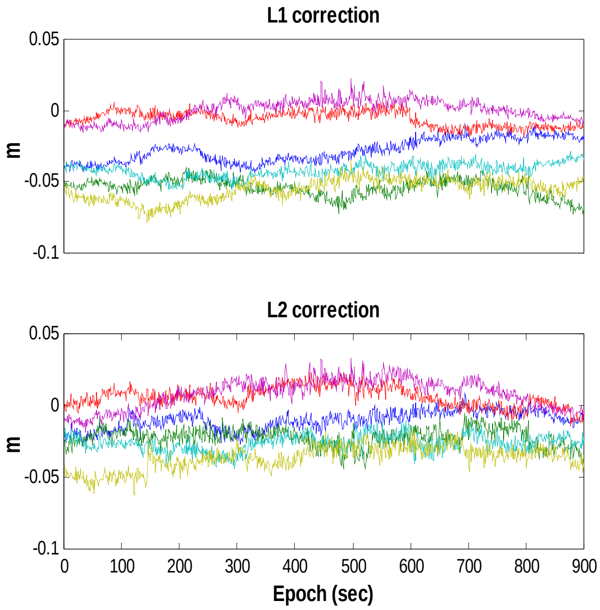

Fig. 3 Network corrections for L1 and L2 carrier phases measurements

3.2 Testing Results

First of all, a comparison between single and multiple reference station solutions was made in order to study potential benefits of using the additional reference stations in the carrier phase-based GPS kinematic positioning. Figures 4 and 5 depict the standard deviations of L1 and L2 carrier phases showing the precision (e.g., noise level) of these measurements and the positioning accuracy.

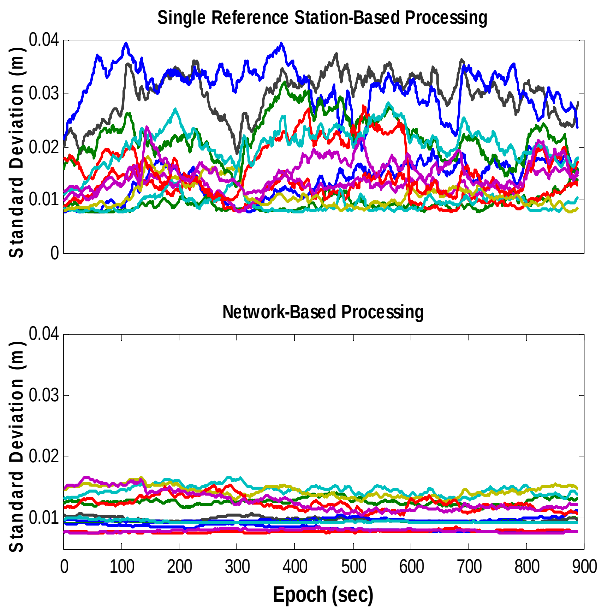

Fig. 4 Standard deviation of L1 and L2 carrier phases measurements

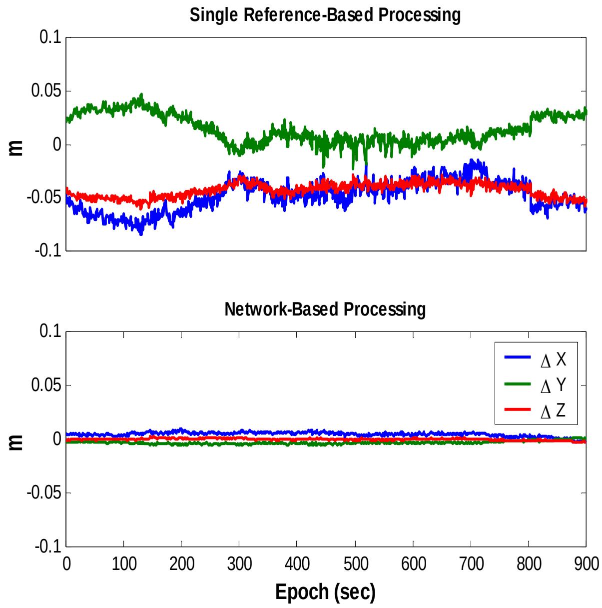

Fig. 5 Accuracy of positioning solution

As already mentioned, the additional reference GPS station installed within the test area with baseline length approximately 100m provided the reference trajectory since its accuracy can be at the few centimetre level with correct integer ambiguity (e.g., short-range kinematic positioning). Hence, it is possible to use the trajectory for evaluating positioning accuracy of the test results. It can be recognised from these results that the application of the network corrections significantly reduces the measurement errors and consequentially improves positioning accuracy, demonstrating the main advantage of the GPS network for medium and long baseline kinematic positioning.

In order to clearly demonstrate effectiveness of the online stochastic modelling method for network-based GPS kinematic positioning, a comparison of the online modelling method with a model based on an apriori assumption of the measurement precision was made. For convenience, they are referred to as ‘Preset’ (based on the error propagation law – see Musa et al., 2004) and ‘Estimated’ (i.e., Equation 4). Figure 6 shows the aposteriori variance value changes obtained from the two different stochastic models. The values should have unity according to the least squares estimation theory (Cross, 1983) if both the functional and stochastic models are correctly defined. In other words, if the variance is significantly different from unity, it is suspected that outliers exist in the measurements, or there is a problem with the fidelity of the stochastic and/or functional models (ibid). The figure shows that the variance values from the preset model are more variable than those from the estimated model. This may be due to the fact that the stochastic model does not realistically reflect the residual biases of the measurements which are mainly caused by the functional model uncertainty. On the other hand, the estimated results show that the variance values are stable and very close to '1', demonstrating that the residuals are appropriately considered by the introduced online modelling method.

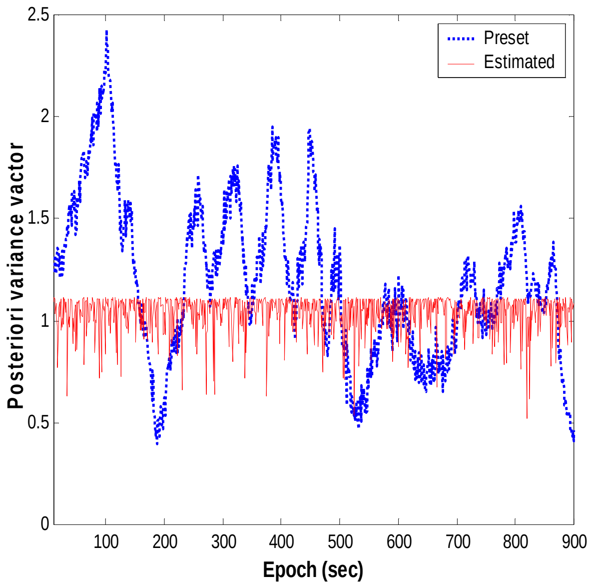

Fig. 6 Posterior variance changes

Figures 7 and 8 illustrate the standard deviations for the DD C1 code and L1 carrier phase observations, respectively. It is evident that big differences exist between the two stochastic models. In reality, the accuracy of the measurements may be influenced by many factors, which must be considered by the appropriate stochastic model. As mentioned early, there are three factors causing the uncertainty (i.e., residual biases) of GPS observations after applying the corrections in the network-based approach. Even though it goes without saying that the uncertainty should be variable due to the satellite and receiver dynamics, the results with the preset stochastic model are almost constant values, which are unrealistic. In contrast, the fluctuations in the estimated results using the proposed on-line stochastic modelling method indicate that such uncertainties are reflected in the stochastic model, making them more realistic.

The determination of the integer ambiguities, commonly referred to as ambiguity resolution (AR), is the most critical data analysis step for high precision GPS based positioning. With fixed integer ambiguities, the carrier phases can be used as unambiguous precise range measurements. A realistic estimation of measurement covariance matrices can provide reliable statistics for ambiguity resolution. To demonstrate this more clearly, both the solutions with the preset and estimated covariance matrices were generated. The ADOP (Ambiguity Dilution of Precision) measure defined by Teunissen and Odijk (1997) is designed to describe the impact of the receiver-satellite geometry on the precision and the correlation of ambiguity parameters. The calculated ADOP values are depicted in Figure 9, indicating significant improvement of the precision and the correlation of the estimated float ambiguities. As a consequence of this it is expected that the ambiguity search volume is reduced and its shape becomes more like a sphere, speeding up the ambiguity searching process (ibid).

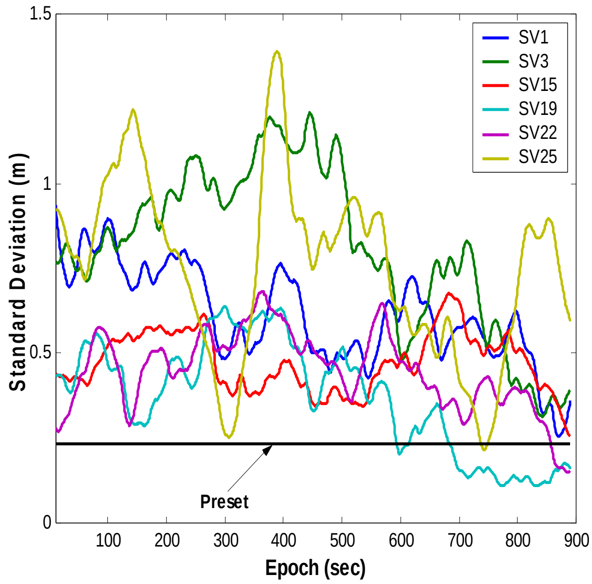

Fig. 7 Standard deviation of C1 pseudo-range measurements

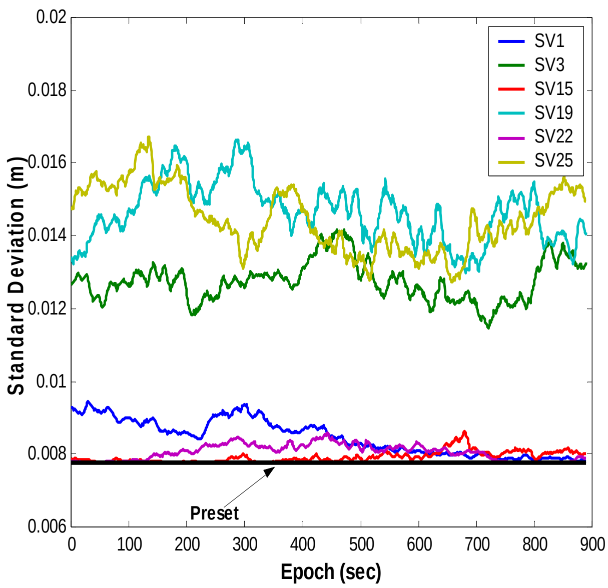

Fig. 8 Standard deviation of L1 carrier phase measurements

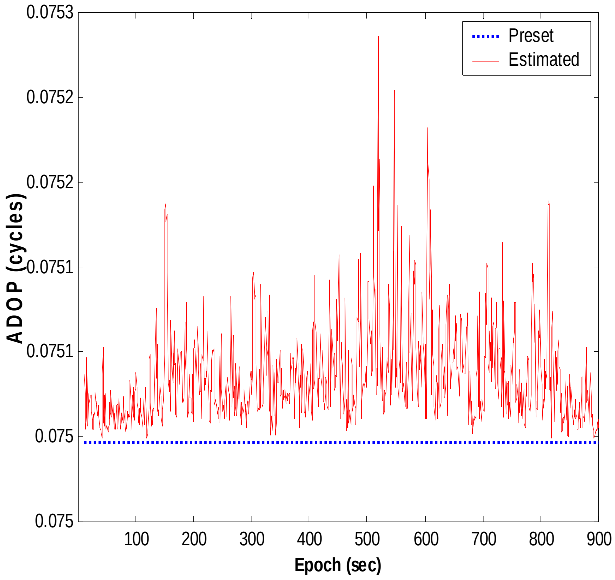

Fig. 9 Ambiguity Dilution of Precision (ADOP)

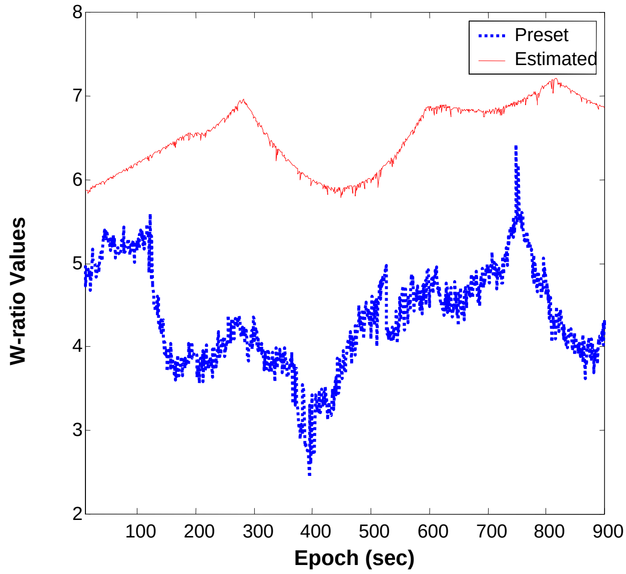

Fig. 10 W-ratio values

It is crucial to ensure that the most likely integer ambiguity combination is statistically better than the second best combination, as defined by the second minimum quadratic form of the least squares residuals, the so-called the *'ambiguity validation test'*. The improved float ambiguity estimates are of great importance for the ambiguity validation test (Wang, 2000; Wang et al., 2003; Lee, 2004). First of all, in order to check the correctness of the best ambiguity combination, the reference ambiguity combination was computed from the reference trajectory obtained from the short-range positioning. A comparison of the best combination with the reference one indicated their ambiguities were exactly the same, hence being able to be considered as the correct ambiguity set. Therefore, it is anticipated that the larger validation test statistics, the higher the probability of validating the correct ambiguity combination.

Figure 10 depicts the validation test statistics using the *W*-ratio proposed by Wang et al. (1998). These results shown in Figure 10 indicate that the ambiguity validation test statistics with the 'Estimated' measurement covariance matrices are much larger than those of the preset measurement covariance matrices. Consequentially, the best ambiguity combination has more probability to be validated by the test, making it possible to reliably resolve correct ambiguities through avoiding type I errors in the statistical hypothesis test.

4. Concluding remarks

Although the impact of baseline length-dependent GPS errors, such as orbit uncertainty, and atmospheric effects, constrains the applicable baseline length between reference and mobile user receiver to perhaps 10-15km, the development of the network-based approaches makes it possible to overcome this constraint. However, the positioning performance is largely affected by the residual biases due to imperfect network mathematical models. These residual biases contribute to the noise terms and make it difficult to define a functional model that can deal with them.

In this paper, an online stochastic modelling method that reflects all the uncertainties of the network-based GPS RTK positioning has been introduced. This stochastic modelling method estimates the covariance matrix of observations at the current epoch based on the estimation residuals from the previous positioning results. In addition, field experiments were carried out to evaluate the performance of the modelling technique. Test results indicate that the proposed technique improves: (a) the covariance matrix of the observations; (b) the model fidelity of the least squares estimation; and (c) the performance and reliability of the ambiguity resolution for network-based GPS kinematic positioning.

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