On a New Equation for Critical Current Density Directly in Terms of the BCS Interaction Parameter, Debye Temperature and the Fermi Energy of the Superconductor

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ABSTRACT

Recasting the BCS theory in the larger framework of the Bethe-Salpeter equation, a new equation is derived for the temperature-dependent critical current density $j_c(T)$ of an elemental superconductor (SC) directly in terms of the basic parameters of the theory, namely the dimensionless coupling constant $[N(0)V]$, the Debye temperature $\theta_D$ and, additionally, the Fermi energy $E_F$—unlike earlier such equations based on diverse, indirect criteria. Our approach provides an ab initio theoretical justification for one of the latter, textbook equations invoked at $T = 0$ which involves Fermi momentum; additionally, it relates $j_c$ with the relevant parameters of the problem at $T \neq 0$. Noting that the numerical value of $E_F$ of a high-$T_c$ SC is a necessary input for the construction of its Fermi surface—which sheds light on its gap-structure, we also briefly discuss extension of our approach for such SCs.

Keywords: Critical Current Density; BCS Parameters; Fermi Energy; Elemental/Non-Elemental Superconductors

1. Introduction

The critical current density ($j_c$) of a superconductor (SC) is the maximum current density that it can carry beyond which it loses the characteristic of superconductivity. It is an important parameter because greater its value, greater is the practical use to which the SC can be put. The basic relation between $j_c$ and the critical velocity ($v_c$) of Cooper pairs (CPs) at any temperature $T$ and an applied field $H$ is:

$$ j_c(T, H) = n_s(T, H) e^* v_c(T, H), \quad (v_c = P_c / 2m^*) \quad (1) $$

where $n_s$ is the number of CPs, $e^*$, $P_c$, and $(2m^*)$ are, respectively, the charge, the critical momentum, and the effective mass of a CP. We note that, since formation of CPs in the BCS theory is synonymous with the formation of their condensate [e.g., 1], $P_c$ in (1) may also be defined as the minimum momentum that causes dissociation of the condensate.

As alternatives to (1), several derived relations for $j_c$ can be found in the literature [2-5], some of which have been reproduced in Table 1. Salient features of these relations are: 1) They are obtained via indirect approaches based on diverse criteria such as the type of SC being dealt with (type I or II) and its geometry; 2) They lead to values of $j_c$ that are generally much greater than the experimental values; and 3) Only one of them involves the Fermi energy $E_F$ (via Fermi momentum) of the SC—this will be further discussed below.

$E_F$ of an SC is an important parameter too because, as has been remarked [6], “There is every evidence that the remarkable low value of $E_F$ (<100 meV) and the strong coupling of carriers with high-frequency phonons is the cause of high $T_c$ in all newly discovered superconductors.” Furthermore, the input of the numerical value of $E_F$ is essential to construct the Fermi surface $E_j(k)$ of an SC via $E_j(k) = E_F$, from which it is seen that [7; p. 117] the whole process of determining theoretically the shape of the Fermi surface involves calculating $E_j(k)$ over the entire Brillouin zone and then constructing the particular constant energy surface that corresponds to $E_F$. However, this assumes that the actual numerical value of $E_F$ is available, which may well not be the case. The importance of the Fermi surface stems from the fact it sheds light on the gap-structure of the SC since it marks the boundary between the occupied and the unoccupied parts of the band $j$. This explains the considerable experimental effort that has been expended on constructing the
Fermi surfaces of a variety of high-\(T_c\) SCs as reported in [e.g., 8,9] and, more recently, in [10-12]. In particular, in the latter of these references, the gaps observed in iron-pnictide SCs as nodes or line nodes on the Fermi surface have evinced considerable interest. For a quantitative account of the \(T_c\) and the multiple gaps of a prominent member of the iron-pnictide family, namely \(\text{Ba}_0.6\text{K}_0.4\text{Fe}_2\text{As}_2\), in the framework of the generalized-BCS equations (GBCSEs) [13]—which will be further discussed below, we draw attention to [14].

The purpose of this note is to present an approach in which \(P_c(T)\) (denoted as the momentum at which the binding energy of the CPs vanishes (this is equivalent to the vanishing of the gap [13])—is calculated via the dynamics of CPs. As will be seen, we are then led via (1) to an equation for \(j_c(T)\) directly in terms of the familiar BCS parameters, namely the dimensionless coupling constant \([N(0)V]\), the Debye temperature \(\theta_D\) and, additionally, \(E_F\) of the SC. The framework employed by us is that of the Bethe-Salpeter equation (BSE) for reasons to be spelled out shortly.

The paper is organized as follows. In the next section, we obtain equations for \(P_c(T, H = 0)\) and \(P_c(T = 0, H = 0)\) for a simple SC. The solutions of these equations for Sn are obtained in Section 3 and compared with similar results obtained by a different method. Extension of our approach for non-elemental SCs is presented in Section 4. In Section 5 we make four brief comments. The final section sums up our conclusions.

### 2. Equations for \(P_c(T, H = 0)\) and \(P_c(T = 0, H = 0)\) for a Simple SC

Our starting point is the \(T = 0, H = 0\) BSE [15] for the bound states of particles \(a, b\) bound via the interaction kernel \(I_{a,b}^c\) in the ladder and instantaneous approximations:

\[
F_a F_b \Psi'(p_{\mu}) = (2\pi)^{-1} \int d^4 q \Psi'(p_{\mu} + q_{\mu}) I_{a,b}^c(q). \tag{2}
\]

Customization of this equation for CPs requires that \(a, b\) should be electrons. We then have

\[
F_a^{-1} = \frac{1}{(\gamma^\mu_{\mu} p_{\mu}^2/2 + \gamma^\nu_{\nu} p_{\nu} - m + i\epsilon)}
\]

\[
F_b^{-1} = \frac{1}{(\gamma^\mu_{\mu} p_{\mu}/2 - \gamma^\nu_{\nu} p_{\nu} - m + i\epsilon)}, \tag{3}
\]

where \(m\) is the electron mass, \(\gamma^\mu_{\nu}\) are the Dirac matrices, \(\pm p_{\mu}\) are 4-momenta of the two electrons in the centre of mass (c.m.) frame, and \(P_{\mu}\) is the 4-momentum of the c.m. of the CP in the laboratory frame.

In our earlier work [13] based on (2), it sufficed to set

\[
P_c = (E, 0); (E = 2E_F + W) \tag{4}
\]

where \(E\) is the total energy of a CP; it then turned out that \(W ≅ \Delta\). The BCS interaction kernel in (2) then was

\[
I_{a,b}^c(q - p) = -\frac{V}{(2\pi)^2} \Theta(V > 0) \tag{5}
\]

\[
E_F - \omega_D \leq \frac{p^2}{2m} \leq E_F + \omega_D
\]
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[Note: We use natural units: mass, momentum, energy, etc. in eV, \( c = \hbar = 1 \).]

The role of the 4th dimension in (2) is simply to provide the means to temperature-generalize the theory at the outset via the Matsubara recipe. Thus, following the steps that have been detailed in \([13,16]\) we obtain from (2) the 3-dimensional equation

\[
S(p) = (-2\pi i)^{-1} \int d^3 q \left[ -i \phi_{q,p} + \psi \right] - i \phi_{q,p} + \psi \right] \right] \left[ B(p) - 2m \right], \tag{7}
\]

\[
A(p) = E/2 - p^2/2m, \\
B(p) = A(p), \\
I_{ab}'(q - p) = \frac{\gamma_q \gamma_p}{2} I_{ab}(q - p),
\]

and

\[
J(q) = \int \frac{dq_x}{D(q)} \tag{8}
\]

where

\[
D(q) = A(q) + B(q) = (E - q^2/2m), \tag{9}
\]

and \( \beta = 1/k_B T, k_B \) being the Boltzmann constant.

Since critical velocity is defined as the velocity of CPs at which \( W = 0 \), we now need to consider (2) for the case of moving CPs. Hence (4) is replaced by

\[
P_m = (E, p) \tag{11}
\]

where \( p \) is the 3-momentum of the c.m. of a CP. It is pertinent at this stage to draw attention to the interaction Hamiltonian corresponding to (2), which is actually apparent from the structure of the equation:

\[
H_m = g \psi \phi
\]

where \( \psi \) is the electron field and \( \phi \) the phonon field; exchanges of the latter field between the electrons with coupling strength \( g \) being responsible for pairing. Both for elemental and non-elemental SCs, one is now enabled to calculate not only the \( T_c \)s and \( \Delta_s \)--as has been shown \([17,18]\), but also \( P_c(T) \) of the pairs as will be seen below. Because the BSE formalism accommodates CPs having non-zero c.m. momentum, it constitutes a larger framework than the original BCS formalism which restricts the Hamiltonian at the outset to comprise of terms corresponding to pairs having zero c.m. momentum.

Since energies of the electrons forming a CP now take on the values \( (P/2 + \pm p \pm q) \right| \right| 2m \), the BCS model interaction given in (5) gets replaced by

\[
I_{ab}'(q - p) = \frac{V}{(2\pi)^3} \left( V > 0 \right) \text{ for } E_F - \hbar \omega_d
\]

\[
\leq \frac{(P/2 + p + q)^2}{2m}, \frac{(P/2 - p + q)^2}{2m} \leq E_F + \hbar \omega_d, \tag{12}
\]

\[
= 0 \text{ (otherwise)}.
\]

Substituting (9)-(12) into (6), we obtain

\[
S(p) = \int \frac{d^3 q}{(2\pi)^3} \frac{T_1(p) + T_2(p)}{De(q)} S(q) \tag{13}
\]

where

\[
T_1(p) = \tanh \left[ \beta \left( E + (P/2 + q)^2 \right)/2m \right], \\
T_2(p) = \tanh \left[ \beta \left( E + (P/2 - q)^2 \right)/2m \right], \tag{13a}
\]

\[
De(q) = \left[ E - \frac{(P/2 + q)^2}{2m} - \frac{(P/2 - q)^2}{2m} \right]
\]

and it is seen that, as is well known for a constant kernel, the wave function for the pair is a constant; the limits (L, U) will be dealt with shortly. Putting

\[
\frac{T_1(p) + T_2(p)}{De(q)} = \varphi(q)
\]

in (13), multiplying the resulting equation with

\[
\int_L^U d^3 p/(2\pi)^3
\]

and simplifying, we obtain

\[
1 = \frac{V}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{T_1(p) + T_2(p)}{De(q)} \tag{14}
\]

with

\[
d^3 p = p^2 dp sin \theta d\theta d\phi, \xi = p^2/2m - E_F,
\]

so that, since the integration range for \( \xi \) is \( E_F \),

\[
p \right| \right| (2mE_F)^{1/2}, p^2 dp \right| \right| m(2mE_F)^{1/2} d\xi, \tag{15}
\]

we obtain from (14) the equation

\[
1 = \frac{V}{8} \left[ (2m)^{3/2} E_F^{3/2} \right] \int_L^U d\xi \frac{X}{Y} \tag{16}
\]

where


\[ X = A_1 + A_2 \]

\[ A_1 = \tanh \left[ \frac{\beta}{2} (\xi + P\alpha x + \frac{P^2}{8m} - \frac{W}{2}) \right] \]

\[ A_2 = \tanh \left[ \frac{\beta}{2} (\xi - P\alpha x + \frac{P^2}{8m} - \frac{W}{2}) \right] \]

\[ Y = \xi + \frac{P^2}{8m} - \frac{W}{2} \]

\[ \alpha = \left( \frac{E_F}{2m} \right)^{1/2}, \quad x = \cos(P, p) \]

and (13a) and the definition of \( E \) in (4) have been used. In the natural units employed by us, both \( m \) and \( E_F \) are in eVs; the second pre-factor within the square brackets on the RHS of (16) is therefore recognized as the 3-dimensional density of states at the Fermi surface (with the dimensions of \((eV^{-1}cm^{-3})\)) in the units customarily employed in the BCS theory. Henceforth we denote this factor by \( N(0) \).

Note that the term corresponding to \( P \) \( x/2m \) in the expansion of \((P^2 + P)^2/2m \) has been written as \( P\alpha x \) by using (15) and the definitions of \( \alpha \) and \( x \) that follow (16).

We now specify the limit \( L \). It follows from (12) that

\[ E_F - \omega_D \leq \frac{P^2}{8m} + \frac{P^2}{2m} + P\alpha x, \quad E_F - \omega_D \leq \frac{P^2}{8m} + \frac{P^2}{2m} - P\alpha x \]

where (15) has been used. These relations may be written as

\[ -\omega_D - \frac{P^2}{8m} - P\alpha x \leq \frac{P^2}{2m} - E_F = \xi, \]

\[ -\omega_D - \frac{P^2}{8m} + P\alpha x \leq \frac{P^2}{2m} - E_F = \xi. \]

If we denote \(-\omega_D - \frac{P^2}{8m} - P\alpha x\) by point A, and \(-\omega_D - \frac{P^2}{8m} + P\alpha x\) by point B on the energy axis, then it follows that \( \xi \) should always lie to the right of both A, and B. Thus \( L \) is fixed as

\[ L = -\omega_D - \frac{P^2}{8m} + P\alpha x. \]

Similarly,

\[ U = \omega_D - \frac{P^2}{8m} - P\alpha x. \]

We now put \( W = 0 \) in (16) in order to determine the critical momentum \( P_c(T) \) at any temperature. Simultaneously, we neglect \( P_c^2/8m \) everywhere—a posteriori justification to follow, excepting in the denominator of the integrand where it must be retained so as to avoid the singularity at \( \xi = 0 \). It is then seen that it is an excellent approximation to write (16) as:

\[ 1 = \frac{[N(0)V]}{2} \int_0^\infty \frac{d\xi}{\xi} \psi(x, \xi, P) \]

where

\[ \psi(x, \xi, P) = \tanh \left[ \frac{\beta}{2} (\xi + P\alpha x) \right] + \tanh \left[ \frac{\beta}{2} (\xi - P\alpha x) / (8m) \right] \]

\[ = f_1(x, \xi, P) + f_2(x, \xi, P) \]

\[ U' = \omega_D - P\alpha x. \]

Equation (19) affords a consistency check of our procedure so far. Putting \( P_c = 0 \) causes the \( x \)-integral to yield unity, and the two \( \tanh \)-functions to add up, leading to the correct BCS equation for \( T_c \). Note that when \( T = 0 \) \((\beta = \infty), f_2(P_c, \xi, x) = 1\), whereas the value of \( f_1(P, \xi, x) \) depends on whether \( \xi \) is less or greater than \( P\alpha x \). Therefore, when \( T = 0 \), we can write (19) as

\[ 1 - \frac{\lambda}{2} (I_1 + I_2) = 0, \quad (21) \]

where

\[ I_1 = \int_0^\infty \frac{d\xi}{\xi} \int_0^\infty \frac{d\xi}{\xi + E_1}, \quad (22) \]

\[ I_2 = \int_0^\infty \frac{d\xi}{\xi} \int_0^\infty \frac{d\xi}{\xi + E_2}, \quad (23) \]

\[ \lambda = [N(0)V], E_1 = \omega_D, E_2 = P_0\alpha, \quad P_0 = P_c(T = 0), E_A = P_0^2/8m. \]

Carrying out the elementary integrations in (22) and (23), (21) yields

\[ 1 - \lambda \left[ \frac{E_1}{E_2} \ln \left( \frac{E_1}{E_2} \right) + \frac{E_2}{E_1} \ln \left( \frac{E_2}{E_1} \right) + \ln \left( \frac{E_1 - E_2 + E_3}{E_2 + E_3} \right) \right] = 0. \]

Since, as will be seen below, \( E_3 \cap E_1, E_2 \), we may write it more compactly as

\[ 1 - \lambda \left[ y \ln \left( \frac{y}{y-1} \right) + \ln (y-1) \right] = 0, \quad (25) \]

where the dimensionless parameter

\[ y = E_1/E_2 = \omega_D/P_0\alpha. \]

3. Solutions of (25) and (19) for \( S_n \) and Comparison of Results for \( j_c \) via (1) with Those Obtained via an Alternate, Indirect Approach

We deal with \( S_n \) because superconducting properties based on its \( j_c \) have been discussed in standard texts such as
and $\Delta/m_p^* v_F = 1.46 \times 10^6 \text{ cm} \cdot \text{sec}^{-1}$; 
\[ E_F = (1/2)m^* v_F^2 = 1.74 \text{ eV}, \]  
Using (27) and the experimental value of $v_F$ taken at the Fermi edge to be $6.97 \times 10^7 \text{ cm/sec}$ \cite{3, p. 248}, we have 
\[ n_s = 8.50 \times 10^{21} \text{ cm}^{-3}, \]  
which, it has been remarked \cite{19}, is appreciably less than one electron per atom, but not unreasonable in view of the complicated band structure of tin, which has been discussed in \cite[7, p. 294]{7}.

In our approach, we first need the value of $\lambda$ to solve (25). Substituting the experimental value of $T_c$, quoted above and $\theta_D = 195 \text{ K}$ in the BCS equation for $T_c$: 
\[ \lambda = -1/\ln(T_c/1.14\theta_D), \]  
we obtain $\lambda = 0.2445$, whence \[ y = o_0 \frac{p_0}{P_0} \frac{k_0 \theta_D}{P_0} \sqrt{2m^*} \frac{2m^*}{E_F} = 22.48, \]  
where the definition of $\alpha$ given after (16) has been used. Using (29) and (1), we have 
\[ v_0 = \frac{P_0}{2m^*} = A \sqrt{\frac{B}{E_F}}; \]  
\[ A = \frac{k_0 \theta_D}{2m^* c v_F} = 17.402 \text{ cm sec}^{-1}, \]  
\[ B = 2 \times 1.26 \times 0.5110 \times 10^6 \text{ eV} \]  
and 
\[ j_0 = n_s e^* A \sqrt{\frac{B}{E_F}}, \]  
where $2m^*$ is the effective mass of a CP and $m^*$ has been taken to be 1.26 times the free electron mass as before, $e^*$ is twice the electronic charge and the value of $E_F$ is in eV.

Since we have already determined $A$ via dynamics of the problem, $e^*$ and $B$ are known constants and $j_0$ is known from experiment, (31) involves two unknowns: $n_s$ and $E_F$; knowledge of either of them enables one to calculate the other. Guided by textbook wisdom, if we use the values of $j_0$ ($2 \times 10^7 \text{ Amp cm}^2$) and $E_F$ (given in (27)), we obtain from (30) and (31) the following results 
\[ \alpha = 1.162 \times 10^{-3}, P_0 = 0.643 \text{ eV} \]  
\[ E_1 = 1.68 \times 10^{-3} \text{ eV}, E_2 = 7.476 \times 10^{-4} \text{ eV}, \]  
\[ E_3 = 8.029 \times 10^{-8} \text{ eV} \]  
\[ v_0 = 1.50 \times 10^4 \text{ cm sec}^{-1} \]  
\[ n_s(CPs) = 4.17 \times 10^{21} \text{ cm}^{-3}. \]  
The values of $E_1$, $E_2$ and $E_3$ in (32) justify the approximation made in reducing (24) to (25). The result in (33) is almost identical with the value obtained via (26) and quoted in (27), while the result in (34) translates into $8.34 \times 10^{21} \text{ cm}^3$ for the number of super electrons which, again, is in excellent agreement with the value quoted in (28). It is thus established that the approach followed in this paper provides an \textit{ab initio} theoretical justification for the textbook equation (26) valid at $T = 0$; \textit{additionally}: 1) it relates $j_c$ with the relevant parameters of the problem at $T \neq 0$ via (19) and 2) it can easily be extended to bring non-elemental high-$T_c$ SCs under its purview as will be discussed in the next section.

With $P_0$ known, it is convenient to solve (19) in terms of the reduced (or normalized) variables defined as $t = T/T_c$ and $p(t) = P(t)/P_0$. \textbf{Figure 1} gives the result of this exercise for $0 \leq t \leq 1$. We have also studied the variation of $p$ with $t$ for five other elements: Pb, Hg, In, Tl, and Nb—by taking for their $E_F$s the values given by the free electron model \cite[p. 248]{20}, and found it to be similar to that of Sn.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Variation of reduced critical momentum with reduced temperature for Sn obtained via (19) with the input of $\lambda = 0.2445$, $\theta_D = 195 \text{ K}$ and $E_F = 1.74 \text{ eV}$.}
\end{figure}
4. Equations for $Pc(T, H = 0)$ and $Pc(T = 0, H = 0)$ for a Non-Elemental SC

The $T_s$ and the multiple gaps of several non-elemental high-$T_c$ SCs (other than iron-pnictide SCs) have been dealt with in [17,18] via GBCSEs. We recall from [13,16] that these equations constitute a generalization of the BCS equations because: 1) they incorporate the mechanism of multi-phonon exchanges for the formation of Cooper pairs besides the usual one-phonon exchange mechanism; and 2) they invoke more than one Debye temperature—which is another way to specify the mass-dependent Debye frequency of an ion species—to characterize the SC.

In order to calculate $P_c$ in the scenario in which CPs are bound via say, two-phonon exchange mechanism in a CS $A_xB_{1-x}$, we need to generalize (19) and (24). This is accomplished by replacing the propagator in (12) by a superpropagator [13]:

$$I_{ab}^c (q - p) = \frac{1}{(2\pi)^3} \left( V_{1,2}^c + V_{2,1}^c \right),$$

range of $V_{1,2}^c$: $E_F - (\omega_D)^{1,2}$

$$\leq \frac{1}{2m} \left[ \left( \frac{p + p \text{ or } q}{2m} \right)^2 - \left( \frac{p - p \text{ or } q}{2m} \right)^2 \right]$$

$$\leq E_F + (\omega_D)^{1,2}$$

$$= 0, \text{(otherwise)}$$

where $V_{1,2}^c > 0$ are the BCS model interactions for the species of phonons belonging to $A, B$ in the combined state of the constituents $A$ and $B$, to be distinguished from $V_{1,2}^c$, which are the free state interactions of $A, B$; $(\omega_D)^{1,2}$ are to be similarly distinguished from $(\omega_D)^{1,2}$. Following now the sequence of steps between (8) and (24), we obtain the generalized version of (19) as:

$$1 = \int_0^1 dx \left[ J_1 (x) + J_2 (x) \right],$$

where

$$J_1 (x) = \frac{\lambda_1^c \nu_1^c}{2} \int_0^\infty d\xi \psi(x, \xi, P_c) \left[ \frac{\beta}{\xi + p_c^2/8m} \right]$$

$$J_2 (x) = \frac{\lambda_2^c \nu_2^c}{2} \int_0^\infty d\xi \psi(x, \xi, P_c) \left[ \frac{\beta}{\xi + p_c^2/8m} \right]$$

$$\psi(x, \xi, P_c) = \tanh \left[ \frac{\beta}{2} (\xi + P_c \alpha x) \right] + \tanh \left[ \frac{\beta}{2} (\xi - P_c \alpha x) \right]$$

$$\lambda_{1,2}^c = \left[ N(0)V_{1,2}^c \right], U_{1,2} = (\omega_D)^{1,2} - P_c \alpha x.$$  (38)

Equation (24) now goes over to

$$\lambda_1^c \phi_1 ((\omega_D)^1, \alpha, p_0) - \lambda_2^c \phi_2 ((\omega_D)^2, \alpha, p_0) = 0,$$  (39)

where 

$$\phi_1 = \left[ \frac{E_1}{E_2} \ln \left( \frac{E_1}{E_2} \right) + \frac{P_0}{8m\alpha} \ln \left( \frac{P_0}{8m\alpha} \right) + \frac{E_1 - E_2}{E_2} \right],$$

$$E_1 = (\omega_D)^1, E_2 = P_c \alpha.$$  (40)

The solution of (39) for MgB$_2$, for example, requires the inputs of $\lambda_1^c$ and the two Debye temperatures: $(k_i \theta_0)^{1,2}$; in addition, we require $E_F$ of the CS. Such solutions will be addressed elsewhere.

5. Discussion

We have dealt above with equations that were obtained via positive energy projection operators (PEPOs). This suffices for the problem addressed because $P_c$ corresponds to the situation when $W = 0$; in this limit, it has been shown in [21] that the equation obtained via the negative energy projection operators is identical with the one obtained via the PEPOs; also that: 1) CPs formed via electron-electron and hole-hole scatterings make equal contributions to the BS amplitude; and 2) the amplitudes for the formation of CPs corresponding to the mixed energy projection operators are zero.

Note that if we concern ourselves with the ratios of $j_c$ at different temperatures, which seems to be a realistic application of our equations, then the choice of the effective mass of the electron in (1) becomes immaterial.

Even a cursory survey of the literature shows that $j_c$ of an elemental/non-elemental SC can vary between wide limits—depending upon the shape, size and alloying materials of the sample. The study presented here suggests that this variation comes about because each sample has its own set of intrinsic parameters: $T_s$, $\theta_0$, and $E_F$. Substituting these into the equation for $j_c$ (which is known from experiment) leads to a relation involving $n_s$ and $E_F$. Knowledge of either of them then determines the other.

We finally note that the equations for $j_c(T)$ presented in this paper can be generalized to include an external magnetic field via the Landau quantization scheme—as has been done to obtain dynamics-based equations for critical magnetic fields for both elemental and non-elemental
SCs in [22].

6. Conclusions

Equation (19) for an elemental SC and (36) for a non-elemental SC are the new results of this paper: they enable one to calculate the critical momentum \( P_c \) of the SC at any \( T \) in zero-external magnetic field directly in terms of the familiar parameters \([N(0)V], \theta_D\) and \(E_F\) that characterize it. Substitution of these values of \( P_c \) into (1) then constitutes a direct approach for the calculation of the critical current densities.

\( T = 0 \) limits of both (19) and (36) were obtained—leading to (24) and (40), respectively. While it was further shown that (24) can justifiably be reduced to (25), we note that caution needs to be exercised should one seek to carry out a similar reduction of (40).

A necessary input for the calculation of \( P_c \) (and hence \( j_c \)) of an SC is its \( E_F \), which is a parameter that is seldom available for the high-\( T_c \) SCs. It therefore seems to us that an immediate and realistic application of the approach presented here is to calculate the \( E_F \) of such SCs via the input of their \( j_c \)'s which are readily available in the literature. The importance of \( E_F \) of the high-\( T_c \) SCs is borne out by the studies reported in [6] and [8-12].

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