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Underlying Assets Distribution in Derivatives: The BRIC Case

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Abstract

This paper addresses one of the main issues regarding numerical derivatives valuation, particularly the search for an alternative to the normality assumption of underlying asset returns, to obtain the price by using numerical techniques. There might be difficulties in making normality assumptions, which could produce over-valuated or sub-valuated prices of derivatives. Under this consideration, the Generalized Hyperbolic family has been proven to be a proper selection to model heavy tailed distribution behavior. The Normal Inverse Gaussian (NIG) distribution is a member flexible enough to model financial returns. NIG distribution can be used to model distribution returns under different states of nature. The indexes of the Brazil, Russia, India and China (BRIC) economies were studied at different time-periods using return data series from 2002 to 2005, 2006 to 2010 and 2011 to 2015, in such a manner to demonstrate with statistical criteria that NIG fits the empirical distribution in the three periods; even throughout economic downturn. This result may be used as an improvement in derivatives valuation with indexes as underlying assets.

Keywords

Derivatives, Normal Inverse Gaussian, Valuation, Returns, BRIC

1. Introduction

One of the main concerns regarding financial analysis, derives from an inadequate study of the returns of several financial variables; since it is imperative to model the dynamics of these returns. With the continuous development of mathematical tools as well as computational capacity, different approaches have attempted to improve model adjustment by combining numerical and analytical frameworks. In particular, the stochastic processes modelling returns have been the core of the researching agenda development since the thesis was proposed by Bachélier in 1900 [1]. Premise, in which normality is assumed, has been proven to be erroneous by application of diverse statistical tests. This issue is transferred to another branch: the derivatives valuation. In numerous cases, the valuation of different derivatives is tested by Monte Carlo simulation as they do not have a close valuation formula; *i.e.* there is not an exact equation, as in the case of European Options with the well-known Black, Scholes and Merton model. For Monte Carlo's application, it is necessary to investigate, for example, stochastic returns dynamics and their respective simulation algorithm.

In this context, the importance of derivatives valuation has become a central issue to consider. According to data from World Bank, the estimated global GDP in 2016 was placed at 75.543 billion dollars. In addition, the BIS published their semiannual derivative statistics, in which the approximate notional amount of outstanding OTC derivatives contracts equaled $542 trillion dollars at the end of June 2017 [2]. Noting this disparity, the relevance of studies around derivatives valuation in modern world becomes quite evident.

In this regard, multiple studies researched for better approximations to empirical returns density and have demonstrated that the members of the Generalized Hyperbolic family have a better fit to observations [3] [4]. One of the main properties regarding this distributions set is the presence of heavier tails than those of Normal distribution; which is an important property of returns' behavior [5]. Another property is the flexibility of the functions, since these distributions are determined by five parameters. The adjustment has a better opportunity to reflect the nature of the empirical data; property that according to Cont, is necessary to approach this kind of distributions [5].

Although the Normal Inverse Gaussian (NIG) distribution has verified to be an excellent selection for stock returns distribution, it may also be used inclusively for indexes' returns of financial markets [6] [7].

In particular, this paper focuses on the probability distribution model of the main indexes of the BRIC economies in different periods of financial conditions (the case of South Africa is excluded because it was not until 2010, that it became part of this group).

By dividing this study into three time intervals, the period from 2002 to 2015, we reviewed the capability of NIG to fit distribution returns even in extreme periods, such as the downturn of the financial crisis. By doing so, it may be possible to use Monte Carlo simulations with the proper distribution and parameters in order to obtain a better price of derivatives which relies on index as underlying asset.

The organization of the paper follows the next sequence: in Section 2, we discuss the Generalized Hyperbolic and Normal Inverse Gaussian distributions. The methodology applied to the data is presented in Section 3; then, Section 4 shows the results obtained. Finally, the conclusions are stated in Section 5.

2. Literature Review

The Generalized Hyperbolic family was firstly proposed by Barndorff-Nielsen in 1977 [8]. This class of distributions is defined by five parameters; by fixing the parameter $\lambda = -1/2$, the NIG distribution is obtained. He exposed the capability of the NIG distribution to model heavier tails than those of the Normal distribution, a fact which is commonly found in returns data series. These characteristics became of interest due to the consequences in risk management applications and other branches of finance. Particularly Eberlein & Keller used the DAX index in a three-year period from 1989 to 1992 to perform statistical test to compare the fit of empirical data with adjusted Normal and Hyperbolic distributions [3]. Their results concluded that, for the studied period, the hyperbolic distribution is a better option to model returns.

However, later studies Barndorff-Nielsen used NIG distributions as it is better to model heavy tailed observations [9]. Under this assumption he used statistical tests as well, in order to compare how Hyperbolic and NIG distributions fit to empirical data. Finally, he concluded that NIG performs better as a model for the same data used by Eberlein and Keller [9]. Later, Rydberg proved (using data from Denmark, Germany and United States' stock markets) that the NIG distribution is a better function to model the returns [10].

Trejo, Núñez and Lorenzo developed a study around the usage of NIG to model the stock return distribution in the Mexican market, as well as the IPC and S & P500—indexes from Mexico and United States respectively [11]. By studying the Mexican market, they were able to show that NIG distribution has a better fit for stocks and index return distribution than a Normal one; so tools like the Brownian Process are not the best to simulate Mexican financial series.

Recent studies concerning the Generalized Hyperbolic family of returns in prices of commodities such as gold and petroleum have been done. Mota and Mata use historical prices from Brent, WTI and Mexican mix, to adjust the parameters of a HG distribution [12]. For this purpose, they took two-time period intervals to determine if those fit can be used in different states of nature; using the period from 2010 to 2013 a higher price, and from 2014 to 2015 a period in which the international price plummeted. Mota and Mata's results demonstrated that these types of distributions are able to better fit empirical returns of such commodities.

Relative to gold return, the studies have been conducted around the possibility to model distribution in order to obtain a better Value at Risk (VaR) measure [13] [14]. Using the time interval from 1991 to 2017, the studies use different classes of the Generalized Hyperbolic family, in order to compare them within risk management applications and techniques.

3. Methodology

For the purpose of this study, we used the daily data collected from Bloomberg of the BRIC indexes: IVOB, NIFTY50, SHCOMP and RTSI. Through the selection of data series, we intended to represent three periods of time, in a manner to demonstrate that NIG distribution could fit indexes return data series in all nature states; the periods were named as pre-crisis (2002-2006), crisis (2007-2010) and post-crisis period (2011-2015). These representing periods were carefully chosen in order to shelter against other impacts not considered as part of the study. In particular, the crisis of 2001 emerged after the terrorist attack of the Twin Towers, and 2016 as the year in which capital flows returned to developed countries—mainly the continental United States—with the imminent interest rate normalization period [15] [16]. The nature of these shocks is distinct, but this work focuses only on the crisis included in the period from 2007 to 2010, thus naming the studied periods.

From each index series we calculated the logarithmic return as daily data could be used as an approximation to a continuous series; the equation states as follows:

$$r_i = \ln P_i - \ln P_{i-\Delta t}, \quad \forall i = \{1, \cdots, n\} \tag{1}$$

where:

r_i is the return of the index on the day.

P_i is the closing level of the index at day *i*.

P_{i-1} is the closing level of the index at day $i-1$.

3.1. Descriptive Statistics

Skewness and kurtosis were calculated for each index series in order to validate distributions which have higher values of skewness, whether positive or negative; so we can expect that the empirical data does not correspond to a Normal distribution.

The statistics of these series are presented in the following **Table 1**.

Table 1. Descriptive statistics.

Time Period	Index	Mean	Variance	Skewness	Kurtosis
2002-2005	IBOV	0.0009	0.0003	−0.2645	0.6944
	NIFTY50	0.0011	0.0002	−1.0271	9.0648
	SHCOMP	0.0004	0.0002	0.6407	4.6071
	RTSI	0.0015	0.0003	−0.6751	4.1838
2006-2010	IBOV	0.0006	0.0004	−0.0031	6.1839
	NIFTY50	0.0004	0.0004	0.0780	7.1262
	SHCOMP	0.0000	0.0004	−0.2815	2.1921
	RTSI	−0.0001	0.0007	−0.3191	10.7541
2011-2015	IBOV	−0.0004	0.0002	−0.0468	1.5607
	NIFTY50	0.0002	0.0001	−0.1697	1.5807
	SHCOMP	0.0002	0.0002	−0.8859	6.2777
	RTSI	−0.0007	0.0004	−0.3010	6.7127

*High Kurtosis values appeared in every data series distribution.

By analyzing the excess of kurtosis, a different behavior is notable from the Normal distribution, so the presence of heavy tails is expected.

3.2. Normality Test

Using the Anderson-Darling and Shapiro-Francia normality test, it is possible to reject the null hypothesis of normality. In this case the proposed NIG distribution becomes a candidate to fit the empirical data. Both normality tests are assumed as follows:

H_0: Sample resulting from a normal distribution is confirmed.

H_a: Sample which does not come from a normal distribution, H_0 is discarded.

For the acceptance of the null hypothesis, the p-value of each of the data series was obtained using both tests (Anderson-Darling and Shaphiro-Francia), with a level of significance of 0.05, so that, if p-value ≥ 0.05, the null hypothesis is accepted, otherwise it is rejected and the alternative hypothesis is accepted.

3.3. Shapiro-Francia Test

The normality test developed by Shapiro and Francia as an approximate and simplified version of the Shapiro Wilk test to prove the normality of a larger series of data [17]. The test parameter is obtained by calculating the slope of the regression line by simple least squares, *i.e.*,

$$W = \left(\sum_{i=1}^{n} b_i y_i\right)^2 \Big/ \sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2 . \tag{2}$$

3.4. Anderson Darling Test

The Anderson-Darling criteria are used to test the hypothesis that a series of data comes from a population that adheres to a continuous Cumulative Distribution Function (CDF) [18]. The test is performed as follows:

$$W_n^2 = -n - \frac{1}{n} \sum_{j=1}^{n} (2j-1)\left[\ln\left(u_j\right) + \ln\left(1 - u_{n-j+1}\right)\right]. \tag{3}$$

3.5. Normal Inverse Gaussian Distribution

As mentioned before, the NIG distribution has been proven in multiples studies to fit the financial series. This kind of distribution is defined by Barndorff-Nielsen as follows:

$$g\left(x;\alpha,\beta,\mu,\delta\right) = a\left(\alpha,\beta,\mu,\delta\right) q\left(\frac{x-\mu}{\delta}\right)^{-1} K_1\left\{\delta\alpha q\left(\frac{x-\mu}{\delta}\right)\right\} \exp\left(\beta x\right) \tag{4}$$

where

$$a\left(\alpha,\beta,\mu,\delta\right) = \pi^{-1}\alpha \exp\left(\delta\sqrt{\alpha^2 - \beta^2}\right) - \beta\mu . \tag{5}$$

And

$$q\left(x\right) = \sqrt{1 + x^2} \tag{6}$$

where K is the modified Bessel function of third order and index 1. Also,

α, β, μ and δ, are parameters, satisfying $0 \le |\beta| \le \alpha, \mu \in \mathbb{R}$ and $0 < \delta$.

The parameters α and β determines the shape, and μ and δ scale the distribution. Parameter α, which takes nonnegative values, denotes the flatness of the density function, *i.e.* a high value of α means a greater concentration of the probability around μ. The parameter β defines a kind of skewness of the distribution. When $\beta = 0$, the NIG distribution is symmetric around the mean. A negative value represents a heavier left tail. The parameter δ describes the scale of the distribution and the parameter μ is responsible for the shift of the distribution density [6].

3.6. Goodness of Fit Tests

By simulating a vector with the obtained parameters, we test the similarity of both distributions with Kolmogrov-Smirnov and Anderson-Darling criteria in which the *p*-values corresponds with the acceptation zone of the null hypothesis. So according to the statistical tests, it is possible to say that NIG distribution is capable of modeling the returns even during a period of economic crisis.

4. Results

Through the results obtained by calculating the descriptive statistics of the series, we concluded that all series have heavy tails and high skewness values, which indicated that the series could be fitted with a member of the Hyperbolic Generalized Family; as the NIG distribution. Before we proceeded to estimate the NIG parameters, we applied the Normality Test (Anderson-Darling, Shapiro-Francia), to confirm that the series were not Normal.

Applying the Normality Test to the data it was possible to reject the null hypothesis of normality in every case. This result is consistent with the stylized features of financial series where higher values of kurtosis do not correspond to the Normal distribution (**Table 2**).

Table 2. Normality test (*p*-value).

Time Period	Index	Anderson-Darling	Shapiro-Francia
2002-2005	IBOV	5.783e−06	1.55e−05
	NIFTY50	<2.2e−16	<2.2e−16
	SHCOMP	<2.2e−16	<2.2e−16
	RTSI	<2.2e−16	<2.2e−16
2006-2010	IBOV	<2.2e−16	<2.2e−16
	NIFTY50	<2.2e−16	<2.2e−16
	SHCOMP	<2.2e−16	1.62e−14
	RTSI	<2.2e−16	<2.2e−16
2011-2015	IBOV	4.85e−09	3.502e−09
	NIFTY50	8.357e−09	5.173e−09
	SHCOMP	<2.2e−16	<2.2e−16
	RTSI	<2.2e−16	<2.2e−16

Both Tests assume a significance level of 0.05, that means that if *p*-value ≥ 0.05 null hypothesis is accepted otherwise is rejected and the alternative hypothesis is confirmed. Own elaboration, data processed in R Software.

Having proven the no-normality of indexes return distribution, and the excess kurtosis obtained from the descriptive statistics of the series, the NIG could be used to model the empirical data to obtain a distribution that better describes the empirical data series.

For the estimation of the NIG parameters we applied Maximum Likelihood Estimation (MLE). Although other methods could have been used, the selected algorithm solves the maximization problem by numerical methods. The parameters are shown in **Table 3** below. Parameters were estimated using R Software.

With the estimated NIG parameters, a series with a particular NIG distribution was simulated in order to make a statistical analysis using Log-likelihood Test to compare the similarity of the empirical data series with the simulated data series. To complete this task, Kolmogorov-Smirnov and Anderson-Darling tests were used.

The results do not refuse the null hypothesis—the statistical similarity of distributions—in all cases except for the SHCOMP index in the pre-crisis period under the Kolmogorov-Smirnov test. However, the Anderson-Darling, considered a more precise test, confirms similarity in every case. Therefore, according to statistical criteria, the NIG distribution can fit the indexes return distribution in the three states of nature; defined as pre-crisis, crisis and post-crisis.

Quantitative results of Likelihood (**Table 4**) between NIG simulation distribution and the empirical data distribution obtained through the analysis could be observed graphically. Qualitative comparison of distributions considered Normal Distribution (red), empirical data distribution (blue) and simulated NIG distribution (green), as well as the QQ Plot is presented in the graphics bellow (**Figures 1-3**).

Table 3. NIG parameters.

p	Index	α	β	δ	μ	n
2002-2005	IBOV	115.3831	−20.6768	0.0325	0.0068	1043
	NIFTY50	79.7261	0.19.269330	0.0136	0.0044	1258
	SHCOMP	56.8119	5.7093	0.0097	0.0006	1303
	RTSI	47.6604	−7.6627	0.0146	0.0039	1303
2006-2010	IBOV	31.8151	−3.1540	0.0131	0.0019	1304
	NIFTY50	35.9781	−2.9873	0.0145	0.0016	989
	SHCOMP	40.5741	−9.4047	0.0175	0.0042	1044
	RTSI	19.5138	−2.9695	0.0139	0.0021	1044
2011-2015	IBOV	99.0900	5.1516	0.0199	−0.0014	1303
	NIFTY50	124.5000	−3.9450	0.0141	0.0007	1238
	SHCOMP	37.4145	−1.1891	0.0081	0.0004	1303
	RTSI	44.6705	−0.9362	0.0158	−0.0003	1303

NIG Parameters obtained for each series at different period times. Own elaboration, data processed in R Software.

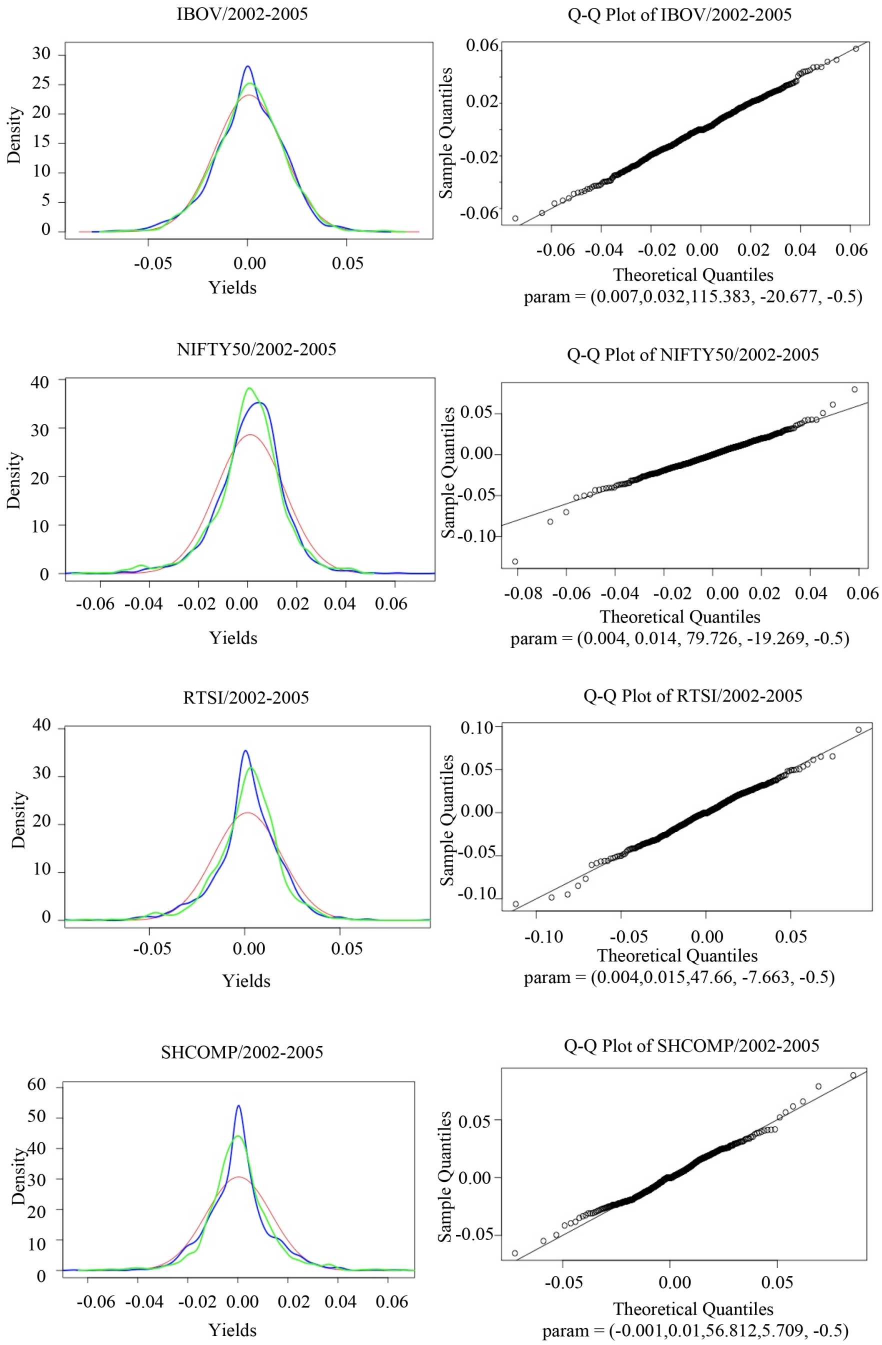

Figure 1. Normal, empirical and NIG data series distribution at pre-crisis scenario. Distribution graphics show normal distribution (Red), empirical data distribution (Blue) and NIG simulated distribution (Green), for interpretation purposes. Graphics were generated in R software.

5. Conclusion

Through this analysis, it can be observed that financial returns are reasonably adjusted by the NIG distribution. Particularly, we can use this fact on derivatives valuation which does not have a closed formula of valuation, and it is necessary to employ numerical techniques. The behavior of underlying assets is then a central

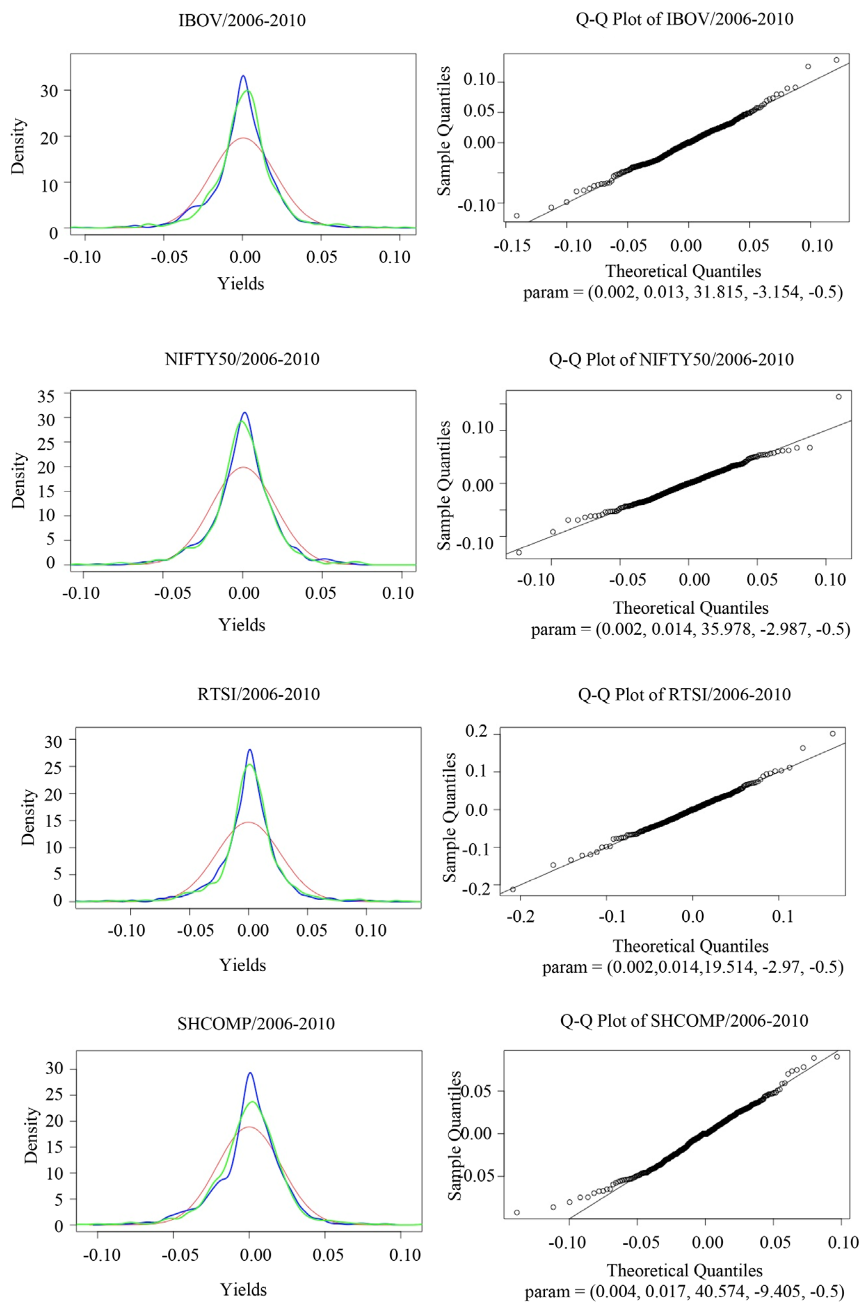

Figure 2. Normal, empirical and NIG data series distribution at crisis scenario. Distribution graphics show normal distribution (Red), empirical data distribution (Blue) and NIG simulated distribution (Green), for interpretation purposes. Graphics were generated in R software.

issue of this topic. To address this problem, we propose to use the NIG as a distribution that has proven to be an adequate method to fit distribution of stock returns. We have demonstrated to fit the returns of indexes of the BRIC economies as well. Under this basis the indexes return from these economies were analyzed in three periods divided according to different states of nature. Those are pre-crisis, crisis and post-crisis scenarios. These results provide evidence that

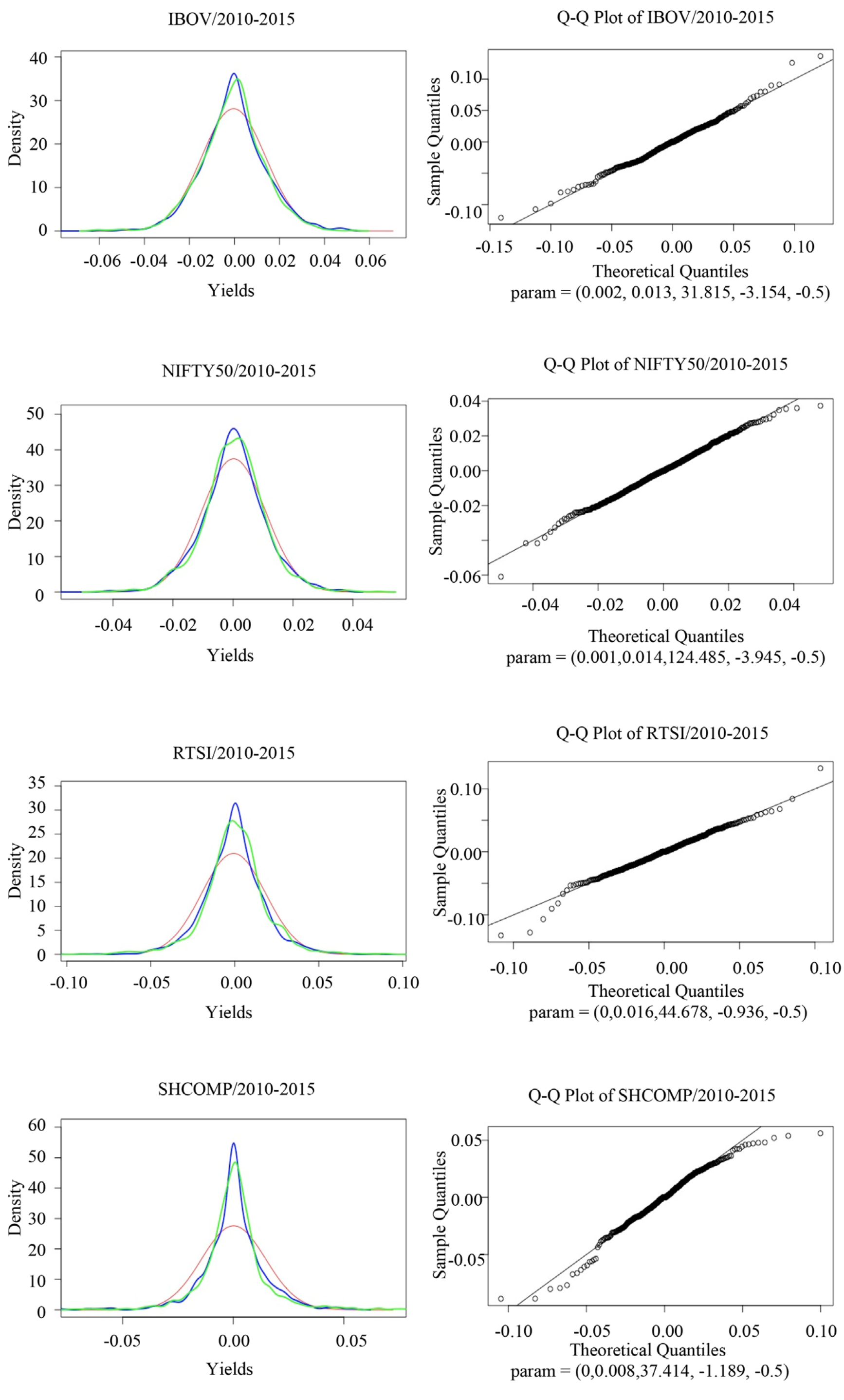

Figure 3. Normal, empirical and NIG data series distribution at post-crisis scenario. Distribution graphics show normal distribution (Red), empirical data distribution (Blue) and NIG simulated distribution (Green), for interpretation purposes. Graphics were generated in R software.

NIG should be used, or at least tested to model indexes returns in different scenarios, because distribution fits better to empirical data series, even at extreme periods like 2008 financial downturn, which deeply affected all of the world

Table 4. Likelihood test (p-value).

Time Period	Index	Kolmogorov-Smirnov	Anderson-Darling
2002-2005	IBOV	0.8758	0.7774
	NIFTY50	0.2142	0.3006
	SHCOMP	0.0465	0.1121
	RTSI	0.0863	0.2552
2006-2010	IBOV	0.3400	0.5358
	NIFTY50	0.6029	0.8094
	SHCOMP	0.0890	0.3554
	RTSI	0.4937	0.3279
2011-2015	IBOV	0.2711	0.5014
	NIFTY50	0.7713	0.7679
	SHCOMP	0.1961	0.2523
	RTSI	0.0681	0.1423

Both tests assume a significance level of 0.05, that means that if p-value ≥ 0.05 null hypothesis is accepted otherwise is rejected and the alternative hypothesis is confirmed. Own elaboration, data processed in R Software.

economies. NIG distribution could be used as outstanding tool for derivatives valuation that uses indexes returns as underlying through Monte Carlo simulations and its variants.

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