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# Optimal Ordering Policy for Deteriorating Items Having Constant Demand and Deterioration Rate

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## Abstract

This study derives an optimal ordering policy for perishable products. Here the demand considered is constant demand and items considered have a constant rate of deterioration. The sensitivity analysis of model has been done and formula for calculating optimal ordering quantity has been derived. The model follows the classical EOQ model, *i.e.* the point of minima is the point where the total ordering cost is equal to the sum of holding costs and deterioration cost.

## Keywords

Perishable Goods, Optimality, Sensitivity, Economic Order Quantity

## 1. Introduction

Resources are scarce and misuse of resources is sin. In the last few decades the study of perishable goods has gained a lot of importance. Ghare & Schrader [1] were the first to apply the concept of deterioration. They established an inventory model with a constant rate of deterioration, which was further extended by Covert & Philip [2] who worked on a model considering a variable rate of deterioration with two parameter Weibull distributions. Chun Tao Chung [3] established an EOQ model for deteriorating items under inflation when the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Shah and Shukla [4] developed a deterministic inventory model in which item is subject to constant deterioration and shortages are allowed. Min, J., Zhou, Y. W., & Zhao, J. [5] established an inventory model for deteriorating items under stock dependent demand and two-level trade credit. Sarker, B. R., Jamal, A. M. M., & Wang, S. [6] derived a supply chain model for perishable products under inflation and permissible delay in payment.

Gour Chandra Mahata [7] had investigated the optimal retailers' replenishment decisions for deteriorating items under two levels of trade credit policy to reflect supply chain management within the economic production quantity (EPQ) framework. Lio, Chung & Huang [8] had designed recently a two-warehouse inventory model for deteriorating items when the supplier offers the retailer a delay period and in turn the retailer provides a delay period to their customers. Lio, Chung and Huang [9] had designed recently a two-warehouse inventory model for deteriorating items when the supplier offers the retailer a delay period and in turn the retailer provides a delay period to their customers. Bhanu Priya Dash, Trailokyanath Singh, Hadibandhu Pattnayak [10] framed an inventory model for deteriorating items with exponential declining demand and time-varying holding cost.

In this study an attempt has been made to frame an inventory policy for deteriorating products with constant demand and deterioration rate. This study derives the mechanism by which the wholesalers/retailers can decide about their cycle period and economic order quantity, without doing any mathematical modeling. The sensitivity analysis of the decision variables with the changes in the parameter values is done and the optimal policy is obtained for the various cases. It has also been proved that the total cost obtained provides optimal solution. The proposed model can be converted into the classical EOQ model by considering nonperishable items. The paper concludes with the concluding remarks.

## 2. Model Description and Analysis

In this model constant demand rate $\beta$ is considered with a constant rate of deterioration $\alpha$, depletion of the inventory occurs due to customers demand as well as due to deterioration of the items. For any prescribed period the inventory at any time t is given by the following nonlinear differential equation

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t}+\alpha I(t)=-\beta,\ 0\le t\le T \tag{1}$$

Solving Equation (1), we obtain I (t) during the time period ( $0\le t\le T$ ) as

$$I(t)=-\frac{\beta}{\alpha}+C\mathrm{e}^{-\alpha t},\ 0\le t\le T \tag{2}$$

Using the condition, at the time, $t=T$ *i.e.* at the end of a cycle, $I(T)=0$, we obtain

$$I(t)=\frac{\beta}{\alpha}\left(\mathrm{e}^{\alpha(T-t)}-1\right),\ 0\le t\le T \tag{3}$$

At time $t = 0$, when the order is received *i.e.* cycle starts, $I(0)=Q$ (Quantity Ordered) and Equation (3) gives the ordered quantity as

$$Q=\frac{\beta}{\alpha}\left(\mathrm{e}^{\alpha T}-1\right) \tag{4}$$

The ordered quantity depends upon the demand, cycle time and the rate of deterioration. To determine the optimal order quantity know Q (Economic Order quantity), we have to derive the total variable cost equation for the proposed model first and also check the feasibility of the derived model.

Since the total demand during cycle period T is $\beta$T, and the quantity ordered is Q, so anything less than Q which is not used is the deteriorated products. Hence, the amount of materials which deteriorates during one cycle is

$$T_D = Q - \beta T = \frac{\beta}{\alpha}\left(\mathrm{e}^{\alpha T} - 1\right) - \beta T \tag{5}$$

The total variable cost will consist of the following

1) The ordering cost of the materials is fixed at S dollars/order for the proposed financial year. It consists of the cost of procurement and inbound logistics costs.

2) The deterioration cost is given by c*$T_D$ using Equation (5) as the number of units deteriorated is D and the cost price of each unit is c, which comes out to be

$$D = c\left[\frac{\beta}{\alpha}\left(\mathrm{e}^{\alpha T} - 1\right) - \beta T\right] \tag{6}$$

3) The holding cost is the function of average inventory, it can be defined as keeping costs, which includes storage cost, insurance cost, cost of capital and it is given by

$$H = h\int_0^T I(t)\,\mathrm{d}t$$

Which, upon simplification, yields

$$H = \frac{\beta h}{\alpha^2}\left(\mathrm{e}^{\alpha T} - 1 - \alpha T\right) \tag{7}$$

Total variable cost function for one cycle is given by the sum of all the three cost, *i.e.* ordering cost, deterioration cost and holding cost which is given by Equations (5), (6) and (7). It is defined as variable cost as it changes with the ordering quantity and the demand.

$$TVC = A + D + H$$

It consists of two components, one is ordering cost and the other is carrying cost, carrying cost consists of holding and deterioration cost in this model. If the items considered are non-perishable items than the total variable cost will consist of only two costs, *i.e.* ordering cost and holding cost, which are the costs considered in the classical EOQ model. Hence the above model will represent the classical EOQ model, where EOQ will be given by the point where the holding cost and the ordering cost per cycle are equal.

$$TVC = S + \frac{c\beta}{\alpha}\left(\mathrm{e}^{\alpha T} - 1 - \alpha T\right) + \frac{h\beta}{\alpha^2}\left(\mathrm{e}^{\alpha T} - 1 - \alpha T\right) \tag{8}$$

Considering there are n cycles in a year, then the Total Variable Cost per annum, TC, is simply given by

$$TC = \frac{S}{T} + \frac{c\beta}{\alpha T}\left(e^{\alpha T} - 1 - \alpha T\right) + \frac{h\beta}{\alpha^2 T}\left(e^{\alpha T} - 1 - \alpha T\right) \tag{9}$$

This study aim is to derive the optimal order quantity (EOQ) which results in the minimum inventory cost. Once, we obtain the cycle time, we can easily derive the economic order quantity and the number of cycles required per annum. To obtain that we have to do the analysis of the total variable cost per annum.

**Analysis of Total Variable Cost**

The total variable cost is given by Equation (9), which consists of the exponential function of $\alpha$T that can be expanded in powers of $\alpha$T. As $\alpha$ is the deterioration rate, it is assumed to be very small and in most of the practical cases $\alpha$ is assumed to be less than 10%. Hence, the higher power of will be very small converging towards zero, therefore neglecting higher powers of $\alpha$, the total variable cost equation is given by

$$TC = \frac{S}{T} + c\beta\left(\frac{\alpha T}{2} + \frac{\alpha^2 T^2}{6}\right) + h\beta\left(\frac{T}{2} + \frac{\alpha T^2}{6} + \frac{\alpha^2 T^3}{24}\right)$$

Taking the first order derivative of the total cost equation with respect to the cycle time, T, we obtain

$$\frac{\mathrm{d}TC}{\mathrm{d}T} = \frac{-S}{T^2} + c\beta\left(\frac{\alpha}{2} + \frac{\alpha^2 T}{3}\right) + h\beta\left(\frac{1}{2} + \frac{\alpha T}{3} + \frac{\alpha^2 T^2}{8}\right)$$

Taking the second order derivative of the total cost equation with respect to the cycle time, T, we obtain

$$\frac{\mathrm{d}^2 TC}{\mathrm{d}T^2} = \frac{2S}{T^3} + c\beta\left(\frac{\alpha^2}{3}\right) + h\beta\left(\frac{\alpha}{3} + \frac{\alpha^2 T}{4}\right)$$

which is convex, hence the optimal solution exists for the proposed model.

Considering the principal of maxima and minima and putting the first derivative of the total variable cost equation equal to zero, which gives the point of minima. Putting the value in second derivative would give the positive value as all the variables considered in the equation are positive. Hence, it is proved the point obtained is a point of minima.

The above model can be converted into the classical EOQ (Economic order quantity) model if nonperishable items are considered. Otherwise, also it follows the classical EOQ principle which states that the optimal order quantity is obtained when ordering cost is equal to the carrying cost.

On the simplification of first order derivative obtained above, the following equation is derived

$$3\beta h\alpha^2 T^4 + 8\beta\alpha\left(h + c\alpha\right)T^3 + 12\beta\left(h + c\alpha\right)T^2 - 24S = 0$$

which is a bi-quadratic equation and can be solved for the given values of the constraints to get the optimal cycle time. Hence, the economic order quantity (EOQ) can be obtained using Equation (4).

## 3. Sensitivity Analysis of the Model

Assuming, the deterioration rate, $\alpha = 0.05$ *i.e.* 5%, yearly demand, $\beta = 1000$ units, the unit cost of the item, c = \$100. We have considered three cases, taking the holding costs as \$1, \$2 and \$3 and varying the ordering cost from \$100 to \$500 (Tables 1-3).

**Case I**: Considering the above assumptions and holding cost, h = \$1

**Case II:** Considering the holding cost, h = \$2

**Case III:** Considering the holding cost, h = \$3

## 4. Findings

The following observations can be made from the above tables, as the ordering cost increases the number of inventory cycles decreases. The point, at which optimal solution is obtained, is the point where the ordering is equal to the sum of holding and deterioration cost, which is aligned to the EOQ (Economic order quantity) model result, where optimal order quantity is obtained at the point where ordering cost is equal to the holding cost for that cycle.

**Table 1.** Assumptions and holding cost, h = \$1.

| Ordering Cost | Number of Cycles | Cycle Time (Days) | Total Variable Cost(\$) |
|---|---|---|---|
| 100 | 5.5 | 99 | 1097.11 |
| 200 | 3.875 | 66 | 1552.53 |
| 300 | 3.175 | 114 | 1902.36 |
| 400 | 2.747 | 131 | 2197.55 |
| 500 | 2.652 | 136 | 2654 |

**Table 2.** The holding cost, h = \$2.

| Ordering Cost | Number of Cycles | Cycle Time (Days) | Total Variable Cost (\$) |
|---|---|---|---|
| 100 | 5.917159763 | 61 | 1184.88 |
| 200 | 4.201680672 | 86 | 1676.65 |
| 300 | 3.424657534 | 106 | 2054.389 |
| 400 | 2.96735905 | 122 | 2373.1 |
| 500 | 2.652519894 | 136 | 2654.1 |

**Table 3.** The holding cost, h = \$3.

| Ordering Cost | Number of Cycles | Cycle Time (Days) | Total Variable Cost (\$) |
|---|---|---|---|
| 100 | 6.33 | 57 | 1266.58 |
| 200 | 4.484 | 81 | 1792.18 |
| 300 | 3.676 | 98 | 2196 |
| 400 | 3.154 | 114 | 2537 |
|  | 2.881 | 125 | 2837 |

## 5. Concluding Remarks

In this study an optimal ordering policy has been proposed which equips wholesaler/retailers to decide their Economic order quantity and the inventory cycle time, which can help them in reducing their operating costs drastically. This model provides a simple framework to wholesalers/retailers to derive optimal inventory levels without doing any mathematical modeling. The sensitivity analysis of various costs and cycle time has been done. Also, the convexity of the total cost has been validated and the cycle time formula has been derived.

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## Notations

The following notations are used in developing the model for deteriorating items having constant demand and deterioration rate.

$T_D$: Amount of material deterioration during a cycle time, $T$

$\alpha$: Deterioration rate, a fraction of the on-hand inventory

$c$: The unit cost per item (dollars/unit)

$S$: The ordering cost of inventory (dollars/order)

$\beta$: The demand rate (units per unit time)

$T$: Replenishment cycle time

$H$: The total cost of holding inventory per cycle

$D$: Total deterioration cost per cycle

$I(t)$: The inventory level at time $t$