A Comprehensive Analysis of the Response of Private Consumption to Government Spending

Yoshito Funashima

Faculty of Economics, Tohoku Gakuin University, Sendai, Japan
Email: funashima@mail.tohoku-gakuin.ac.jp

Abstract
This paper considers a simple model in which government spending is productive and has a complementary relationship with private consumption to study the response of the latter to government spending. We discuss how these two characteristics can yield empirical observations that indicate a positive response of private consumption to government spending. By assuming some plausible parameter settings, we demonstrate that these dual aspects of government spending, which are normally treated separately in the literature, are inseparably linked. Our findings reveal that in addition to the presence of complementarity, productivity—even if minimal—increases the likelihood of generating a positive consumption response.

Keywords
Private Consumption, Government Spending, Productivity, Complementarity

1. Introduction
Canonical models in modern macroeconomics predict the negative response of private consumption to government spending owing to a negative wealth effect (e.g., Aiyagari et al. [1]; Baxter and King [2]). Only recently, Ramey [3] presented the evidence consistent with this prediction. However, several other studies report the positive impact of government spending shock on private consumption (e.g., Gali et al. [4]; Beetsma and Giuliodori [5]). In a very influential article published in 2002, Blanchard and Perotti [6] stated that it is difficult to reconcile the positive response of consumption with the neoclassical approach.

Multiple hypotheses have been advanced to bridge the gap between theory and empirical evidence for explaining the positive response of consumption to government spending. Among these, there are two prominent hypotheses regarding
characteristics of government spending.

On the one hand, some predecessors look at the demand side and focus on the complementarity between government spending and private consumption by applying Bailey’s [7] formulation. From a purely theoretical standpoint, Ganelli and Tervala [8] indicate that an increase in government spending can cause a rise in private consumption. On the other hand, some of the previous work in this area takes particular note of the supply side, that is, the productivity of government spending was formulated by Barro [9]. According to this viewpoint, government spending enhances the productivity of private firms and increases output by varying degrees, as shown in the large body of literature spearheaded by Aschauer [10]. In the present context, Linnemann and Schabert [11] examined the effects of productive government spending on some key variables in a New Keynesian framework, and they noted that private consumption can positively respond to productive government spending.

Additionally, the production elasticity of government spending is also a significant consideration in studying the impact of government expenditure on production. In this context, empirical works project mixed estimates. At 40 percent, Aschauer [10] projected the ceiling estimate (also see, Glomm and Ravikumar [12]). By using a simple model, Tervala [13] exclusively studied lump-sum taxes and documented that productive government expenditures are unlikely to cause a positive consumption response when the production elasticity of government spending is small, as empirically observed in previous works. This conclusion is different from Linnemann and Schabert [11], who show that even when the production elasticity of government spending is minimal; it contributes towards generating a positive private consumption response.

This paper works in the direction of showing that the dual aspects of public expenditure, which are normally treated separately in the literature, are inseparably linked under some plausible parameter settings. This has been achieved by combining the standpoints of Ganelli and Tervala [8] and Tervala [13] for creating a simple, minimal model in which government spending has both productivity and complementarity. In particular, we show that the sole presence of complementarity is less likely to generate a positive response for private consumption, and that productivity is also required, even if it takes minimal values. Specifically, even when government spending is a perfect complement to private consumption, private consumption does not respond positively to government spending provided the productivity of government spending is zero and the Frisch elasticity of labor supply is equal to or less than one.

The rest of the paper is organized as follows. The model has been described in Section 2 and results have been presented and discussed in Section 3. Section 4 concludes.

2. The Model

We consider a model economy in which the foregoing dual aspects of govern-
ment spending (i.e., productivity and complementarity) are incorporated. As mentioned in detail below, productivity pertains to production function and complementarity to utility function.

In the literature, Linnemann and Schabert [11] and Tervala [13] assume that government spending contributes to private production, as in Barro [9]. Based on this, it is assumed that the representative firm produces a single final good according to the available technology:

$$Y = NG^\alpha$$  \hspace{1cm} (1)

where $Y$ is output level, $N$ is labor input, $G$ is government spending, and $\alpha (\geq 0)$ is the production elasticity of government spending. Let $P$ and $W$ denote the price and wage, respectively. The profits are given by the following expression:

$$\pi = PY - WN$$

and maximization with respect to $N$ results in the following:

$$P = \frac{W}{G^\phi}$$  \hspace{1cm} (2)

We now turn to household behavior. There is a representative household that seeks to maximize the utility function, with which the possible complementarity between government spending and private consumption is incorporated. Specifically, as in Ganelli and Tervala [8], the preference is formulated by the following additive-separable utility function:

$$U = \ln(C + \psi G) - \frac{N^{1+\phi}}{1+\phi}$$

where $C$ is private consumption, $N$ is labor supply, and $\phi (\geq 0)$ is the elasticity of the marginal disutility of labor supply. Following Tervala [13], we refer to $1/\phi$ as the Frisch elasticity of labor supply. Furthermore, $\psi$ is a key factor for our purpose. If $\psi$ is negative (positively), complementarity (substitution) between government spending and private consumption holds. Above all, $\psi = -1 \quad (\psi = 1)$ refers to perfect complementarity (substitution) between government spending and private consumption. If $\psi$ is zero, the household feels that government spending is not related to utility as a whole, and it is of no concern in consumption decision-making.

Noting that the profits of the firm are zero, the budget constraint is

$$PC = WN - P\tau$$

where $\tau$ denotes real lump-sum taxes to which the government has access, and the government spending is financed entirely by $\tau$ in a balanced budget, such that $\tau = G$. Utility maximization with respect to $C$ and $N$ yields the following:

$$1 \over {C + \psi G} = \frac{PN^\theta}{W}$$  \hspace{1cm} (3)

We now log-linearize the model as in Ganelli and Tervala [8] and Tervala [13]. While specifying an initial steady state, they assume that initial government spending is zero. In the same fashion, the log-linearized system of Equations (1)-(3)
and the good market clearing condition (i.e., \( Y = C + G \)) are expressed as follows:

\[
\begin{align*}
\dot{Y} &= \hat{N} + \alpha \hat{G} \\
\dot{P} &= -\alpha \hat{G} \\
\hat{C} + \psi \hat{G} &= -\phi \hat{N} - \hat{P} \\
\dot{Y} &= \hat{C} + \hat{G}
\end{align*}
\]  

(4) \hspace{1cm} (5) \hspace{1cm} (6) \hspace{1cm} (7)

where hats refer to percentage deviations from the initial steady state.

3. Results and Analysis

This section focuses on both productivity and complementarity of government spending and examines how these two characteristics can yield empirical observations, especially with regard to the response of private consumption to government spending.

3.1. Government Spending Multipliers

The main indications of the present model can be inferred from the above Equations (4)-(7). By solving the equations for \( \dot{C} \) and \( \dot{Y} \) as a function of \( \dot{G} \), we obtain \( \dot{C} = m_c \hat{G} \) and \( \dot{Y} = m_y \hat{G} \), where

\[
\begin{align*}
m_c &= \frac{\alpha + \alpha \phi - \psi}{1 + \phi} \\
m_y &= \frac{\alpha + \alpha \phi - \psi + 1}{1 + \phi}
\end{align*}
\]  

(8) \hspace{1cm} (9)

which are the same as the results of Tervala [13] when \( \psi = 0 \).

These two equations imply that the impact of government spending on private consumption and output is determined by three parameters: \( \alpha \), \( \phi \), and \( \psi \). First, as is clear from (7), \( m_y \) is larger than \( m_c \) and they differ by one: \( m_y - m_c = 1 \).

Moreover, the marginal changes of \( m_y \) and \( m_c \) are equivalent, such that

\[
\begin{align*}
\mu_y &= \frac{\partial m_y}{\partial \alpha} = \frac{\partial m_y}{\partial \psi} = 1 \\
\mu_y &= \frac{\partial m_c}{\partial \psi} = \frac{\partial m_y}{\partial \psi} = -\frac{1}{1 + \phi} \\
\mu_y &= \frac{\partial m_c}{\partial \phi} = \frac{\partial m_y}{\partial \phi} = \frac{\psi - 1}{(1 + \phi)}
\end{align*}
\]

As detailed in subsequent subsection, since we consider the case where \( \phi \geq 0 \) and \( \psi \in [-2, 1] \), it follows that \( \mu_y \) is negative and \( \mu_\psi \) is equal to or lower than zero. Our main interest is in assessing the effects of \( \alpha \) and \( \psi \) on the two multipliers. Evidently, as emphasized in previous works, both parameters play an important role. First, without relying on any parameters, the multipliers are linear-proportional to \( \alpha \). This implies that the productivity of government spending can not only positively enhance the response of output, but also private
consumption, to a rise in government spending, as shown in Linnemann and Schabert [11] and Tervala [13]. Moreover, it can be ascertained that the multipliers are inversely proportional to $\psi$, implying that the complementarity (substitution) between private consumption and government spending can generate (inhibit) positive responses, as shown in Ganelli and Tervala [8].

Up to this point, we have given a simple quantitative rating of the government spending multipliers. In what follows, we focus primarily on the qualitative aspect. For investigating whether or not private consumption and output respond positively to a rise in government spending, we only have to observe their sign.

From an aforementioned condition ($\phi \geq 0$), (8) and (9), we readily obtain the following propositions.

**Proposition 1.** Government spending leads to a rise in private consumption if and only if

$$m_c > 0 \iff \alpha + \alpha \phi - \phi - \psi > 0.$$  

**Proposition 2.** Government spending leads to a rise in output if and only if

$$m_y > 0 \iff \alpha + \alpha \phi - \psi + 1 > 0.$$  

Proposition 1 asserts that private consumption can respond positively to a rise in government spending only when $\alpha$ is large or $\psi$ is small, or both are satisfied. On the contrary, Proposition 2 suggests that the positive output response requires easier conditions.

### 3.2. Numerical Characterization

We further explore how the government spending multipliers can be positive within some realistic parameter settings. Before considering possible parameter settings, we provide an overview of the estimates in previous empirical works, which are relevant to the following exploration.

There are three key parameters for determining the sign of government spending multipliers as shown in (8) and (9). We review the estimates of $\alpha$ and $\phi$ according to Tervala [13]. First, for the production elasticity of government spending, $\alpha$, majority studies in the empirical literature report the estimates that are less than 40 percent (for example, see Aschauer [10]; Glomm and Ravikumar [12]). More importantly, it is noteworthy that they are not estimated on the basis of flow (i.e., government spending) as in the present model, but on the basis of stock (i.e., government capital, also called public capital). On narrowing our focus to the empirical papers that estimate production elasticity by using the flow data, as stated in Tervala [13], we find that there is no such work except for Evans and Karras [14]. They use a panel of the U.S. states in the period from 1970 to 1986 and point out that among the various items of government spending, only the government educational services are productive, and the production elasticity is estimated to be approximately 0.04. Therefore, in our analysis, $\alpha$ should be considered to be quite small and government spending has mild productivity.

The second parameter is the Frisch elasticity of labor supply, denoted earlier...
by $1/\phi$. Till date, many empirical studies have attempted to estimate this value (e.g., MaCurdy [15]; French [16]). While these estimates vary, we are aware of a notable study by Domeij and Flodén [17] who state that the elasticity for men is mostly estimated between 0 and 0.5 in the existing literature, and find that it is estimated to be 50 percent lower than the true value when standard econometric techniques are employed. In other words, there is a strong possibility that the true value can be up to one. This conclusion is not inconsistent with that of Rotemberg and Woodford [18]. As stated in Ganelli and Tervala [8], Rotemberg and Woodford find that the value is between 0.47 and 1.66, and the straightforward average is calculated to be 1.065. This is supported by a relatively recent study by Kimball and Shapiro [19] who provide evidence that the elasticity is approximately one.

The third parameter is $\psi$. While Ganelli and Tervala [8] and Tervala [13] do not mention what value of $\psi$ is empirically supported, the degree varies across countries. Karras [20] estimated $\psi$ for 30 countries by supposing the utility function that is similar to our specification. Table 3 and Table 4 in Karras [20] reported that most of the estimates are in the range of −2 to 1. The average for 30 countries in Table 3 (Table 4) is −1.309 (−1.024) and the median is −1.050 (−0.901), and the approximate value of $\psi$ is considered to be −1.

Summarizing the above-mentioned empirical evidence, it seems reasonable to assume that approximately, $\alpha$ is minimal, $\phi = 1$, and $\psi = −1$. In what follows, these are taken into account.

Figure 1 captures the main implication of our analysis. The figure presents a view of the threshold surface, on which $\alpha + \alpha \phi - \phi - \psi = 0$ holds, with responses of $\hat{C}$ to a rise in $\hat{G}$. In the space below (above) the surface, $\alpha + \alpha \phi - \phi - \psi > 0$ ($<0$) holds, and accordingly, from proposition 1, government spending leads to

Figure 1. Threshold surface on the response of private consumption to government spending.
an increase (decrease) in private consumption. For comparison with Tervala [13], who focuses on only $\phi$ and $\alpha$, and does not consider $\psi$, several iso-$\psi$ contours are also shown in a certain $(\phi, \alpha)$ plane in the figure.

We begin by reconfirming the case of Tervala [13], which corresponds to a contour passing through point $(\phi, \alpha, \psi) = (0, 0, -1)$ in Figure 1. To do this, we now assume that $\psi = 0$, that is, government spending neither complements nor substitutes for private consumption. It turns out at once that if the Frisch elasticity of labor supply is in the vicinity of the value one, as suggested by Kimball and Shapiro [19], the production elasticity of government spending must be higher than 0.4 ($\alpha > 0.4$) in order to satisfy the condition, $\alpha + \alpha\phi - \phi > 0$. Contrary to the results of Linnemann and Schabert [11], it is Tervala’s [13] sum and substance that the positive response of private consumption to government spending cannot occur solely in the presence of productivity, as long as government spending is mildly productive ($0 < \alpha < 0.4$).

Next, we consider the case of Ganelli and Tervala [8] where although complementarity exists between private consumption and government spending, government spending is absolutely not productive ($\alpha = 0$). We observe in Figure 1 that even when certain strong complementarity is assumed, the condition $\phi + \psi < 0$ is less likely to be satisfied in the absence of productivity. Specifically, when government spending is a perfect complement to private consumption ($\psi = -1$), the equation $\phi + \psi = 0$ holds if productivity of government spending is zero and the Frisch elasticity of labor supply is equal to one ($\phi = 1$). As a result, the multiplier $m_c$ becomes zero and thereby, the positive consumption response is not explained. Additionally, if the Frisch elasticity of labor supply is less than one, ($\phi > 1$), then $\phi + \psi > 0$, and private consumption negatively responds to government spending. These examples suggest that the sole presence of complementarity also rules out the positive response of $\hat{C}$ to a rise in $\hat{G}$, in this simple framework.

In summary, it is shown that neither productivity nor complementarity can solely generate positive consumption response in the present model based on Tervala [13]. Instead, we abstract from an alternative possibility that seems a plausible explanation for the positive response of $\hat{C}$ to a rise in $\hat{G}$. To show this, we now consider the case where government spending has both productivity and complementarity, and highlight some of their combinations within the realistic parameter settings as before.

For simplicity, we again assume the plausible case that government spending is a perfect complement to private consumption ($\psi = -1$) and the Frisch elasticity of labor supply is equal to one ($\phi = 1$). Additionally, mildly productive government spending is introduced, and we assume that $\alpha$ runs from higher than 0 through 0.4 at most. Given such parameter settings, there is a definite possibility of the presence of $(\phi, \alpha, \psi)$ in the space below the threshold surface in Figure 1. This illustrates that mildly productive government spending can serve as a crucial factor even if it takes very minimal values (e.g., $\alpha = 0.01$). Likewise,
from Figure 1, we can confirm that multiple combinations of \((\phi, \alpha, \psi)\) satisfy the inequality sign such that \(\alpha + \alpha\phi - \phi - \psi + 1 > 0\), on allowing for mildly productive government spending and complementarity to some extent.

Incidentally, mildly productive government spending may have relevance to the positive output response. Similar to Figure 1, a view of the threshold surface with responses of \(\hat{Y}\) to a rise in \(\hat{G}\) is depicted in Figure 2. \(\alpha + \alpha\phi - \psi + 1 > 0\) (\(<0\) holds in space below (above) the surface, and from proposition 2, government spending leads to an increase (decrease) in output. It follows from Figure 2 that even if \(\alpha = 0\), the positive output response is explained for any value of \(\psi (\in [-2,1])\), except for the case of perfect substitution where \(\psi = 1\) is satisfied. In other words, even if perfect substitution is assumed, the sole presence of mildly productive government spending could cause positive output response.

4. Conclusions

This paper took particular note of a critical gap between theory and empirical evidence in macroeconomics and proposed some new implications. It dealt with the dual aspects of government spending, which are normally treated separately in the literature, and demonstrated that they are inseparably linked. In this context, reconciliation of the empirical evidence that private consumption positively responds to a rise in government spending is necessary. Some plausible parameter settings are considered in this study, and unlike Linnemann and Schabert [11] and Ganelli and Tervala [8], the analysis based on a minimal model formulated by Tervala [13] arrives at a conclusion: the empirical observation cannot be understood in the presence of not only productivity of government spending, but also complementarity between government spending and private consumption.

Figure 2. Threshold surface on the response of output to government spending.
Moreover, mildly productive government spending can be crucial for explaining the empirical observation. Once an appreciable extent of complementarity is assumed, the empirical observation can be explained by productivity, even if it takes some minimal values. In contrast to private consumption, the output in general is likely to respond positively to a rise in government spending only in the presence of mild productivity.

Policymakers are prone to increase government expenditures during an economic downturn. However, from a welfare perspective, not only output but also consumption is relevant. Our results suggest that policymakers need to recognize the importance of quality of government spending rather than quantity when stimulating an economy without a decline in consumption.

**Acknowledgements**

The author thanks an anonymous referee and Hiroshi Ohta for the helpful comments. Financial support from Grants-in-Aid for Scientific Research (C) (25380368, 17K03770) is gratefully acknowledged.

**References**


