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Revenue Sharing in a Sports League with an Open Market in Playing Talent: A Comment

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Abstract

Szymanski [1] and Szymanski and Késenne [2] showed that, in the standard model of a sports league, gate revenue sharing will tend to increase competitive imbalance between weak and strong teams, a seemingly perverse result. Dobson and Goddard [3] claim that "this analysis is flawed. If the revenue function is specified appropriately, gate revenue sharing always reduces competitive inequality." This comment points out the analytical error in their paper which leads to their erroneous conclusion. Once their error is corrected, it is shown that the earlier results stand.

Keywords

Professional Team Sports, Revenue Sharing, Competitive Inequality

1. The Issue

Competitive balance is considered an important issue in sports league. Many people believe that in the absence of a sufficient degree of competitive balance among teams the outcome of league competition will become too predictable; fans will lose interest; and the league will collapse. Redistributing revenues from strong teams to weak teams is widely advocated for overcoming this problem. However, the effectiveness of these schemes depends crucially on the revenue sharing mechanism. In a series of papers Szymanski [1], Szymanski and Késenne [2] and Szymanski [4] (henceforth "SK") showed that in the standard theoretical model of sports league competition, gate revenue sharing (allocating a fixed percentage of ticket revenue to the visiting team) has the perverse effect of increasing competitive imbalance (I refer to this as the SK result).

Dobson and Goddard [3] (henceforth "DG") claim that the SK result is a consequence of "an inappropriately specified revenue function". SK assume that team revenues are a concave function of team win percentage, at first increasing and eventually decreasing. Win percentage in turn depends on the share of talent employed, and hence only relative shares matter. DG assume in addition that revenues are increasing in the absolute quantity of talent in the league, and they claim that adding this to model reverses the SK result. This comment shows that their results are in fact a consequence of an analytical error, and once the error is corrected, the SK result holds.

2. The Model and the Error

The standard model in the sports literature assumes that there are two profit maximizing teams that each chooses a quantity of talent which can be hired at a constant marginal cost per unit of talent. One team, the large market team, is assumed to have a larger revenue generating capacity for any given win percentage. DG (p412) define the revenue functions for teams 1 and 2 as

$R_1 = \sigma\left(T - \gamma T^2\right)\left(w_1 - 0.5w_1^2\right)$ and $R_2 = \left(T - \gamma T^2\right)\left(w_2 - 0.5w_2^2\right)$, where $T = t_1 + t_2$ is the total amount of playing talent in the league, w is win percentage, $\sigma(>1)$ is the parameter characterizing the larger market size supporting team 1 and γ is a parameter that captures decreasing returns to league quality. Win percentage is assumed to be a function of the share of total talent employed: $w_1 = t_1/\left(t_1 + t_2\right)$ and $w_2 = 1 - w_1$. Note that for either team win percentage is bounded between 0 and 1.

A condition for equilibrium is that the marginal revenue of talent of team 1 and team 2 are equalized. This condition can be defined as

$$\frac{(1+\alpha)}{2}\frac{\partial R_1}{\partial t_1} + \frac{(1-\alpha)}{2}\frac{\partial R_2}{\partial t_1} = \frac{(1+\alpha)}{2}\frac{\partial R_2}{\partial t_2} + \frac{(1-\alpha)}{2}\frac{\partial R_1}{\partial t_2} \tag{1}$$

α is the degree of revenue sharing; $\alpha = 1$ implies no revenue sharing; $\alpha = 0$ implies equal revenue sharing. DG identify the four derivatives $\partial R_1/\partial t_1$, $\partial R_2/\partial t_1$, $\partial R_2/\partial t_2$ and $\partial R_1/\partial t_2$ in the middle of page 412. It is in the middle two equations listed in the paper, $\partial R_2/\partial t_1$ and $\partial R_1/\partial t_2$ that the mistakes occur.

In DG the derivatives are stated as follows:

$$\frac{\partial R_2}{\partial t_1} = \left(1 - 2\gamma T\right)\left(w_2 - 0.5w_2^2\right) - \left(1 - \gamma T\right)\left(1 - w_2\right)w_1 \tag{2}$$

$$\frac{\partial R_1}{\partial t_2} = \sigma\left(1 - 2\gamma T\right)\left(w_1 - 0.5w_1^2\right) - \left(1 - \gamma T\right)\left(1 - w_1\right)w_2 \tag{3}$$

where $T = t_1 + t_2$.

The second term in each of these equations is incorrect. The derivative we are looking for in the second term of $\partial R_2/\partial t_1$ is $\partial\left(w_2 - w_2^2/2\right)/\partial t_1$ where $w_2 = t_2/\left(t_1 + t_2\right)$. This derivative is $-\left(1 - w_2\right)t_2/\left(t_1 + t_2\right)^2 = -\left(1 - w_2\right)w_2/T$.

Likewise, the derivative we are looking for in the second term of $\partial R_1/\partial t_2$ is $\partial\left(w_1 - w_1^{\,2}/2\right)/\partial t_2$ where $w_1 = t_1/\left(t_1 + t_2\right)$. This derivative is $-\left(1 - w_1\right)\, t_1/\left(t_1 + t_2\right)^2 = -\left(1 - w_1\right)w_1/T$.

Correcting these errors, Equations (2) and (3) are restated as follows

$$\frac{\partial R_2}{\partial t_1} = \left(1 - 2\gamma T\right)\left(w_2 - 0.5w_2^2\right) - \left(1 - \gamma T\right)\left(1 - w_2\right)w_2 \tag{2'}$$

$$\frac{\partial R_1}{\partial t_2} = \sigma\left(1-2\gamma T\right)\left(w_1 - 0.5w_1^2\right) - \left(1-\gamma T\right)\left(1-w_1\right)w_1 \qquad (3)'$$

These errors have important implication for the derivation of the equilibrium condition. After some manipulation it can be shown that DG Equation (3) should read

$$\begin{aligned} MR_1 = MR_2 = 0 &=> \left(w_1/w_2\right)^2 \\ &= \sigma + \left(1-\alpha\right)/\left(1+\alpha\right)\left(\sigma-1\right)w_1/w_2 \\ &\quad + \alpha/\left(1+\alpha\right)\left(1-2\gamma T\right)/\left(1-\gamma T\right)\left(\sigma w_1 - w_1^2 - w_2 + w_2^2/2\right) \end{aligned} \qquad (3')$$

By inspection it should be clear that this formulation (3') encompasses DG Equation (1), which emerges as a special case where absolute talent has no impact.

Once the correct specification (3') is applied, the claims of the authors are no longer tenable. First, it is impossible to derive any general conclusions about the equilibrium values of w_1 and w_2 from what is a complex polynomial. To prove the existence of an interior solution the authors would need to show that the second order conditions for a maximum are satisfied for values of w_1 in the relevant range, not just the first order condition. The authors do not report anything about the second order conditions.

I have, however, attempted to find simulation solutions choosing particular values of γ and T. In all cases I found there only existed corner solutions where $w_1 = 1$, whether there is revenue sharing or not. When there is no revenue sharing $(\alpha = 1)$ I found that $MR_1 > 0$ and $MR_2 = 0$ when $w_1 = 1$, while under equal sharing $(\alpha = 0)$ I found that $MR_1 > MR_2$ for all values of $w_1 < 1$. Both of these findings imply a corner solution at $w_1 = 1$. I was also unable to find an interior equilibrium for any value of $0 < \alpha < 1$ (a file with numerical examples is available from the author on request).

Thus the claim of the authors to have shown that "Competitive inequality is lower with equal revenue sharing $(\alpha = 0)$ than it is with no revenue sharing $(\alpha = 1)$" is false, and the model they propose does not appear to have an interior solution.

Intuitively the problem with the specification advanced by the authors is that both teams stand to gain by increasing total quality, but since the revenue of team 1 is larger than the revenue team 2 for any given value of own win percentage, team 1 typically has the greater incentive to invest in talent, and this turns out to be true even under revenue sharing.

Finally, it is worth commenting briefly on the authors assertion that the "If revenues depend upon relative team quality only, however, the equal revenue sharing solution $(\alpha = 0)$ is degenerate". The critique is that a solution cannot be defined when $T = 0$ (since win percentage itself is not defined) and hence, they argue the model is inappropriate. First, note that the first order conditions are not defined when $T = 0$ in their model as well, since this is a property of the contest success function which they and previous authors have used. Second, as previous authors have pointed out, reducing talent to zero would also imply paying zero wages, which is not credible in a world where workers have an outside option. The logic of the previous models is that joint profit maximizing owners would reduce wages to the lowest possible level consistent with fielding a team, which would require a strictly positive quantity of talent at a strictly positive wage rate.

3. Conclusion

This comment addresses an error in the mathematical derivation of Dobson and Goddard [3]. These authors claimed to show that gate revenue sharing would increase competitive balance in a model in which demand was increasing in the aggregate quality of players. In fact, their result vanishes once the error has been corrected and the perverse result identified in Szymanski and Késenne [2] still holds. In the standard model, gate revenue sharing implies less, not more, competitive balance.

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