The Panzar-Rosse Statistic Revisited

Hiroshi Gunji1, Yuan Yuan2

1Faculty of Economics, Daito Bunka University, Tokyo, Japan
2Faculty of Economics and Management, East China Normal University, Shanghai, China
Email: hgunji@ic.daito.ac.jp


Received: August 22, 2016
Accepted: December 9, 2016
Published: December 14, 2016

Copyright © 2017 by authors and Scientific Research Publishing Inc. This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Abstract

In this note, we prove that even if the technology of firms exhibits increasing returns to scale, the Panzar-Rosse statistic in a monopolistic competitive market is still available and has a negative value. Further, we show that the statistic would be greater than unity if firms with increasing-returns-to-scale technology were to choose a saddle point under certain conditions. This implies that the value of greater than unity is not actually observed.

Keywords

Panzar-Rosse Statistic, Monopolistic Competition, Increasing Returns to Scale

1. Introduction

The competitiveness of financial intermediaries has attracted the attention of many economic researchers. Bikker and Haaf [1] investigate the competitiveness of the banking industry in 23 countries using the method proposed by Panzar and Rosse [2], and conclude that the industry in most countries exhibits monopolistic competition. Gelos and Roldós [3] and Claessens and Leaven [4] also use the Panzar-Rosse method to estimate the competitiveness of the banking system. In the former, the banking industry is studied in eight emerging markets, and in the later is 50 countries. Moreover, Yuan [5] uses the same method to investigate China’s banking system, and concludes that the market was highly competitive during the period 1997-2000.

One of the reasons why attention is given to the financial market might be that many researchers study and discuss whether economic growth and business cycles can be stimulated by the financial markets' activities1. For instance, Claessens and Leaven [7] investigate the relationship between competition in the financial system and industrial growth, using the Panzar-Rosse statistic as a proxy for competitiveness in the system. Further, Gunji et al. [8] show that competition in the banking industry leads to smaller monetary policy effects on bank lending.

1For recent developments in the literature, see Wachtel [6].

DOI: 10.4236/tel.2017.71004 December 14, 2016
The method suggested by Panzar and Rosse [2] attempts to infer whether the industry is competitive, using the sum of the factor price elasticities of the reduced form revenue equation, denoted by \( \psi \). The estimate of \( \psi \), for example, can be easily obtained from the following log-linear equation:

\[
\ln R_i = \beta_0 + \beta_i \ln w_{i1} + \cdots + \beta_K \ln w_{iK} + X_i \gamma + \varepsilon_i \quad (i = 1, \ldots, n)
\]

where \( R_i \) denotes total revenue, \( w_{ik} \) is a factor price of \( k \)th input \((k = 1, \ldots, K)\), \( X_i \) is a vector of control variables, and \( \varepsilon_i \) is a disturbance. Since \( \beta_k \) indicates the \( k \)th-price elasticity of \( R \), the estimated Panzar-Rosse statistic is defined as \( \hat{\psi} = \sum b_k \) where \( b_k \) denotes the ordinary least squares estimate for \( \beta_k \). Panzar and Rosse [2] show that \( \psi \) should be less than or equal to zero in monopoly (collusive) equilibrium, less than or equal to unity in monopolistic competition, and equal to unity in long-run competitive equilibrium. However, the significance of a value of greater than unity, and under what conditions this happens, have not been proved before.

In this note, we consider the Panzar-Rosse statistic in a simple version of a monopolistic competition model. It is natural to study this model, since it is exploited in a large number of economics fields: economic growth, business cycles, monetary economics, spatial economics, and so on. We prove that, in the market, the Panzar-Rosse statistic is negative, and available even when firms have an increasing-returns-to-scale technology. From this point of view, the Panzar-Rosse methodology is superior to others. For example, the method of Hall [9] requires that the technology of firms exhibits constant returns to scale. We also show that, if firms with increasing-returns-to-scale technology were to choose a saddle point under certain conditions, the Panzar-Rosse statistic would be greater than unity. Since this case is unrealistic, the statistic is never actually observed to be greater than unity.

2. The Model

Suppose that there are a single final good, \( Y \), and intermediate goods, \( y(i) \) for \( i \in [0,1] \). A firm which produces the final good has a production technology,

\[
Y = \left[ \int_0^1 y(i)^\lambda \, di \right]^{1/\lambda}
\]

(1)

where \( \lambda \in (0,1) \) is the elasticity of substitution. The profit of this firm is

\[
\Pi = PY - \int_0^1 p(i) y(i) \, di
\]

(2)

where \( P \) denotes the price of the final good and \( p(i) \) denotes the price of the intermediate good \( i \). The firm maximizes Equation (2) subject to Equation (1). From the first order necessary condition for this problem, the demand for the \( i \)th intermediate product is

\[
y(i) = \left[ p(i)/P \right]^{(\lambda-1)/\lambda} Y
\]

(3)

The firm which produces the \( i \)th intermediate good has Cobb-Douglas production technology.

\(^2\)One of the advantages of this method is that it allows researchers to obtain an unbiased estimate of \( \psi \) in a finite sample if the disturbance satisfies the assumption of strict exogeneity.
\[ y(i) = k(i)^\alpha l(i)^\beta \]  \hspace{1cm} (4)

where \( \alpha > 0 \) and \( \beta > 0 \). Note that we do not assume anything about returns to scale. For simplicity, we assume without loss of generality that these goods are produced using two inputs, i.e., capital stock \( k(i) \) and labor \( l(i) \). The profit of the intermediate firm is

\[ \pi(i) = R(i) - C(i) \] \hspace{1cm} (5)

where the revenue and cost functions are

\[
R(i) = y(i) p(i)/P, \\
C(i) = r(i) k(i)/P + w(i) l(i)/P.
\]

The firm producing the \( i \) th intermediate good maximizes Equation (5) with respect to \( k(i) \) and \( l(i) \) subject to Equations (3) and (4).

**Assumption 1.** \( \lambda(\alpha + \beta) < 1 \).

This inequality assures these intermediate firms of positive profits. (See also the second-order conditions shown below.) The technology of the firm may exhibit increasing returns to scale, e.g., \( \alpha + \beta > 1 \), as long as Assumption 1 holds.

### 3. Panzar-Rosse Statistic

**Definition 1.** The Panzar-Rosse statistic in the model with two inputs is

\[
\psi = \frac{\tilde{r}(i) \partial R'(i)}{R'} + \frac{\tilde{w}(i) \partial R'(i)}{R'},
\]

where \( R'(i) \) denotes the firm’s reduced form revenue function, \( \tilde{r}(i) \equiv r(i)/P \), and \( \tilde{w}(i) \equiv w(i)/P \).

Substituting (3) and (4) into (5), we have

\[ \pi(i) = k(i)^\alpha l(i)^\beta y^{1-\gamma} - \tilde{r}(i) k(i) - \tilde{w}(i) l(i) \] \hspace{1cm} (6)

The first-order necessary conditions for this problem are

\[
\lambda \alpha y^{1-\gamma} k'(i)^{\alpha-1} l'(i)^{\beta} = \tilde{r}(i), \\
\lambda \beta y^{1-\gamma} k'(i)^{\alpha} l'(i)^{\beta-1} = \tilde{w}(i).
\]

These equations are rewritten as

\[
k'(i) = \lambda^{1-\gamma} y^{1-\gamma} \left[ \frac{\alpha}{\tilde{r}(i)} y^{\gamma} \right]^\frac{1-\beta}{1-\gamma} \left[ \frac{\beta}{\tilde{w}(i)} y^{\gamma} \right]^\frac{\beta}{1-\gamma},
\]

\[
l'(i) = \lambda^{1-\gamma} y^{1-\gamma} \left[ \frac{\alpha}{\tilde{r}(i)} y^{\gamma} \right]^\frac{1-\beta}{1-\gamma} \left[ \frac{\beta}{\tilde{w}(i)} y^{\gamma} \right]^\frac{\beta}{1-\gamma},
\]

where \( \gamma = \lambda(\alpha + \beta) \). Therefore, we obtain the reduced form revenue function

\[
R'(i) = \lambda^{1-\gamma} y^{1-\gamma} \left[ \frac{\alpha}{\tilde{r}(i)} y^{\gamma} \right]^\frac{1-\beta}{1-\gamma} \left[ \frac{\beta}{\tilde{w}(i)} y^{\gamma} \right]^\frac{\beta}{1-\gamma}.
\]

Provided \( Y \) and \( P \) as given, the first derivatives of \( R'(i) \) with respect to the factor prices are
\[ \frac{\partial R^*(i)}{\partial \bar{r}(i)} = \frac{\lambda \alpha \ R^*(i)}{1 - \gamma \bar{r}(i)}, \]
\[ \frac{\partial R^*(i)}{\partial \bar{w}(i)} = \frac{\lambda \beta \ R^*(i)}{1 - \gamma \bar{w}(i)}. \]

Hence, we have the Panzar-Rosse statistic,
\[ \psi = \frac{-\gamma}{(1 - \gamma)}. \]

Since \( 0 < \gamma < 1 \) from \( \alpha, \beta > 0 \) and Assumption 1, \( \psi \) is less than zero.

**Proposition 1.** In a monopolistic competitive market under Assumption 1, even if the production technology exhibits increasing returns to scale, the Panzar-Rosse statistic is available, and has a negative value.

It is important to note that Proposition 1 depends critically on Assumption 1. If Assumption 1 is violated, i.e., \( \gamma > 1 \), then the Panzar-Rosse statistic must be greater than unity. Further, from Equation (6) and \( \alpha, \beta > 0 \), the technology of the firms exhibits increasing returns to scale. Note, however, that the second-order sufficient condition for maximization of the firms’ problem is that the Hessian is definitely negative, that is,
\[ \pi_{\alpha} < 0 : \lambda \alpha \ (\lambda \alpha -1) Y^{\alpha-1} k(i) Y^{\alpha -2} l(i) < 0, \]
\[ \pi_{\beta} < 0 : \lambda \beta (\lambda \beta -1) Y^{\alpha -2} l(i) Y^{\alpha -2} < 0, \]
\[ \pi_{\alpha} \pi_{\beta} > \pi_{\alpha}^2 : \lambda (\alpha + \beta) < 1. \]

So the firms need to satisfy \( \lambda \alpha < 1, \lambda \beta < 1 \) and \( \gamma < 1 \). Hence, when \( \gamma > 1 \), we obtain a saddle point from the solution of the first-order conditions. If \( \gamma > 1 \) and if firms with increasing-returns-to-scale technology were to choose a saddle point, then the Panzar-Rosse statistic would be greater than unity. Needless to say, the set-up where each firm chooses a saddle point instead of a maximum is extremely unrealistic. In applications, such a value is not estimated if the regression models that researchers use are appropriately specified. Therefore, once one obtains an estimate that is significantly greater than unity, the robustness of the model must be thoroughly checked.

4. **Conclusion**

This note shows that the Panzar-Rosse statistic is available even when production technology exhibits increasing returns to scale, and that it would be greater than unity if the firms were to choose a saddle point. Thus, the statistic actually would not be estimated to be statistically significantly greater than unity.

**Acknowledgements**

This work was supported by JSPS KAKENHI Grant Number JP22730260. The authors are grateful to Masayoshi Tsurumi and seminar participants at Hosei University for their comments. Of course, all remaining errors are our responsibility.

**References**


Submit or recommend next manuscript to SCIRP and we will provide best service for you:

Accepting pre-submission inquiries through Email, Facebook, LinkedIn, Twitter, etc.
A wide selection of journals (inclusive of 9 subjects, more than 200 journals)
Providing 24-hour high-quality service
User-friendly online submission system
Fair and swift peer-review system
Efficient typesetting and proofreading procedure
Display of the result of downloads and visits, as well as the number of cited articles
Maximum dissemination of your research work
Submit your manuscript at: http://papersubmission.scirp.org/
Or contact tel@scirp.org