Theoretical Economics Letters, 2012, 2, 230-231
http://dx.doi.org/10.4236/tel.2012.22042 Published Online May 2012 (http://www.SciRP.org/journal/tel)

The Policy Role in the Stock Markets

Moawia Alghalith, Esha Ramlogan, Martin Franklin
Economics Department, University of the West Indies, St. Augustine, USA
Email: malghalith@gmail.com

Received April 5, 2012; revised April 20, 2012; accepted April 27, 2012

ABSTRACT

This note is an attempt to model the role of the policymaker in stabilizing the stock markets. In doing so, we present an elasticity formula that links the risk-free interest rate to the value of the stock index.

Keywords: Portfolio; Stock; Risk-Free Interest Rate; Policy

1. Introduction

The theoretical literature on portfolio selection is vast (see, for example, [1-3] among many others. However, the empirical applications of these models have not been extensive. Moreover, the policy role in the financial markets is neglected by the literature, given the extreme importance an the timeliness of the topic in a very volatile financial environment.

To our knowledge, this note is the first methodological and empirical attempt to model the role of policy (the Central Bank) in stabilizing the financial markets. More specifically, we introduce a novel method of measuring the preferences of investors. In doing so, we used historical observed data to compute the values of the unobserved preferences. The results of this estimation are immensely valuable to policy makers because they reveal to them important information about the financial markets. Most importantly, we develop an elasticity formula that links the risk-free interest rate to the value of the portfolio. This formula is very useful to policymakers, since they control the risk-free interest rate. That is, they can adjust this rate to offset a decline in the value of the stock index. This note is organized into two main sections: the development of the model and the empirical analysis using data for the Trinidad and Tobago stock market.

2. The Model and Method

We develop our model using the stochastic portfolio model (see, for example, [4] among many others). The dynamics of the risky asset price are given by

$$dS_s = S_s\left\{\mu_s ds + \sigma_s dW_s\right\}, \tag{1}$$

where the superscript s denotes time, $\mu_s \in C_b^2(R)$ and $\sigma_s \in C_b^2(R)$ are the rate of return and the volatility, respectively.

The wealth process is given by

$$X_T^{\pi} = x + \int_t^T \left\{r_s X_s^{\pi} + (\mu_s - r_s)\pi_s\right\} ds + \int_t^T \pi_s \sigma_s dW_s,\quad t \le s \le T, \tag{2}$$

where x is the initial wealth, π_s is the portfolio process, with $E\left[\int_t^T \pi_s^2 ds\right] < \infty$. The portfolio $\pi_s \in \mathcal{A}(x)$ is admissible (*i.e.* $X_s^{\pi} \ge 0$) The dynamics of the wealth process are given by

$$dX_s = \left\{r_s X_s^{\pi} + (\mu_s - r_s)\pi_s\right\} ds + \pi_s \sigma_s dW_s$$

The investor maximizes the expected utility of the terminal wealth

$$V(t,x) = \sup_{\pi_t} E\left[U\left(X_T^{\pi}\right)\middle|\mathcal{F}_t\right], \tag{3}$$

where $\mathcal{F}_t$ is the filtration of information, $V(\cdot)$ is the value function, $U(\cdot)$ is a continuous and bounded utility function.

If the value function is smooth, it satisfies the Hamilton-Jacobi-Bellman partial differential equation

$$V_t + r_t x V_x + \sup_{\pi_t}\left\{\pi_t(\mu_t - r_t)V_x + \frac{1}{2}\pi_t^2 \sigma_t^2 V_{xx}\right\} = 0, \tag{4}$$

Hence, the optimal solution is

$$\pi_t^* = -\frac{(\mu_t - r_t)V_x}{\sigma_t^2 V_{xx}}. \tag{5}$$

The crucial relationship for policymakers is the relationship between the optimal portfolio (index) and the Treasury Bill rate (the risk-free rate). This relationship is given by the following elasticity formula

$$\frac{\partial \pi_t}{\partial r_t}\frac{r_t}{\pi_t}=\frac{V_x}{\sigma_t^2 V_{xx}}\frac{r_t}{\pi_t}. \tag{6}$$

3. Empirical Results

However, we need to to generate a data series for $V_x/V_{xx} \equiv \beta$. In particular, we used daily data (2005-2011) for the Trinidad and Tobago Composite Index (π), the discount rate on the Trinidad and Tobago Treasury bills (r) and the return on the Trinidad and Tobago Composite Index (u). We also generated data for the volatility of the index as follows

$$\sigma_t^2=\frac{1}{3}\sum_{t=1}^{3}\left(S_t-S_{t-1}\right)^2.$$

Using (5), we generate the following data series for β by direct calculations. Thus, we can calculate (6), using the average values of β, π, σ, and r, as follows

$$\frac{\partial \pi_t}{\partial r_t}\frac{r_t}{\pi_t}=\left(\frac{-250000}{2526.411071}\right)\left(\frac{6.855857143}{917.19}\right)=-0.73.$$

Thus a 1% decrease in the Treasury bill rate will increase the portfolio by 0.73%. This relationship enables the policymakers to offset swings in the portfolio and thus stabilize the stock market.

REFERENCES

[1] M. Alghalith, "General Closed-Form Solutions to the Dynamic Optimization Problem in Incomplete Markets," *Applied Mathematics*, Vol. 2, No. 4, 2011, pp. 433-435. doi:10.4236/am.2011.24054

[2] S. E. Shreve and H. M. Soner, "Optimal Investment and Consumption with Transaction Costs," *The Annals of Applied Probability*, Vol. 4, No. 3, 1994, pp. 609-692. doi:10.1214/aoap/1177004966

[3] G. Yin and X. Y. Zhou, "Mean Variance Portfolio Selection under Markov Regime: Discrete Time Models and Continuous Time Limits," *Proceedings of the 15th International Symposium on Mathematical Theory of Networks and Systems*, Leuven, 5-9 July 2000, pp. 1-6.

[4] J. Cvitanic and F. Zapatero, "Introduction to the Economics and the Mathematics of Financial Markets," MIT Press, Cambridge, 2004.