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What Is Number?

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Abstract

What is number? This question is difficult to answer. Because the number is one of the most basic concepts, it is difficult to define the natural number with other concepts. Still, this problem is worth trying to answer. Now, everything is digitized and processed on computer. The importance of the number is increasing day by day. Now is time to consider what number is. Throughout the history of humankind, the ancient Greeks considered this question most profoundly. In particular, Plato defined the natural number one. The natural number one is equal, invariable and indivisible. These properties are intuitively acceptable. However, we have never seen or touched the natural number one itself. How can we know it? Socrates said that we know it before birth. This claim is called anamnesis. In this paper, we use a method, in which Socrates' anamnesis is studied by the contemporary science. From a modern viewpoint, we could take Socrates' anamnesis to mean that the natural number one is written in our genes. This article considers whether there is a biological entity corresponding to the natural number one. As a result, we find that a life itself is the prototype of the natural number one, and then properties of life make a critical base of DNA similar to the natural number one through natural selection. A life is an integrated and indivisible system, which resists the law of entropy. Furthermore, the basic properties of life enable natural selection, which conserves genetic information despite the law of entropy. The source of the power, which enables life to resist the law of entropy, is the genetic information. In conclusion, a life is a prototype of the natural number one. Furthermore, a life recognizes nature using natural numbers and resists the law of entropy using natural numbers.

Keywords

Gene, Entropy, Life, Natural Number One, Natural Selection

1. Introduction

What is number? This question is difficult to answer. Even though many philosophers have discussed it, there is no conclusion. However, everyone knows and uses numbers. Furthermore, everything is quantified in the capitalist society. Anything can be converted into money. You can buy goods, medical care, entertainment, information, and so forth with money. Furthermore, we spend a considerable time in our lives working to gain money. It seems that numbers dominate us.

Moreover, today is the computer age. Computer signals consist of 0 s and 1 s. Although computers could initially only handle numbers and were used solely for calculations, they now handle language, voice and images, and can even process music and movies. Everything is digitized; anything can be represented by numbers in the computer.

Not only are we surrounded by numbers, but numbers are within us. On this front, neuroscience has made great progress. The neural mechanism of the recognition of numbers has been gradually elucidated (Butterworth, 1999; Dehaene, 2011). It has been shown that animals also have the ability to count and recognize numbers (Devlin, 2005). Recently, neurons responding to numbers were found in the monkey's brain (Nieder et al., 2002, 2004; Sawamura et al., 2002). Even more progress in this field is expected. Now is the time to think about what number is. In human history, the ancient Greeks considered this question most profoundly. Thus, we start from Plato's dialogue.

Plato defined the natural number one. However, the natural number one cannot be perceived by any sense. Then, Socrates pointed out that we have to know the natural number one before birth. This idea is called the anamnesis. That is, there must be a biological entity corresponding to the natural number one. In this paper, we shall search the biological entity of the natural number one.

2. The Natural Number One

It is difficult for philosophers and mathematicians to define the natural number one. Too basic concepts are difficult to define. The axiomatism is invented in order to avoid this difficulty. Peano defined natural numbers without defining the natural number one. Peano's axioms are as follows (Kennedy, 1980).

1) One is a number.

2) The sign + placed after a number produces a number.

3) If a and b are two numbers, and their successors are equal, then they are also equal.

4) One is not the successor of any number.

5) If s is class containing one, and if the class made up of the successors of s is contained in s, then every number is contained in the class s.

Peano left the natural number one as an undefined term. According to axiomatism, Peano's axioms rule the relationship among primitive notations, which are undefined. This method, axiomatism, is the mainstream of modern mathematics. Thus, the majority of mathematicians do not discuss much about what the natural number one is.

However, the ancient Greeks, the founders of philosophy, constantly discussed such problems. Although many of their arguments have been lost, Plato's dialogue remains. The ancient Greeks thought of one as a basic concept. Plato emphasized the importance of one, such as in the following excerpt (Plato, 2007b).

> I mean, as I was saying, that arithmetic has a very great and elevating effect, compelling the soul to reason about abstract number, and rebelling against the introduction of visible or tangible objects into the argument. You know how steadily the masters of the art repel and ridicule anyone who attempts to divide absolute unity when he is calculating, and if you divide, they multiply, taking care that one shall continue one and not become lost in fractions.
>
> That is very true.
>
> Now, suppose a person were to say to them: O my friends, what are these wonderful numbers about which you are reasoning, in which, as you say, there is a unity such as you demand, and each unit is equal, invariable, indivisible, -what would they answer?

The above is a dialogue between Socrates and Guraukon in *The Republic*, in which the properties of the natural number one are explicitly stated. The natural number one has three properties: equality, invariability and indivisibility. Further, Socrates elaborated that the pure one must have no sensible property. That is, the natural number one has no tangible or physical properties; it is abstract. An important feature of numbers is their lack of sensible properties, which allows us to quantify everything with them.

In seeking out the true identity of the natural number one, we have an advantage over the ancient Greeks since we can use modern science and technology. The starting point of the configuration of numbers is natural numbers, which are sets of the natural number one. What is the natural number one? We explore this question in the following sections.

3. Anamnesis

The natural number one does not tangibly exist anywhere. This leads to an important question: where did the natural number one come from? A hint of the answer is in Plato's dialogue. In *Phaedo* (Plato, 2007a), Socrates said that the complete equality does not exist in reality. That is, we know no viewable and tangible objects that are equal to each other. Yet, we know the complete equality. Where did such an abstract equality come from? Socrates said as follows.

> Then before we began to see or hear or perceive in any way, we must have had a knowledge of absolute equality, or we could not have referred to that the equals which are derived from the senses?—for to that they all aspire, and of that they fall short?

Socrates said that we must know the equality before birth. This claim is very bold, but stands to reason.

We shall reconsider properties of the natural number one. Socrates said that completely equal objects do not exist. Likewise, an indivisible object does not exist. Even a diamond, which is the hardest material, can be divided. Thus, any tangible object cannot be indivisible. Likewise, an invariable object does not exist. Even a stone is eroded by weather. Any tangible object cannot be completely invariable for a long time.

Using the same logic as Socrates, we must know indivisibility and invariability before birth. Necessarily, we know the natural number one itself before birth. Now, we seek a biological entity of the natural number one.

4. The Blank Canvas

When we consider this problem, Schrodinger's thinking is helpful. It seems that human beings have different selves. Schrodinger said in the epilogue of "What is life?" (Schrodinger, 1967b):

> Yet each of us has the indisputable impression that the sum total of his own experience and memory forms a unit, quite distinct from that of any other person. He refers to it as "I" and What is this "I"?
>
> If you analyse it closely you will, I think, find that it is just the facts little more than a collection of single data (experiences and memories), namely the canvas upon which they are collected. And you will, on close introspection, find that what you really mean by "I" is that ground-stuff upon which they are collected.

Schrodinger stated that differences among selves are due to individual experiences and memories, and thus, a newborn baby's self is a blank canvas. Surely, a newborn baby has no experience and no memory. In this sense, newborn babies' selves are equal to each other. That is, he claimed the unity of selves. However, there are genetic differences among neonates' brains. Let us proceed to a deeper discussion.

Schrodinger's idea can be generalized. As applied to general life, experiences and memories correspond to genes. If we excluded all genes from modern living organisms, basic properties of the life would remain. This operation will extract the minimum requirement of life.

Firstly, there is an indivisible unit of life. If we repeatedly divide any life, we will reach the indivisible unit of life. Even, a multicellular organism or a syncytium can be divided into cells, which are indivisible. For example, human cells cannot live alone by themselves, but individual cells can be artificially cultured. Furthermore, if a cell is forcedly divided, the cell will die. A cell is indivisible as long as it lives. Then, we need to define what is alive. From the standpoint of physics, living is to resist the law of entropy (Schrodinger, 1967a).

Secondly, a cell is an integrated system, which resists the law of entropy. For example, the homeostasis means that a living organism keeps its internal environment constant: pH, salt concentration, temperature, etc. However, if a living organism dies, it will be rapidly degraded. According to the law of entropy, a dead cell will approach the equilibrium. Thus, a living cell configures a highly ordered system, which resists the law of entropy. Since the system is highly integrated and elaborated, when the system failure occurs, it will rapidly decay. We will discuss how a life resists the law of entropy in later sections.

Finally, a cell is indivisible and resists the law of entropy. This is the basic property of a life. In this sense, all modern living organisms are equal. That is, if we regard basic properties of a life as a blank canvas, all modern living organisms are paintings on the same blank canvas. This leads to the idea that a life is the prototype of the natural number one. When we focus on the basic property of a cell, all cells are equal, indivisible and invariable.

5. If It Can Go Wrong, It Will Go Wrong

Life can be thought of as a prototype of the natural number one, but genetic information has properties closer to the natural number one. For example, histones, which bind to nuclear DNA, are very important proteins for survival. In particular, histone H4 is the most highly conserved protein.

Famously, the sequence of histone H4 differs on only two sites across the total 102 amino acids between a pea and a calf (DeLange et al., 1969). It is assumed that animal and plant diverged from the common ancestor about 1.6 billion years ago (Wang et al., 1999). This conservation of information is astounding. During the period, continents have drifted (Scotese, n.d.) and geographic features on the surface of the earth have changed. Necessarily, any object could not remain on the surface of the earth during the period.

Why and how is the above conservation possible? The major obstacle is the law of entropy. Then, we need to review the second law of thermodynamics. The law of entropy means that a thermodynamic system approaches equilibrium, which is the state of maximum entropy. The equilibrium is the macroscopic static state. At the microscopic level, two opposite reactions are balanced. An example is shown in the next paragraph.

First, we put water into an empty container and seal it. Second, the system, which consists of water and air, approaches equilibrium. Water evaporates until the water vapor in the air reaches saturation. Finally, the macroscopic change stops at equilibrium. From the microscopic viewpoint, the rate of the evaporation of the water equals the rate of the devolatilization of the water vapor in equilibrium. That is, the equilibrium is the balance of two reversible reactions.

The law of entropy means that nothing is unbreakable because the shape of what we see is a biased distribution of matter. Inevitably, when we try to preserve information for a long time, it deteriorates and is eventually lost. For example, when we try to record characters on paper, it can be easily burned or torn. Even if we carve characters in stone, it will be eroded by weather. Surely, any information cannot be preserved permanently.

The law of entropy is applied to a copy so the complete copy is impossible. We assume that the accuracy of a copy is r and the number of copies is n. Then, as n

increases, r^n approaches 0 without limit:

$$\lim_{n \to \infty} r^n = 0 \quad (r < 1). \tag{1}$$

Equation (1) is the result of the law of entropy.

If natural selection did not exist, Equation (1) could be applied to living organisms. The error rate of the DNA replication is about 1 base per 10^9 bases (Bruce et al., 2002b). Under suitable conditions, Escherichia coli (*E. coli*) divides within 30 minutes (Todar, n.d.). If *E. coli* divided at this rate without natural selection, all bases of *E. coli's* DNA would change within one million years. In this case, life would be extinct rapidly.

6. The Conservation of Genetic Information despite the Law of Entropy

Why can living organisms conserve genetic information despite the law of entropy? Natural selection, the great discovery of Darwin (Darwin, 1876) and the driving force of evolution, enables the conservation. We consider how natural selection conserves genetic information despite the law of entropy.

The important point is that lethal genes are irreversibly removed by natural selection. As death is an irreversible process, natural selection can protect information against entropy. When there is an irreversible process in a system, the second law of thermodynamics cannot be applied to the system. As a result, notwithstanding repeated copying, the genes necessary for survival do not change.

The evolution rate of a protein is determined by natural selection according to the neutral theory of molecular evolution (Kimura, 1983). If a protein is important for survival, it must be conserved. Then, it changes slowly. In contrast, if a protein has no function, it changes rapidly. This logic can be applied to each amino acid residue of a protein. That is, the important amino acid residue for a function of a protein is conserved by natural selection. In hemoglobin, the surface portion is less important than the heme pocket, which is vitally important for oxygen transport. Amino acid residues on the surface of the hemoglobin change ten times faster than amino acid residues in the heme pocket (Kimura, 1973).

If a protein has no function, it is not the subject of natural selection. Then, it will be one of the fastest changing proteins. In reality, fibrinopeptides have no function (Bruce et al., 2002b). When bleeding occurs, fibrinopeptides are discarded from fibrinogen. Then, fibrins are generated, playing the central role in blood clotting. The amino acid sequence of discarded fibrinopeptides does not affect the survival rate of the living organism. That is, any amino acid residue is not the target of natural selection. Therefore, the fibrinopeptide is one of the fastest evolving proteins.

If any change of the amino acid residues of a protein is fatal, the copying accuracy of amino acid residues of the protein is 1 because any miscopy is eliminated by death. Necessarily, if copies are repeated any times, the accuracy is constantly 1:

$$\lim_{n\to\infty} r^n = 1 \quad (r = 1). \tag{2}$$

Equation (2) can be applied to an idealized example. Conserved amino acid residues of histone H4 are close to an idealized example. Hence, conserved amino acid residues of histone H4 have close properties to the natural number one because they remained identical during 1.6 billion years. Each conserved amino acid residue has the properties of equality, invariability and indivisibility. Their identity depends on natural selection, which requires the irreversibility of death.

Of course, the above argument can be applied to DNA. Especially, the DNA sequence of ribosomal RNA gene is universal among three domains. Focusing on this fact, Carl Woese used the small subunit of the ribosome for classifying three domains (Woese et al., 1977). Woese said that genes of the translation system were fixed before any bifurcation of the universal tree of life because genes of the translation system are conserved among three domains (Woese, 1998). Woese named this fixation "crystallization." As a result, critical bases of the small subunit of the ribosomal RNA are highly conserved (Bruce et al., 2002a). Necessarily they have similar properties of the natural number one. They are indivisible and equal to each other. Furthermore, they are invariable over three billion years. Therefore, life itself is the prototype of the natural number one, and life recognizes nature using genes. As a result, critical bases of DNA for survival have very similar properties of the natural number one.

7. Discussion

Firstly, considering the natural number one, we can intuitively accept Plato's definition of one. However, we cannot logically derive the definition of the natural number one. Regardless, we are able to use the natural number one. Since the natural number one is too fundamental a concept, it is difficult to derive it from other concepts. As described above, because natural numbers come from life itself, it is the starting point of all other concepts.

Next, we shall consider the relationship between DNA and natural numbers. For example, let us consider Darwinian evolution of E. coli. The genome size of *E. coli.* is 4×10^6 bases (Bruce et al., 2002a), while the error rate of the bacterial DNA replication is one base per 10^9 bases (Bruce et al., 2002b). Hens, the replication of *E. coli's* DNA is almost correct. Usually, a cell division of an *E. coli* generates two exact copies. Occasionally, a mutation occurs. Usually, the mutation is the change of only one base.

In the above example, the mutated base serves as a label for an individual of Escherichia coli. If the mutation is highly advantageous, the property of the mutated base will approach the natural number one through the natural selection of mutants. The property as a natural number one extracted from an individual mutant is given to the mutated base.

From the above example, we can realize that genetic information is refined by natural selection. If the genetic information system itself is subject to natural selection, the genetic information system will be gradually refined. However, the current genetic information system will not change so easily, because it has been almost completed. Then, Woese's genetic annealing model is helpful. Woese claimed as follows (Woese, 1998):

> A genetic annealing model for the universal ancestor of all extant life is presented; the name of the model derives from its resemblance to physical annealing. The scenario pictured starts when "genetic temperatures" were very high, cellular entities (progenotes) were very simple, and information processing systems were inaccurate. Initially, both mutation rate and lateral gene transfer levels were elevated.

When we accept Woese's model, genetic information systems could be primitive prior to Darwinian evolution.

Therefore, genetic information has properties as natural numbers because a life is the prototype of the natural number one. As a prerequisite, life must be sufficiently complex for the irreversibility of death, and there is a rudimentary genetic information system, which is digital information system and can make a rough copy of a life. Then, if there is an invisible life resisting entropy, evolution will start with a rudimentary genetic information system. Once evolution begins, eventually the critical genetic information for survival will have properties as natural numbers.

Next, we shall discuss the significance of this paper. Firstly the practical social significance is considered. The fact that life is equal to each other provides the concrete basis for humanism and democracy. Particularly, the equality of lives leads to philanthropism for all lives. Secondly, the academic significance is considered. The fact that a life is the prototype of the natural number one provides the concrete basis for mathematics. Based on this fact, there is the possibility of rebuilding mathematics.

Finally, the future direction of the research is considered. We considered the relationship among information, entropy and life, and then we show that the natural number one is key concept. As the direction of further research, the relationship between information and entropy should be clarified more. Furthermore, the role of information in the beginning of life is also the goal of major research. Additionally, there is much room for research in neuroscience, but this paper did not mention it. We shall point out only one fact that is the basis of the future research.

The concept of the natural number one is deeply rooted in the essence of life. However, the concept of the natural number one was created by the human brain. Here, Socrates' anamnesis leads us. We know the pure natural number one before birth. Thus, there should be something corresponding to the natural number one in our brain.

The most likely candidate is the action potential of the neuron, which is the major signal among neurons. The electrical voltage and the duration time of the action potentials of neurons are constant. A remarkable feature of neurons is their universality (Nicholls et al., 2002). For example, there are many kinds of sensory signals, including pain, temperature, and touch. However, they are represented by the number of the same action potentials of neurons. The motor command is also the same signal. Further, there is almost no difference between species. Human beings, earthworms, mosquitos, and octopuses have almost the same action potential of neurons. This means that the action potential of the neuron has been almost invariable during the evolution period of the nervous system. Therefore, the action potential of the neuron resembles the natural number one.

8. Conclusion

The origin of the natural number one is life itself. The genetic information system is constructed from the basic properties of life required for natural selection. It is the result of natural selection that the most conserved bases of DNA have close properties of the natural number one. In concussion, life can be regarded as recognizing nature by natural numbers.

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