Maintaining an Optimal Flow of Forest Products under a Carbon Market: Approximating a Pareto Set of Optimal Silvicultural Regimes for Eucalyptus fastigata

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A competitive co-evolutionary Multi-Objective Genetic Algorithm (cc-MOGA) was used to approximate a Pareto front of efficient silvicultural regimes for Eucalyptus fastigata. The three objectives to be maximised included sawlog, pulpwood and carbon sequestration payment. Three carbon price scenarios (3CPS), i.e. NZ $25, NZ $50 and NZ $100 for a tonne of CO2 sequestered, were used to assess the impact on silvicultural regimes, against a fourth non-carbon Pareto set of efficient regimes (nonCPS), determined from a cc-MOGA with two objectives, i.e. competing sawlog and pulpwood productions. Carbon prices included in stand valuation were found to influence the silvicultural regimes by increasing the rotation length and lowering the final crop number before clearfell. However, there were no significant changes in the frequency, timing, and intensity of thinning operations amongst all the four Pareto sets of solutions. However, the 3CPS were not significantly different from each other, which meant that these silvicultural regimes were insensitive to the price of carbon. This was because maximising carbon sequestration was directly related to the biological growth rate. As such an optimal mix of frequency, intensity, and timing of thinning maintained maximum growth rate for as long as possible for any one rotation.

Keywords: Optimal Control; Competitive Co-Evolutionary Multi-Objective Genetic Algorithm (cc-MOGA); Pareto Front; Forest Holding Value; Kruskal-Wallis Test; Multiple Comparison Procedure

Introduction

The central focus of our analysis was to approximate a set of optimal silvicultural regimes for a Eucalyptus fastigata forest stand under a carbon market. Each estimated regime was expressed as a set of values that included an initial planting stocking, frequency of thinning, timing of thinning, intensity of thinning, final crop number prior to clear-felling, and rotation length. We, therefore, crafted a three-objective optimisation problem, which simultaneously maximised, sawlog, pulpwood and carbon sequestration payment (under three different payment scenarios). This optimisation problem, described later, was based on a two-objective optimisation problem that was successfully solved by simultaneously optimising competing sawlog and pulpwood products (Chikumbo & Nicholas, 2011). The results from the two-objective and three-objective optimisation runs were statistically analysed to decipher the nuances of silvicultural strategies under a carbon market.

Assumptions and Forest Holding Value

To carry out the analysis, we assumed fixed prices for liquid fuels and fossil fuel-based fertilisers such that the carbon price would remain static over the rotation period. This assumption was based on the observation of the EU Emissions Trading Scheme where the carbon price was heavily influenced by fossil fuel prices, which tend to be volatile (White, 2007). The carbon price would in turn influence the forest holding value, and ultimately impact the stand silvicultural regimes of E. fastigata. Forest holding values enable valuation of timber as real property, where timber for immediate harvesting has a liquidation value and the immature resource has a holding value (Mayo & Straka, 2007).

Any forest has an immediate liquidation value if the existing timber is clearfelled and sold along with the land. The forest holding value is the present value of holding the forest until the optimal rotation age (maximum present value) and then selling the timber and land (Klemperer, 1996). The concept is consistent with standard forestry valuation concepts such as land expectation value (Faustmann, 1995; De Jong, Tipper & Montoya-Gómez, 2000). Note that the forest is financially immature for as long as the forest holding value exceeds the forest liquidation value. Therefore, the rotation age should be allowed to increase until the two values are equal (Mayo & Straka, 2007). Thus, the forest holding value provides an ideal financial criterion to evaluate the impact of carbon sequestration payments on the optimal rotation length.

Contribution to Forest Literature

There is an expectation that any forester/land owner wishing to engage in forest-carbon trading, in order to take advantage of a new income stream from carbon sequestration, would want to know the ideal/optimal silvicultural regimes for his/her crop that will not only maximise carbon sequestration (for maximum
pay out), but also maximise production of sawlog and/or pulpwood (De Jong, Tipper & Montoya-Gómez, 2000). However, we do know that many forest analysts have shown that increasing the rotation length would be the sensible thing to do (Appel, 2001; Asante, Armstrong & Adamowicz, 2011; Gutrich & Hoshwarth, 2007). What is scarce in literature is:

1) How the frequency, intensity and timing of thinning for a silvicultural regime are affected;
2) Whether the longer rotation length is linked to a higher or relatively lower final crop number before clearfell;
3) The ability to simultaneously cater for pulpwood and sawlog products under a prevailing carbon market from a single regime instead of different regimes where each specifically caters for a unique product; and
4) How the silvicultural regimes are affected by different carbon prices.

Our paper addresses these issues. The specifics of determining the optimal initial planting stocking, optimal rotation length, frequency, timing and intensity of thinning, tree species and site, have been modeled by forest analysts using multi-stage optimisation since the 60s’ (Hool, 1965), with mixed successes. The reasons for these mixed successes boiled down to the use of inappropriate growth functions, and an inability to do an exhaustive search for all possible states in a dynamic programming formulation (Chikumbo, 1996; Chen, Rose & Leary, 1980). Bellman (1957) coined this exhaustive search problem, the “curse of dimensionality”.

### Historical Background of Problem Solving

A pulpwood production was characterised by short rotations and a relatively longer rotation with thinning (i.e., partial harvesting) for a sawlog production regime (Newman & Wear, 1993). To overcome the curse of dimensionality, the determination of optimal silvicultural regimes for separate pulpwood and sawlog production was pursued using a specialised mathematical formulation, i.e. a combined optimal control and mathematical formulation, i.e. a combined optimal control and optimal parameter selection optimisation (Chikumbo & Mareels, 2003). The growth dynamics of a tree crop stand were described with difference equations in discrete-time and the complete mathematical formulation was as follows:

\[
\min_{u,z} \left\{ J_0(u,z) = \phi_0(x(T),z) + \sum_{t=0}^{T} J_0(t,x(t),u(t),z) \right\}
\]

subject to the growth dynamics,

\[
x(t+1) = f(t,x(t),u(t),z), \text{ (which is the state variable)},
\]

where,

- \( J_0 \) is the cost functional (i.e. a function of state and control variables or simply the objective function);
- \( \phi_0 \) = continuously differentiable function;
- \( J_0 \) = continuously differentiable function with respect to the state and control variables;
- \( t = 0, 1, \ldots, T - 1 \) (for time in years);
- \( T \) = rotation age; and
- \( z = \hat{z} \) = estimated parameter(s) independent of time,

\[
u(t) = [u_1(t), \ldots, u_j(t)],
\]

which represented the control vector over the rotation length at one year intervals expressed as the number of trees harvested per hectare, i.e.

\[
spb(t) = spb(t-1) - u_i(t-1),
\]

where, \( spb(t) \) is the number of standing trees at time, \( t \), and

\[
u_i(t) \leq u_i(t) \leq u_{i^*},
\]

where for each \( u_i \), there are Lower and Upper bounds.

The state variable consisted of the mean stand height, which was the mean height in metres per hectare (\( mht(t) \)), the stand basal area in square metres per hectare (\( sph(t) \)), and the stand volume in cubic metres per hectare (\( vol(t) \)), i.e.

\[
x(t) = \begin{bmatrix} mht(t) \\ sph(t) \\ vol(t) \end{bmatrix},
\]

where,

\[
mht(t) = a_{1sw}mht(t-1) + a_{2sw}mht(t-2) + b_{sw},
\]

\[
a_{1sw} = f(spwb(t)), a_{2sw} = f(spwb(t)),
\]

\[
b_{sw} = f(spwb(t)),
\]

was a discrete-time dynamical model/difference equation (Ljung, 1987) of mean stand height with parameters, \( a_{1sw}, a_{2sw}, \) and \( b_{sw} \) and that were functions of the number of standing trees, \( spb(t) \),

\[
sph(t) = a_{1sw} sph(t-1) + a_{2sw} sph(t-2) + b_{sw},
\]

\[
a_{1sw} = f(spwb(t)), a_{2sw} = f(spwb(t)),
\]

\[
b_{sw} = f(spwb(t)),
\]

the stand basal area, also a discrete-time dynamical model with parameters \( a_{1sw}, a_{2sw}, \) and \( b_{sw} \) that were functions of the number of standing trees, \( spb(t) \),

\[
vol(t) = a_{1vol} vol(t-1) + a_{2vol} vol(t-2) + b_{vol},
\]

\[
a_{1vol} = f(spwb(t)), a_{2vol} = f(spwb(t)),
\]

\[
b_{vol} = f(spwb(t)),
\]

the stand volume function, a discrete-time dynamical model with parameters, \( a_{1vol}, a_{2vol}, \) and \( b_{vol} \) that were functions of the number of standing trees, \( spb(t) \),

\[
z = \left[ z_{0**(1)}, z_1 \right]
\]

the system parameters independent of time, where,

\[
z_{0**(1)} = \text{estimated initial planting stocking;}
\]

\[
z_1 = \text{estimated rotation length},
\]

subject to Lower and Upper bounds,

\[
z_0^{1**} = z_{0**(1)} \leq z_0^{2**}, \text{ and } z_1^1 \leq z_1 \leq z_1^2
\]

The problem (1)-(10) was solved using Pontryagin’s Maximum Principle (PMP) (Chikumbo & Mareels, 2003). Only a single objective problem was solved at any one time, i.e. either a value production or a volume production cost functional. It is possible to solve a multi-objective optimal control problem using PMP (Malinowska & Torres, 2007) for a finite number of cost functionals, but this was never meant to be because of one problem. Trying to estimate the optimal rotation length as a system parameter led to ill-conditioning (i.e. an accumulation
Research Focus and Problem Solving

A switch to a single objective genetic algorithm eliminated
the phase-1 ill-conditioning problem (Chikumbo, 2009a). Chi-
kumbo and Nicholas (2011) demonstrated a two-objective ge-
etic algorithm that simultaneously optimised for a value and
volume production for *Eucalyptus fastigata*. Figure 1 shows
a summary of a generic genetic algorithm and how it works.

A genetic algorithm is initialized with a population of ran-
domly generated individuals which is a guided process of “se-
lection”, “crossover/recombination” and “mutation”. Individu-
als are selected on the basis of their fitness for reproduction.
The parent individuals are recombined to produce offspring
where only some of them are mutated with a certain probability.
The fitness of the offspring is then computed, resulting in the
parents being replaced, thus producing a new generation. If
the criteria of the objective function(s) are not met, this cycle is
performed again until the optimisation criteria are reached
(Polheim, 2006).

Therefore, a set of efficient silvicultural regimes that satis-
fied carbon sequestration payments, sawlog/value production,
and pulpwood/volume production at different levels of "trade-
offs" was estimated using genetic algorithms, eliminating the
need for determining separate regimes for sawlog and pulp-
wood under a carbon market. Such regimes give the forestier/
land owner the ability to satisfy the market with all products
whilst maximizing carbon sequestration payments, with the
flexibility of meeting an increased/decreased supply of any one
of the products as dictated by demand.

The critical part of approximating the set of efficient thin-
ning regimes was to find “trade-off” solutions (i.e. non-domi-
nated solutions) where for each solution an improvement in one
objective did not lead to worsening in the other (Osborne &
Rubenstein, 1994). The set of solutions to the three-objective
problem was determined using a competitive co-evolutionary
genetic algorithm with five subpopulations of 100 individuals
each, computed over 1000 generations. These sub-populations
were evolved independently for a certain number of generations
(isolation time). After the isolation time a number of individu-
als were distributed between the sub-populations (a process
called migration). Each sub-population exerted selective pres-
sure on the other, thereby maintaining diversity a lot longer
than each sub-population would do solitarily, thereby guarding
against premature convergence. When competition was super-
imposed between the sub-populations, the ones with higher
mean fitness values were allowed to maintain larger sub-popu-
lation sizes and received more capable individuals, since they
had more chances of finding the global optimum (Chikumbo,
2009b; Chikumbo, 2012).

The Fonseca and Fleming ranking scheme (Fonseca & Flem-
ing, 1993) was used to determine the non-dominated solutions,
also referred to as the Pareto front. A conflict in the objectives
results in a trade-off set (i.e. Pareto), which means that the so-
lutions in the set are optimal in the wider sense that no other
solutions in the search space are superior to them when all the
objectives are considered.

Fonseca and Fleming called their ranking scheme, Multi-
Objective Genetic Algorithm (MOGA) and it involves assign-
ing an individual’s rank (in the objective function space) equal
to the number of population individuals that dominated that
individual. What this means is that ranking of the individuals
prior to selection for recombination is done according to the
degree of domination; the more members of the current popula-
tion that dominate a particular individual, the lower its rank.
MOGA, therefore, uses fitness sharing in the objective function
space and recombination is also restricted. Reproduction prob-
abilities are determined by means of exponential ranking. Af-
ferwards the fitness values are averaged and shared among
individuals having identical ranks (Zitler, Deb, & Thiele, 2000).
Finally, stochastic universal sampling, which provides zero bias
and minimum spread (i.e. the range of possible values for the number of offspring of an individual), is used to fill the sampling pool. The main strength of MOGA is that it is efficient and relatively easy to implement. It has also been successfully implemented in solving optimal control problems with good overall performance (Coello, 1996).

In this paper we follow the formulation by (Chikumbo & Nicholas, 2011), a competitive co-evolutionary Multi-Objective Genetic Algorithm (cc-MOGA), but with an additional objective, i.e. maximisation of carbon sequestration payment. We focus on the three objectives and the forest holding values (held static because of the assumption of fixed prices of liquid fuel and fossil fuel-based fertilisers) on how they are formulated and discuss the results of the three-objective cc-MOGA. The same species, E. fastigata, and growth functions by (Chikumbo & Nicholas, 2011) are used in our analysis.

**Data**

First, E. fastigata Deane & Maiden is considered to be the most suitable eucalypt for a wide range of sites in New Zealand because it has a wide site tolerance, performing well near sea level to altitudes up to 500 m (Chikumbo & Nicholas, 2011). It can be utilised for solid timber as well as providing short-fibre pulp for the production of fine printing paper (Miller, Hay, & Ecroyd, 2000; Haslett, 1988).

The data used for the growth dynamics of E. fastigata came from a 1979 Nelder trial (Nelder, 1962) established in Kaingaroa Forest (latitude 38°27.6'S, longitude 176°39.9'E and an altitude of 280 m), near Murupara, New Zealand. A Nelder trial consists of trees planted in a series of concentric circles where the growing space available to each tree is determined by the distance to the nearest eight neighbouring trees.

Further detail on the trial is found in (Chikumbo & Nicholas, 2011) and the summary of the data is shown in **Table 1**. The costs and revenue for silvicultural treatments used in determining forest holding values for the valuation of timber products and carbon sequestered were obtained from (Turner, West, Dungey, Wakelin, Maclaren, Adams & Silcock, 2008).

**The Model**

There were two main reasons why cc-MOGA was used to solve the three-objective optimisation problem:

1) A competitive co-evolutionary genetic algorithms maintains diversity and controls the selective pressure by balancing exploration and exploitation in the search space, in a way that avoids premature convergence (Chikumbo, 2009b; Menczer, Degeratu, & Street, 2000); and

2) Splitting the population into diverse sub-populations that communicate through migration may result in parallel speed-ups (Menczer, Degeratu, & Street, 2000).

The three objectives or cost functionals were as follows:

\[ J_i(u) = \max_{u(t)} \sum_{t=1}^{T} \frac{\text{dbh}(t)}{\text{sph}(t)} \cdot \text{sb}(t) \cdot \ldots \]

\[ m_{ht}(t) \cdot f_{v_{2010}}(t) \cdot \frac{\text{vol}(t)}{\text{sph}(t)}, \]

for value production;

\[ J_n(u) = \max_{u(t)} \sum_{t=1}^{T} \frac{\text{dbh}(t)}{\text{sph}(t)} \cdot \text{sb}(t) \cdot \ldots, \]

\[ m_{ht}(t) \cdot f_{v_{2010}}(t), \]

for volume production; and

\[ J_e(u) = \max_{u(t)} \left( \frac{\text{sph}(t) - u(t)}{\text{sph}(t)} \right) \cdot C_f \cdot \ldots \]

\[ \text{vol}(t) \cdot f_{v_{2010}}(t), \]

for carbon sequestration payments.

subject to the constraints,

\[ t \geq T - (n - 1) \]

with upper and lower bounds on the control:

\[ 0 \leq u(t) \leq 300, \forall t \in [t_1, t_2], \]

\[ 0 \leq u(t) \leq 200, \forall t \in [t_3, t_4], \]

and

\[ 0 \leq u(t) \leq 200, \forall t \in [t_5, t_6], \]

where,

\[ t_1 - t_2 = 5 - 10 \text{ year time window for an initial thinning}; \]

\[ t_3 - t_4 = 12 - 15 \text{ year time window for a second thinning}; \]

\[ t_5 - t_6 = 18 - 21 \text{ year time window for a third thinning}; \]

\[ T = \text{rotation length}; \]

\[ f_{v_{2010}}(t) = \text{the forest holding value at time } t \text{ for sawlogs}; \]

\[ f_{p_{2010}}(t) = \text{the forest holding value at time } t \text{ for pulpwood}; \]

\[ f_{c_{2010}}(t) = \text{the forest holding value at time } t \text{ for carbon sequestered}; \]

and

\[ C_f = \text{a factor of 0.23 that converts total carbon sequestered (tCO}_2 \cdot \text{m}^{-3}) \text{ from total stand volume (Meade, Fiuza, & Lu, 2008).} \]

The two-parameter selection constraints were formulated as follows:

\[ 900 \leq \text{sph}(t_0) \leq 2000, \]

for the initial planting stock, and

\[ 25 \leq T \leq 35, \]

for the rotation length, where \( t_0 \) is the initial time.

As for the multi-objective problem with the three cost functionals, \( J_i(u), J_n(u), \text{ and } J_e(u) \), there exists no solution pair \( (x, u) \) that renders a global minimum value to each of the functionals simultaneously. Rather, there exists a finite set of solutions that represent trade-offs. A key concept in determining this set of solutions is that of Pareto optimality:

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**Table 1** Summary of the 286 data points measured from the Nelder trial (Chikumbo & Nicholas, 2011).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>12.2</td>
<td>28.7</td>
<td></td>
</tr>
<tr>
<td>SPH (stems ha(^{-1}))</td>
<td>89</td>
<td>1181</td>
<td>4356</td>
</tr>
<tr>
<td>DBH(^{1}) (cm)</td>
<td>3.6</td>
<td>20.4</td>
<td>61.7</td>
</tr>
<tr>
<td>MD(^2) (cm)</td>
<td>4.2</td>
<td>28.1</td>
<td>62.6</td>
</tr>
<tr>
<td>Mean stand height (m)</td>
<td>3.6</td>
<td>18.7</td>
<td>42.8</td>
</tr>
<tr>
<td>Stand basal area (m(^2) ha(^{-1}))</td>
<td>0.1</td>
<td>28.1</td>
<td>113.6</td>
</tr>
<tr>
<td>Stand volume (m(^3) ha(^{-1}))</td>
<td>0.2</td>
<td>182.1</td>
<td>1075.2</td>
</tr>
<tr>
<td>Volume mean annual increment (m(^3) ha(^{-1}) year(^{-1}))</td>
<td>0.1</td>
<td>12.1</td>
<td>40.2</td>
</tr>
</tbody>
</table>

---

\(^{1}\)Diameter at breast height measured at 1.4 m; \(^{2}\)Mean of 100 largest diameter trees per hectare; \(^{3}\)Mean height of 100 largest diameter trees per hectare.
Assume two solutions \((x, u), (x', y') \in \Omega\), where \(\Omega\) is denoted as the solution space. Then \((x, u)\) is said to dominate \((x', y')\) (also written \((x, u) \succ (x', y')\)) iff
\[
\forall i \in \{1, 2, \ldots, n\} : J_i(x, u) \geq J_i(x', y') \land
\forall m \in \{1, 2, \ldots, n\} : J_m(x, u) > J_m(x', y').
\]
(20)

However, \((x, u)\) is said to cover \((x', y')\) \((x, y) \succ (x', y')\) iff
\[
\forall i \in \{1, 2, \ldots, n\} : J_i(x, u) \leq J_i(x', y') \land
\forall m \in \{1, 2, \ldots, n\} : J_m(x, u) < J_m(x', y').
\]
(21)

In this case \((x, u)\) is non-dominated by \((x', y')\).

Therefore, the set of non-dominated solutions, \(\Omega^v\), within the entire search space, \(\Omega\), is called the Pareto optimal set.

Forest Holding Value and Carbon Sequestration Accounting

The valuation of a stand for a defined as \(fh_{NZD100}(t)\), \(fh_{NZD200}(t)\) and \(fh_{NZD100}(t)\) in the cost functionals (11)-(13), were based on the forest holding value. The holding value is defined as follows:
\[
FVH(t) = PVH(t) - PV_{cost} - LOC,
\]
where,
\[
PVH(t) = \text{holding value at time, } t;
\]
\[
PV_{cost} = \text{Discounted present value of optimal harvest at time, } t;
\]
\[
LOC = \text{Land opportunity cost.}
\]

At any point in time carbon sequestered is a function of the change in biomass and the amount of carbon per m³ of biomass. It is not just the age of the trees per se or standing timber volume that is important, but rather the rate of tree growth (van Kooten, Binkley & Delcourt, 1995). As trees grow they sequester carbon, but once carbon has been sequestered no further benefits are forthcoming. In other words the income generated by sequestering carbon at time, \(t\), is the value of the extra amount of carbon sequestered between \(t\) and \(t-1\) multiplied by the price of carbon, as shown in the following equation:
\[
S_u(t) = \frac{PC^\ast(S(t)-S(t-1))}{(1+r)^t},
\]
where,
\[
S_u(t) = \text{Carbon sequestration payment at time, } t;
\]
\[
PC = \text{Price for sequestering carbon (NZ$ m}^3)\);\]
\[
S(t) = \text{Total carbon sequestered at time, } t; \text{ and}\]
\[
S(t-1) = \text{Total carbon sequestered at time, } t-1.
\]

The present value of the above equation at the time of planting the crop then becomes:

**Table 2.**

<table>
<thead>
<tr>
<th>REGIME</th>
<th>Age at T1 (yrs)</th>
<th>Age at T2 (yrs)</th>
<th>Age at T3 (yrs)</th>
<th>Clear felling age (yrs)</th>
<th>Init. stocking (stems·ha⁻¹)</th>
<th>T1 (stems·ha⁻¹)</th>
<th>T2 (stems·ha⁻¹)</th>
<th>T3 (stems·ha⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>15</td>
<td>18</td>
<td>20</td>
<td>988</td>
<td>287</td>
<td>153</td>
<td>196</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>13</td>
<td>13</td>
<td>35</td>
<td>955</td>
<td>259</td>
<td>196</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>15</td>
<td>18</td>
<td>30</td>
<td>936</td>
<td>224</td>
<td>185</td>
<td>197</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>14</td>
<td>19</td>
<td>35</td>
<td>939</td>
<td>240</td>
<td>190</td>
<td>176</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>15</td>
<td>19</td>
<td>35</td>
<td>937</td>
<td>234</td>
<td>185</td>
<td>197</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>12</td>
<td>18</td>
<td>35</td>
<td>903</td>
<td>228</td>
<td>193</td>
<td>187</td>
</tr>
</tbody>
</table>
Final Crop Number

A $p$-value of 0.0014 suggested that there were differences between the groups as shown in the box plots in Figure 4. The non-carbon scenario showed a higher number of retained crops before final harvesting, than all the carbon scenarios, which showed an overlap of the one standard deviation of their final crop numbers. The mean ranking in Figure 5 showed more clearly the degree of overlap among the three carbon scenarios.

Rotation Length

There was also a difference in the rotation length medians amongst the different groups with a $p$-value of 0.003. The non-carbon scenario showed no variation in the rotation length, which remained at 35 years for all the individual regimes in the Pareto set, whereas the other three carbon scenarios had higher medians, which overlapped at one standard deviation of the medians. The summary is shown in Figure 6. The summary of the mean ranks of the rotation lengths in Figure 7 confirmed the differences between the non-carbon scenario with a lower rotation length, and the three carbon scenarios with higher ranges of rotation lengths. This observation agrees with conventional wisdom that rotation lengths will be longer in order to sequester more carbon. What is interesting with our results here is that the rotation lengths are insensitive to the variations in the carbon price.

We assert that this insensitivity to the price of carbon is because our optimisation model is maxing out the sequestration of carbon as much as the equations for the growth dynamics of *E. fastigata* will allow. This is good news for the forester, in that for any forest signed up to an emissions trading scheme, there is only one set of optimal regimes to consider that will simultaneously satisfy a sawlog and pulpwood market, regardless of fluctuations in the carbon price.
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Figure 4.
Box plots of the four scenarios.

Figure 5.
Wilcoxon rank sum test for the final crop numbers of the different groups/scenarios.

Frequency, Timing and Intensity of Thinning

The frequency, timing and intensity of thinnings did not show any statistical differences for all the four scenarios. Carbon prices do not seem to influence the thinning strategy where it is already optimized for value and volume productions. This might be explained by the fact that though stand volume is reduced through thinning, the sudden availability of more nutrients, light and moisture to the residual trees boosts their growth. It is this growth that will guarantee more sequestration and possibly more payment. Given that the final crop numbers of all the carbon scenarios were lower than those of the non-carbon scenario, and that the initial planting stockings of all the four scenarios were statistically the same, it therefore stands to confirm this assertion. Also with less number of trees as a final crop, it is possible to keep the trees a little longer than one would normally do in a non-carbon market environment because this may guarantee more growth until full-site occupancy is reached. This might encourage fertilisation following a late-age thinning, in order to boost growth and subsequently sequester more carbon. Implications of late-age fertilisation following a thinning may also mean a premium sawlog/veneer product at the end of the rotation.
These results may have been different if the carbon scenarios were matched with forecasted fluctuations in the prices of liquid fuel and fossil fuel-based fertilisers, as this would have meant changes in the forest holding values. It is difficult to imagine how that would have impacted our results. It may well be that we need to take this research further by developing:

1) A dynamic stumpage model; and
2) A fossil fuel-based fertiliser price model, under a carbon trading scheme, and revisit our analysis. Both (1) and (2) will expose the forest holding value to a more realistic output, and maybe provide us with further insight into the impact of carbon prices on the silvicultural regimes of *E. fastigata*.

Further Discussion

It is important to note that although our results are based on a stand level, they are Pareto sets which can still be used at a forest-estate level, where one has hundreds or thousands of stands at different age classes. With each stand with a Pareto set of possible silvicultural regimes, it is possible to optimise at an estate level, assigning the appropriate regime to each stand, which may mean simultaneously meeting harvesting commitments and optimally sequestering carbon under prevailing market constraints. In other words the Pareto optimality at a stand level gives flexibility at an estate level planning. We have not touched on environmental constraints because of the focus of this paper. However, environmental issues are best dealt with at a forest estate level both temporally and spatially, given their long-term gestation period.
Conclusion

The consequences on the silvicultural regimes for *Eucalyptus fastigata*, when the crop is simultaneously managed for, carbon sequestration, sawlog and pulpwood were, decreased final crop numbers, and increased rotation lengths. It will be well worth it to investigate fertilisation following late-age thinning in order to boost growth. This would mean sequestering more carbon and guarantee of a premium sawlog/veneer product at the end of the rotation. The regimes under a carbon market were also found to be insensitive to fluctuations in the price of carbon. Our findings were based on a fixed stumpage used in the calculation of forest holding values.

Acknowledgements

This project was funded by the New Zealand Foundation for Research Science and Technology under Contract No. C04X-0805, “Diverse Forests” and the contribution from Future Forest Research Ltd is gratefully acknowledged.

REFERENCES


Appendix A: NZ $25 Carbon Price Scenario Silvicultural Regimes

Table A1.

Pareto optimal set for the NZ $25 carbon price scenario.

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Appendix B: NZ $50 Carbon Price Scenario Silvicultural Regimes

Table B1. Pareto optimal set for the NZ $50 carbon price scenario.

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Pareto optimal set for the NZ $100 carbon price scenario.

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